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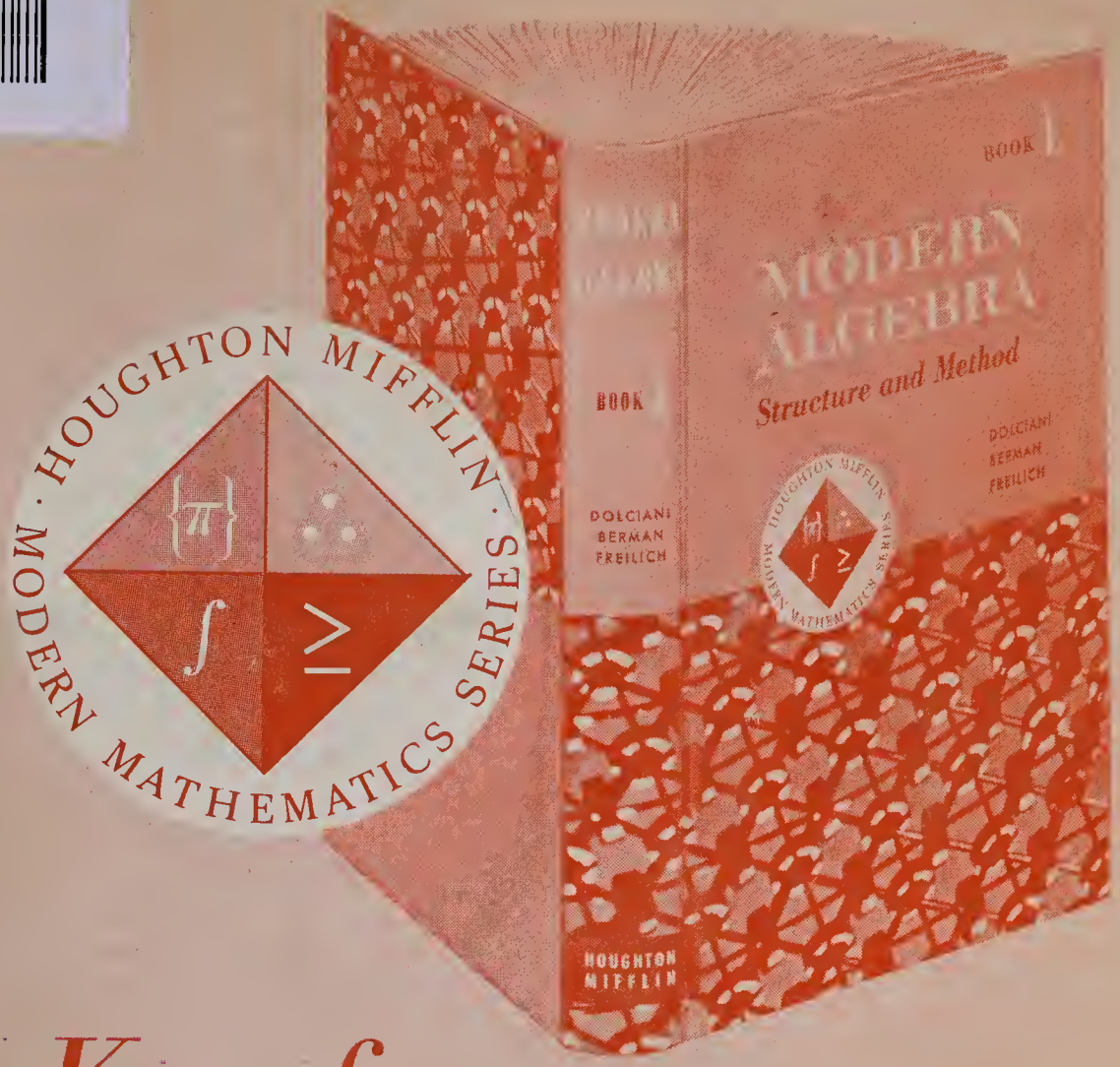

Solution Key for

MODERN ALGEBRA

Structure and Method

BOOK ONE

DOLCIANI • BERMAN • FREILICH

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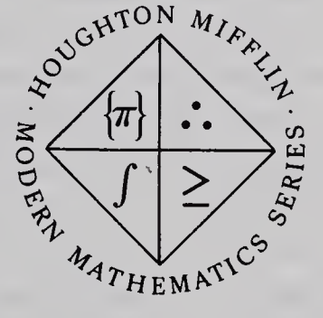
Solution Key for

MODERN ALGEBRA

Structure and Method

BOOK ONE



MARY P. DOLCIANI • SIMON L. BERMAN

JULIUS FREILICH

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FOREWORD

The purpose of this Key is to make the teacher's work easier. It contains references to all written exercises and problems in *MODERN ALGEBRA, Structure and Method: Book One* including these categories: Written Exercises, Problems, Chapter Test, Chapter Review, Cumulative Review, Extra for Experts, Just for Fun, and the chapter entitled Comprehensive Review and Tests. For answers to Oral Exercises the teacher is referred to the annotated Teacher's Edition.

To achieve the purpose of the Key, these criteria were followed in its preparation: (1) To provide a solution for every written exercise and problem in which more than one step is required for solution; (2) To arrange the solutions compactly enough so that the teacher will not be inconvenienced by the size of the Key (which contains the solution to more than 7600 exercises). For this reason once a desirable form of solution was established, certain shortcuts were used which teachers will not want their students to follow: (1) Including more than one equals sign in a mathematical sentence; (2) Condensing sentences such as "Let w = the number of feet in the width."

ABBREVIATIONS

A = acre(s)
add. = addition, additive
amp = ampere(s)
amt. = amount
ans. = answer
arith. = arithmetic
assoc. = associative
avg. = average
bu = bushel(s)
C = Centigrade
cal = calorie(s)
cm = centimeter(s)
coeff. = coefficient
comm. = commutative
comp. = complement
cu = cubic
defn. = definition
denom. = denominator
distrib. = distributive
div. = division
doz = dozen
eqn. = equation
equiv. = equivalent
ex. = exercise(s)
exp. = exponent
F = Fahrenheit, false
ft = foot, feet
g = gram(s)
hr = hour(s)
in = inch(es)
inf. = infinite
K = Kelvin
kg = kilogram(s)
kw-hr = kilowatt-hour(s)
lb = pound(s)
L.C.D. = lowest common denominator
M = meter(s)
max. = maximum
meas. = measure
mi = mile(s)
min = minute(s)
min. = minimum
mph = miles per hour
mult. = multiplication
naut. = nautical
no., nos. = number, numbers
num. = numeral, numerical, numerator
oz = ounce(s)
prop. = property
pt = pint(s)
Q = quotient
qt = quart(s)
reflex. = reflexive
rev. = revolutions
rpm = revolutions per minute
sec = second(s)
soln. = solution
sq = square
subst. = substitution
subt. = subtraction
supp. = supplement
T = true
temp. = temperature
trans. = transitive
transf. = transformation
wk. = week
wt. = weight

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CHAPTER 1. Symbols and Sets

Pages 3–5 · WRITTEN EXERCISES

A **1.** R: $\frac{3}{4}$; S: $1\frac{1}{2}$; T: $1\frac{3}{4}$ **2.** R: $1\frac{2}{3}$; S: 2; T: $2\frac{1}{3}$ **3.** R: $\frac{2}{5}$; S: $\frac{4}{5}$; T: $1\frac{2}{5}$
4. R: $\frac{1}{2}$; S: $\frac{5}{6}$; T: $1\frac{1}{3}$ **5.** R: $\frac{7}{8}$; S: $1\frac{1}{4}$; T: $1\frac{5}{8}$ **6.** R: $\frac{1}{5}$; S: $\frac{7}{10}$; T: $1\frac{3}{5}$
7. 5 **8.** 0 **9.** 2 **10.** 5 **11.** 4 **12.** $\frac{7}{2}$ **13.** $\frac{7}{2}$ **14.** $\frac{4}{3}$ **15.** 8 **16.** 0
17. $\frac{5}{2}$ **18.** $\frac{8}{3}$

B **19.** $\frac{35}{6}$ **20.** $\frac{14}{5}$ **21.** 5 **22.** 2 **23.** 3, 4 **24.** 7, 8 **25.** 3, 4, 5, 6, 7 **26.** 0

C **27.** 3, 6 **28.** None **29.** $\frac{7}{2}$ **30.** $\frac{4}{3}$

Pages 6–7 · WRITTEN EXERCISES

A **1.** $20 = 27$; F **2.** $1.2 = .12$; F **3.** $\frac{5}{6} = \frac{5}{6}$; T **4.** $10 = 10$; T **5.** $\frac{1}{24} = 24$; F
6. $9 = 4$; F **7.** $85 = 85$; T **8.** $0 = 0$; T **9.** $3 = 9$; F **10.** $6\frac{1}{2} = 6\frac{1}{2}$; T

B **11.** $17 + ? = 19$; 2 **12.** $51 + ? = 51$; 0 **13.** $3\frac{2}{3} - ? = 1\frac{1}{12}$; $2\frac{7}{12}$ **14.** $6\frac{1}{2} - ? = 4\frac{1}{6}$; $2\frac{1}{3}$
15. $.4 = 2 \times ?$; .2 **16.** $1.0 = .7 \div ?$; .7 **17.** $.4 = 1 - ?$; .6 **18.** $4 = 1 \div ?$; $\frac{1}{4}$

Pages 9–10 · WRITTEN EXERCISES

A **1.** $4 + 5 = 10 - 1$ **2.** $13 \times 0 < 16 + 0$ **3.** $7 = 7$ **4.** $5 < 6$
5. $0 = 0$ **6.** $\frac{15}{3} = 2 + 3$ **7.** $\frac{8 - 3}{5 - 2} > 1 + .6$ **8.** $6 \times 0 = 0$
9. $5 \times 1 > 5 \times 0$ **10.** $2\frac{1}{8} = 1 + 1\frac{2}{16}$ **11.** $\frac{1}{3} > \frac{1}{6}$ **12.** $\frac{1}{5} + \frac{3}{4} = \frac{6}{8} + .2$
13. $65 \times 1 < 66$ **14.** $\frac{1}{3} + \frac{4}{6} > \frac{5}{9}$ **15.** $3 > 3 \times 0$ **16.** $\frac{1}{2} > \frac{1}{4} + \frac{0}{4}$
17. $2 \times 10 > 17 - 7$ **18.** $.75 + .25 > \frac{3}{4} - \frac{1}{3}$ **19.** $\frac{6}{7} - \frac{2}{3} < \frac{9}{10} + \frac{2}{3}$ **20.** $4 \div 2 = 8 \div 4$
21. $\frac{1}{3} \div 4 < \frac{4}{3}$ **22.** $\frac{1}{3} \times 4 > \frac{3}{4}$ **23.** $\frac{21}{9} - \frac{4}{3} < \frac{3}{3} \times 3$ **24.** $.6 - .2 = .2 \times 2$
25. $4 + 1 = 11 - 6$ **26.** Any num. other than 7 **27.** $5 + 3 = 3 + 5$
28. $8 \times 0 = 0$ **29.** $0 \div 10 = 0$ **30.** $5 + 2 = 7$ **31.** $14 = 7 + 7$
32. $0 + 7 = 7$ **33.** Any num. < 5 **34.** Any num. **35.** $3 \times 5 = 9 + 6$
36. Any num. > 4 **37.** Any num. other than 12 **38.** Any num. ≥ 8 and < 16
39. Any num. > 0 **40.** $8 \div 8 = 1$

B **41.** $\frac{15 + 1}{8} = 2$ **42.** $\frac{24 + 12}{4} = 6 + 3$ **43.** Any num. other than 0
44. $\frac{1}{10} \times 10 = 1$ **45.** Any num. > 1 and < 3 **46.** Any num. $> \frac{1}{3}$ and $< \frac{2}{3}$, e.g., $\frac{1}{2}$
47. Any num. >0 and $<.1$, e.g., .001 **48.** Any num. $> .4$ and $< .5$, e.g., .43
49. Any num. $> \frac{62}{76}$ and $< \frac{82}{76}$, e.g., $\frac{71}{76}$ **50.** Any num. $> \frac{200}{109}$ and $< \frac{200}{107}$, e.g., $\frac{200}{108}$

C **51.** Any num. > 1.53327 and < 1.54327, e.g., 1.53377 **52.** Any num. $> \frac{14867}{23864}$ and $< \frac{14857}{23764}$, e.g., $\frac{1.4868}{2.3864}$
53. Any num. $> \sqrt{6}$ and $< \sqrt{7}$, e.g., $\sqrt{6.3}$ **54.** Any num.

Pages 14–15 · WRITTEN EXERCISES

A **1.** {8, 9}, finite **2.** {0, 1, 2, 3, 4, 5}, finite **3.** {i, e}, finite **4.** $\emptyset$, finite
5. $\emptyset$, finite **6.** {1, 3, 5, 7, 9}, finite **7.** {10, 12, 14, 16, 18}, finite
8. {Feb.}, finite **9.** {1, 2, 3, . . . , 1775}, finite **10.** {202, 204, 206, . . . , 998}, finite

B **11.** {3, 6, 9, 12, 15}, finite **12.** {25, 50, 75, . . .}, inf. **13.** {1964, 1968, 1972, . . .}, inf. **14.** {40}, finite
15. $\{\frac{1}{2}\}$, finite **16.** {1, 3, 5, . . .}, inf. **17.** {2, 4, 6, . . .}, inf. **18.** $\emptyset$, finite

C **19.** {1, 3, 5}, finite **20.** {0, 1, 4, 9, 16}, finite **21.** $\emptyset$, finite **22.** $\emptyset$, finite
23. f **24.** d **25.** h **26.** j **27.** a **28.** g **29.** b **30.** i **31.** c **32.** k **33.** e **34.** l

Page 17 · **WRITTEN EXERCISES**

A

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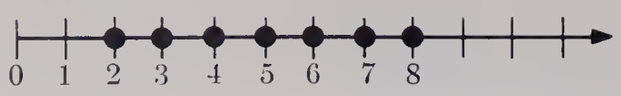

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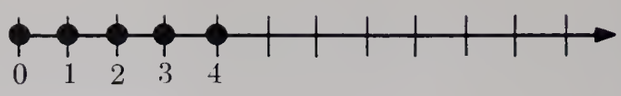

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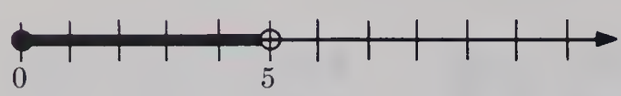

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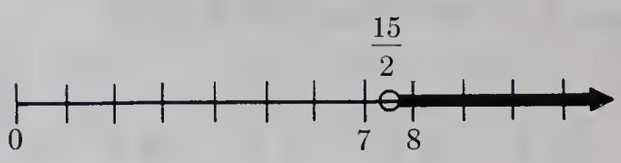

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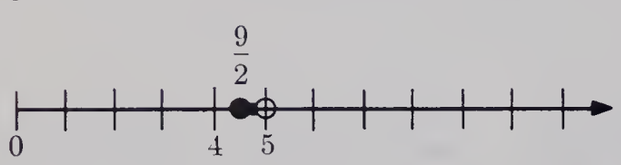

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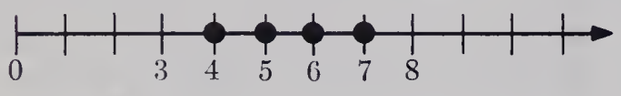

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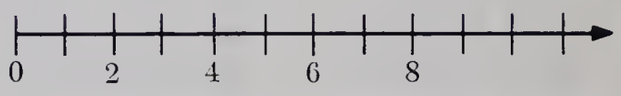

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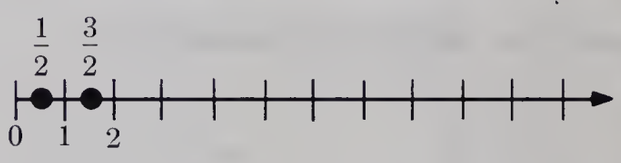

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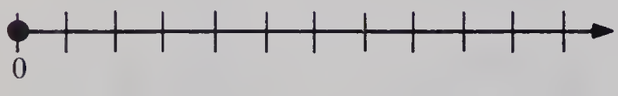

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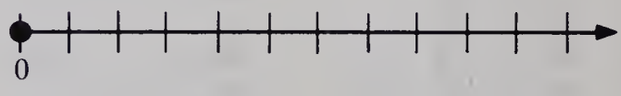

17.

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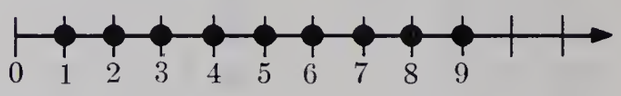


Page 19 · **WRITTEN EXERCISES**

A **1.** {3}, {15}, {10} **2.** {3, 15}, {3, 10}, {10, 15} **3.** {3, 15, 10} **4.** ∅
5. {10}, {15}, {10, 15} **6.** ∅ **7.** {10} **8.** {3}, {15}, {3, 15}
9. {the nos. of arith.} **10.** {the nos. of arith.} **11.** {all fractions} **12.** {all men}
13. {all cities of Alaska} **14.** {the islands of Hawaii}

Page 22 · **WRITTEN EXERCISES**

A **1.** $0 \times [5 + 6] = 0 \times [11] = 0$ **2.** $5 \times [5 - 0] = 5 \times [5] = 25$
3. $17 \times 30 = 510$ **4.** $255 \times 2 = 510$ **5.** $\frac{24}{12} = 2$ **6.** $\frac{16}{2} = 8$
7. $\frac{36}{2} = 18$ **8.** $\frac{36}{18} = 2$ **9.** $(20)18 = 360$ **10.** $(306) + (54) = 360$
11. $(20) \div ([6] \div 4) = (20) \div (\frac{3}{2}) = \frac{40}{3}$ **12.** $[30 \div (10)] \div 3 = [3] \div 3 = 1$

B **13.** $[3 + (36) \div 3] = [3 + 12] = 15$ **14.** $[20 \div 6] \div 13 = [\frac{10}{3}] \div 13 = \frac{10}{39}$
15. $[\frac{36}{18}] \div 12 = \frac{1}{6}$ **16.** $[(5) \div 5] + 1 = [1] + 1 = 2$
17. $[100 + 2] - 51 = [102] - 51 = 51$ **18.** $[\frac{20}{54} - \frac{1}{54}] - \frac{1}{54} = [\frac{19}{54}] - \frac{1}{54} = \frac{18}{54} = \frac{1}{3}$
19. $(27) + [27 + 2] = (27) + [29] = 56$ **20.** $\{12\} - \{\frac{4}{3} \times \frac{1}{8}\} = 12 - \{\frac{1}{6}\} = 11\frac{5}{6}$

C **21.** $[\{(8 - 8) \times 3\} + 5] \times 200 = [\{(0) \times 3\} + 5] \times 200 = [(0) + 5] \times 200 = (5) \times 200 = 1000$
22. $\{(1452 \times 5) - 5\} \times 100 = \{(7260) - 5\} \times 100 = \{7255\} \times 100 = 725{,}500$
23. $[(8 + 60 - 60) + 56] \div 13 = [8 + 56] \div 13 = [64] \div 13 = \frac{64}{13}$
24. $[\{(\frac{1261}{39}) \times 27\} + 27] \div 27 = [\{873\} + 27] \div 27 = [900] \div 27 = \frac{900}{27} = \frac{100}{3}$

Page 24 · WRITTEN EXERCISES

A **1.** $(12) \div 3 + 1 = 4 + 1 = 5$ **2.** $(12) \div (4) = 3$

3. $5(16) \div 4 + 3 = 80 \div 4 + 3 = 20 + 3 = 23$ **4.** $21 + 5(10) - 20 = 21 + 50 - 20 = 51$

5. $7 + 3(4) \div 6 = 7 + 12 \div 6 = 7 + 2 = 9$ **6.** $7 + (12) \div 6 = 7 + 2 = 9$

7. $4 + 2 = 6$ **8.** $30 - 3(5) = 30 - 15 = 15$ **9.** $\dfrac{4+3}{7} = \dfrac{7}{7} = 1$

10. $\dfrac{15+20}{7} = \dfrac{35}{7} = 5$ **11.** $\dfrac{6+21}{3} = \dfrac{27}{3} = 9$ **12.** $\dfrac{65-15}{10} = \dfrac{50}{10} = 5$

13. $27 + 10 - (14) = 23$ **14.** $37 - 11 - 3 = 26 - 3 = 23$ **15.** $5 + 4 - (2) = 7$

16. $9 - 3 + 1 = 6 + 1 = 7$

B **17.** $8 \div 4 \div 2 = 2 \div 2 = 1$ **18.** $72 \div 3 \times 2 \div 48 = 24 \times 2 \div 48 = 48 \div 48 = 1$

19. $6(9) - 3 = 54 - 3 = 51$ **20.** $5(3) \div 3 + 2 = 15 \div 3 + 2 = 5 + 2 = 7$

21. $\dfrac{9-5(1)}{8} = \dfrac{9-5}{8} = \dfrac{1}{2}$ **22.** $\dfrac{4(4)-1}{5} = \dfrac{16-1}{5} = \dfrac{15}{5} = 3$

23. $\dfrac{5+6}{2(11)} = \dfrac{11}{22} = \dfrac{1}{2}$ **24.** $6(14) + 8 - 2 = 84 + 8 - 2 = 90$

25. $(10)(15) = 150$ **26.** $(6 - 4) \div 2 = (2) \div 2 = 1$

27. $(7.3)(9)(9) = 591.3$ **28.** $3.3 + 3(6)(4) - 2 = 3.3 + 72 - 2 = 73.3$

29. $(15 - 3 + 4) \div 8 \times 5 = (12 + 4) \div 8 \times 5 = (16) \div 8 \times 5 = 2 \times 5 = 10$

C **30.** $51 - 16 - 5 = 35 - 5 = 30$ **31.** $\frac{1}{2}(4 \times 4) \div (32 \div 2) = \frac{1}{2}(16) \div (16) = 8 \div 16 = \frac{1}{2}$

32. $\dfrac{20 \times 20 - 3}{13 + 15} = \dfrac{400 - 3}{28} = \dfrac{397}{28}$ **33.** $\dfrac{2 \times 3 - 2 + 16}{20 \div 5} = \dfrac{6 - 2 + 16}{4} = \dfrac{4 + 16}{4} = 5$

34. $\dfrac{1 + 11 + 528}{9 - 1 + 2} = \dfrac{540}{8 + 2} = \dfrac{540}{10} = 54$

Pages 27–28 · CHAPTER TEST

1. $\frac{3}{4}$ **2.** $\frac{5}{4}$ **3.** $\frac{3}{2}$ **4.** 4 **5.** $\frac{7}{2}$ **6.** $\frac{8}{3}$ **7.** $20 = 20$; T **8.** $9 = 12$; F

9. $17 = 17$; T **10.** $12\frac{3}{5} = 13\frac{3}{5}$; F **11.** Any num. other than 2 **12.** Any num. > 20

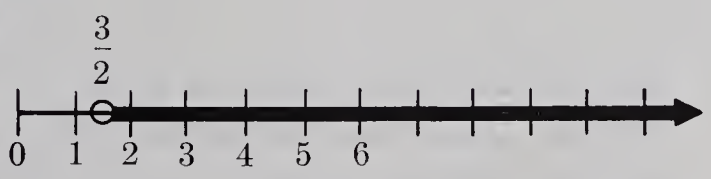
13. Any num. > 2 and < 4 **14.** Any num. $> \dfrac{2700}{245}$ and $< \dfrac{2700}{235}$, e. g., $\dfrac{2700}{237}$ **15.** {6, 12, 18, 24}

16. {100, 101, 102, . . . , 999} **17.** inf. **18.** finite **19.** $\emptyset$

20.

21.

22.



23. {4, 8}

24. $[60 \div 6] \div 10 = [10] \div 10 = 1$ **25.** $10 + 3 = 13$ **26.** $50 - 49 = 1$

Pages 28–30 · CHAPTER REVIEW

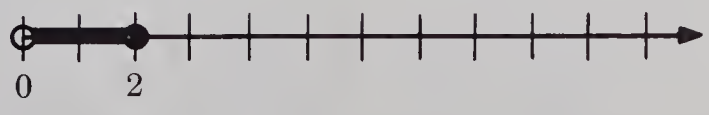
1. number **2.** zero **3.** magnitude **4.** distance **5.** $\frac{3}{4}$ **6.** $\frac{9}{4}$ **7.** E

8. numerical, numeral **9.** equal **10.** $=$ **11.** $\neq$ **12.** greater

13. $5 < 7$ *or* $7 > 5$ **14.** 1, 9 **15.** 5 **16.** set **17.** member *or* element **18.** $\in$

19. not **20.** roster **21.** rule **22.** 12, 15, 18 **23.** infinite **24.** empty

25. finite **26.** empty *or* null **27.** 3 **28.**



29. subset **30.** {20}

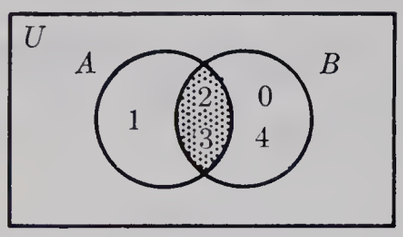
31. brackets, braces, bar **32.** one **33.** $2 + 12 = 14$ **34.** $\dfrac{(2) \div 2}{8 \div (2)} = \dfrac{1}{4}$

35. inclusion, divisions, multiplications, additions, subtractions **36.** $12 - 6 = 6$

37. $3 + 24 = 27$ **38.** $6 - 2 + 15 = 4 + 15 = 19$ **39.** $45 - .55 = 44.45$ **40.** $\dfrac{2 + 12}{2(7)} = \dfrac{14}{14} = 1$

Page 31 · **EXTRA FOR EXPERTS**

1.

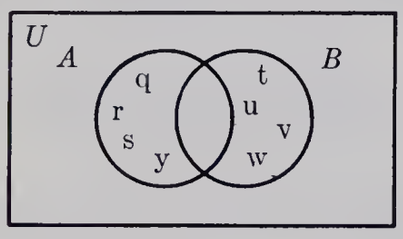


$A \cap B = \{2, 3\}$

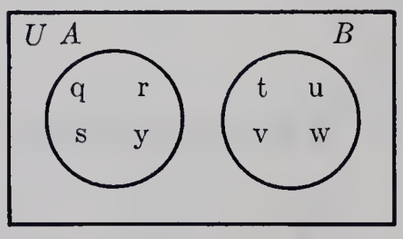
a. $A \cap B$ is a set $C = \{2, 3\}$ such that C is a proper subset of A and of B; because every member of C is also a member of A; likewise, every member of C is also a member of B.

b. When A is a (proper or improper) subset of B

2.



or



$A \cap B = \emptyset$

a. When A and B have no members in common, the intersection of A and B is the empty set.

b. $A \cap B$ is a proper subset of A as well as of B because $\emptyset$ is a proper subset of every set except itself.

c. The membership of each set excludes members of the other set.

Page 33 · **JUST FOR FUN**

Coat: \$59.50; hat: \$8.00; refrig.: \$325.55 **1.** Rain is wet. **2.** Peace is paramount. **3.** Codes are fun.

CHAPTER 2. Variables and Open Sentences

Pages 37–38 · **WRITTEN EXERCISES**

A

1. $2; 6 \cdot 1 + 2 \cdot 3 = 6 + 6 = 12$

2. $2; 3 \cdot 12 - 5 \cdot 5 = 36 - 25 = 11$

3. $1; 2(3 \cdot 1 + 3) = 2(6) = 12$

4. $1; (3 \cdot 12)(3 \cdot 12) = (36)(36) = 1296$

5. $1; (3 \cdot 3)(12 \cdot 12) = (9)(144) = 1296$

6. $1; 2 \cdot 0(12 - 2 \cdot 1) = 2 \cdot 0 \cdot 10 = 0$

7. $2; 2 \cdot 3 \cdot 12 - 4 \cdot 3 \cdot 1 = 72 - 12 = 60$

8. $5; 1 + 1 + 0 + 0 + 0 = 2$

9. $2; 5 \cdot 3 - \frac{1}{2} \cdot 12 = 15 - 6 = 9$

B

10. $2; (5 \cdot 5 - 5) - 5 = (25 - 5) - 5 = 20 - 5 = 15$

11. $1; (2 \cdot \frac{1}{2} - 1)(2 \cdot \frac{1}{2} + 1) = (1 - 1)(1 + 1) = (0)(2) = 0$

12. $1; (3 \cdot 1 + 12)(3 \cdot 1 + 12) = (3 + 12)(3 + 12) = (15)(15) = 225$

13. $1; \dfrac{4 \cdot \frac{1}{2} + 3 \cdot 1}{7} = \dfrac{2 + 3}{7} = \dfrac{5}{7}$

14. $1; \dfrac{6 \cdot \frac{1}{2} + 5}{7 \cdot 5} = \dfrac{3 + 5}{35} = \dfrac{8}{35}$

15. $1; \dfrac{3 \cdot 5 + 12}{5 \cdot 3 - 12} = \dfrac{15 + 12}{15 - 12} = 9$

16. $1; \dfrac{16 \cdot 3 \cdot \frac{1}{2}}{(4 \cdot 3)(2 \cdot 1)} = \dfrac{24}{(12)(2)} = 1$

17. $1; \dfrac{12(2 \cdot 3 + 5)}{12(2 \cdot 3 - 5)} = \dfrac{12(6 + 5)}{12(6 - 5)} = \dfrac{12(11)}{12(1)} = 11$

18. $1; \dfrac{2 \cdot \frac{1}{2}(3 + 1)}{2 \cdot \frac{1}{2}(3 - 1)} = \dfrac{2 \cdot \frac{1}{2}(4)}{2 \cdot \frac{1}{2}(2)} = \dfrac{4}{2} = 2$

19. $1; \dfrac{5 \cdot 12 - 0}{5 \cdot 12 + 0} = \dfrac{60 - 0}{60 + 0} = \dfrac{60}{60} = 1$

Pages 38–39 · **PROBLEMS**

A

1. $\dfrac{4.375 - 3.625}{2} = \dfrac{.750}{2} = .375$ in

2. $\dfrac{5}{12 - 6} = \dfrac{5}{6}$

3. $\frac{9}{5} \cdot 20 + 32 = 36 + 32 = 68°\text{F}$

4. $2(12.250) + (3.142)(3.000) = 24.500 + 9.426 = 33.926$ ft

B

5. $\dfrac{4(6 + 7)}{2} = \dfrac{4(13)}{2} = 26$ sq in

6. $\dfrac{7(7 + 1)}{2} = \dfrac{7(8)}{2} = 28$

7. $\frac{5}{9}(77 - 32) = \frac{5}{9}(45) = 25°\text{C}$

8. $\frac{20}{2}(2 \cdot 6 - 1) = 10(12 - 1) = 10(11) = 110$ ft/sec

9. $\dfrac{2 \cdot 15}{15 - 12} = \dfrac{30}{3} = 10$

10. $\dfrac{7 - 3}{6 - 2} = \dfrac{4}{4} = 1$

11. $180\left(\dfrac{12 - 2}{12}\right) = 180\left(\dfrac{10}{12}\right) = 150°$

12. $(.000023)(10)(80 - 20) = (.000023)(10)(60) = .0138$ ft

C

13. $\frac{22}{7}(24 - 20)(24 + 20) = \frac{22}{7}(4)(44) = \frac{3872}{7} = 553\frac{1}{7}$ sq in

14. $15(15 - 12)(15 - 10)(15 - 8) = 15(3)(5)(7) = 1575$ (sq in)^2

15. $2(7 \cdot 6 + 6 \cdot 5 + 7 \cdot 5) = 2(42 + 30 + 35) = 2(107) = 214$ sq ft

16. $110\left(\dfrac{1}{.10} + \dfrac{1}{.25} + \dfrac{1}{.20}\right) = 110(10 + 4 + 5) = 110(19) = 2090$ amp

17. $\dfrac{12.250}{6.125}\left(\dfrac{1.875 - 1.125}{2}\right) = 2\left(\dfrac{.750}{2}\right) = .750$ in

18. $\dfrac{3(1.5)}{12 + 3(0.1)} = \dfrac{4.5}{12 + .3} = \dfrac{4.5}{12.3} = \dfrac{15}{41}$ amp

19. $\dfrac{7(7 + 1)(2 \cdot 7 + 1)}{6} = \dfrac{7(8)(14 + 1)}{6} = \dfrac{7(8)(15)}{6} = \dfrac{840}{6} = 140$

20. $2 \cdot 14 + 3.25\left(\dfrac{2.5 + 1.2}{2}\right) = 28 + 3.25(1.85) = 28 + 6.0125 = 34.0125$ ft

Pages 42–43 · **WRITTEN EXERCISES**

A

1. b^2 **2.** c^3 **3.** a^3 **4.** d^2 **5.** $7n^3$ **6.** $14m^3$ **7.** n^3 **8.** $8n^3$ **9.** E^4 **10.** F^3

11. R^5 **12.** s^6 **13.** $5y^3$ **14.** $8z^2$ **15.** $\frac{1}{2}g^2$ **16.** $\frac{1}{4}h^5$ **17.** $(2P)^2$ **18.** $(8i)^2$ **19.** $(xy)^3$

20. $(ab)^3$ **21.** $(a - 1)^3$ **22.** $(1 - a)^3$ **23.** $(r + 2)^3$ **24.** $(t + 7)^2$

B

25. $(\frac{1}{2})^2 = \frac{1}{4}$

26. $(\frac{1}{4})^2 = \frac{1}{16}$

27. $4(3)^2 = 4 \cdot 9 = 36$

28. $8(5)^2 = 8 \cdot 25 = 200$

29. $(9 \cdot \frac{1}{3})^2 = 3^2 = 9$

30. $(8 \cdot \frac{1}{4})^2 = 2^2 = 4$

31. $2(3)^2 + 4(3) + 5 = 18 + 12 + 5 = 35$

32. $5(1)^2 - 3(1) + 4 = 5 - 3 + 4 = 2 + 4 = 6$

33. $7(2)^3 + (2)^2 - 2 = 56 + 4 - 2 = 58$

34. $(5)^3 - 2(5)^2 + 5 + 4 = 125 - 50 + 5 + 4 = 75 + 5 + 4 = 84$

35. $(0)^5 + 3(0)^4 - (0)^3 + 0 = 0 + 0 - 0 + 0 = 0$

36. $(0)^{10} + (0)^5 + 0 + 9 = 0 + 0 + 0 + 9 = 9$

C

37. $(5)^2 + (2)^2 + (3)^2 = 25 + 4 + 9 = 38$

38. $5 - (2)^2 + (3)^2 = 5 - 4 + 9 = 1 + 9 = 10$

39. $(5)^2 + 2 + (3)^2 = 25 + 2 + 9 = 36$

40. $5 - 2 + (3)^2 = 5 - 2 + 9 = 3 + 9 = 12$

41. $(5)^2 + (2)^2 - 2(3)^2 = 25 + 4 - 18 = 11$

42. $(5)^2 + 2 - (3)^2 = 25 + 2 - 9 = 18$

43. $(5)^2 - (2)^2 + (3)^2 = 25 - 4 + 9 = 21 + 9 = 30$

44. $\dfrac{2(5)^2 + (5)(2)}{20(3)} = \dfrac{50 + 10}{60} = 1$

45. $\dfrac{(3)^3 - 27}{(5)(2)} = \dfrac{27 - 27}{10} = \dfrac{0}{10} = 0$

46. $\dfrac{(5)^2 + (3)^3}{(2)^2} = \dfrac{25 + 9}{4} = \dfrac{34}{4} = \dfrac{17}{2}$

47. $\dfrac{7(2)^2(3) - 2(5)^2}{(5)(2)} = \dfrac{84 - 50}{10} = \dfrac{34}{10} = \dfrac{17}{5}$

48. $(5 \cdot 3)^3 + (2)^6 = (15)^3 + 64 = 3375 + 64 = 3439$

49. $\left(\dfrac{6 \cdot 2}{3}\right)^3 + 3(5)^2 = (4)^3 + 75 = 64 + 75 = 139$

50. $\dfrac{(5 - 3)^4 - (2)^4}{5 - (2)^2} = \dfrac{(2)^4 - 16}{5 - 4} = \dfrac{16 - 16}{1} = \dfrac{0}{1} = 0$

51. $\dfrac{(2 \cdot 2 - 3)^3 + (2)^3}{(2 \cdot 5 - 3 + 2)^3} = \dfrac{(4 - 3)^3 + 8}{(10 - 3 + 2)^3} = \dfrac{(1)^3 + 8}{(7 + 2)^3} = \dfrac{1 + 8}{(9)^3} = \dfrac{9}{729} = \dfrac{1}{81}$

Pages 43–44 · **PROBLEMS**

A

1. $(15)^2 = 225$ sq cm

2. $(15)^3 = 3375$ cu cm

3. $(254)(300{,}000{,}000)^2 = (254)(90{,}000{,}000{,}000{,}000{,}000) = 22{,}860{,}000{,}000{,}000{,}000{,}000\ \text{g(M/sec)}^2$

4. $(15)^2(.01) = 225(.01) = 2.25\ \text{amp}^2\ \text{ohm}$

5. $\dfrac{(13.7)(25)^2}{2} = \dfrac{(13.7)(625)}{2} = \dfrac{8562.5}{2} = 4281.25$ ft

B

6. $\dfrac{4(3.14)(24)^3}{3} = \dfrac{(12.56)(13824)}{3} = (12.56)(4608) = 57{,}876.48$ cu in

7. $(.000000822)(6000)^4 = (.000000822)(1{,}296{,}000{,}000{,}000{,}000) = 1{,}065{,}312{,}000°\text{K}$

8. $\dfrac{(25)(100)^2}{2} = \dfrac{(25)(10000)}{2} = (25)(5000) = 125{,}000\ \text{g (cm/sec)}^2$

9. $(3.14)(1.25)^2(12.0) = (3.14)(1.5625)(12.0) = 58.875$ cu in

10. $\dfrac{300}{(500)^2} = \dfrac{300}{250{,}000} = \dfrac{3}{2500}$ candle power/sq ft

11. $\dfrac{15(20)^2}{10} = \dfrac{15(400)}{10} = 15(40) = 600\ \text{ft lb/sec}^2$

C

12. $\dfrac{(10.37)(200)}{(25)^2} = \dfrac{2074}{625} = 3.318 \doteq 3.32$ ft/sq mil

13. $\dfrac{(.0000000667)(100{,}000)(900{,}000)}{(1000)^2} = \dfrac{6003}{1{,}000{,}000} = .006003\ \text{(g/cm)}^2$

14. $(0.238)(20)^2(10)(300) = (0.238)(400)(3000) = 285{,}600\ \text{amp}^2\text{(ohm)(sec)}$

15. $(32.2)\left(\dfrac{.50}{2\pi}\right)^2 = (32.2)\left(\dfrac{.2500}{4\pi^2}\right) = \dfrac{8.0500}{4\pi^2} = \dfrac{2.0125}{\pi^2}$ ft

Pages 47–49 · **WRITTEN EXERCISES**

A

1. $1 + 1 = 5$; F
$2 + 1 = 5$; F
$3 + 1 = 5$; F
$4 + 1 = 5$; T
$5 + 1 = 5$; F
$\{4\}$

2. $0 > 3$; F
$2 > 3$; F
$4 > 3$; T
$6 > 3$; T
$8 > 3$; T
$\{4, 6, 8\}$

3. $0 < 1$; T
$1 < 1$; F
$2 < 1$; F
$3 < 1$; F
$4 < 1$; F
$\{0\}$

4. $2 \cdot 2 \neq 8$; T
$2 \cdot 4 \neq 8$; F
$2 \cdot 6 \neq 8$; T
$2 \cdot 8 \neq 8$; T
$\{2, 6, 8\}$

5. $2 \cdot 0 \neq 0$; F
$2 \cdot 2 \neq 0$; T
$2 \cdot 4 \neq 0$; T
$2 \cdot 6 \neq 0$; T
$\{2, 4, 6\}$

6. $2 \cdot 0 + 1 = 5$; F
$2 \cdot 2 + 1 = 5$; T
$2 \cdot 4 + 1 = 5$; F
$2 \cdot 6 + 1 = 5$; F
$\{2\}$

7. $2 \cdot 8 < 20$; T
$2 \cdot 9 < 20$; T
$2 \cdot 10 < 20$; F
$2 \cdot 11 < 20$; F
$\{8, 9\}$

8. $3 \cdot 7 > 25$; F
$3 \cdot 8 > 25$; F
$3 \cdot 9 > 25$; T
$3 \cdot 10 > 25$; T
$\{9, 10\}$

9. a. $10 + (7 + 2) = (10 + 7) + 2$; True; $x \in \{10\}$, $y \in \{7\}$

b. $2 + (0 + 2) = (2 + 0) + 2$; True; $x \in \{2\}$, $y \in \{0\}$

10. a. $10 - (6 + 1) > (10 - 6) + 1$;
$10 - 7 > 4 + 1$; False; $\emptyset$

b. $5 - (3 + 1) > (5 - 3) + 1$;
$5 - 4 > 2 + 1$; False; $\emptyset$

11. $\{3\}$ **12.** $\{1\}$ **13.** $\{11\}$ **14.** $\{3\}$ **15.** $\{0\}$ **16.** $\{0\}$

17.

18.

19.

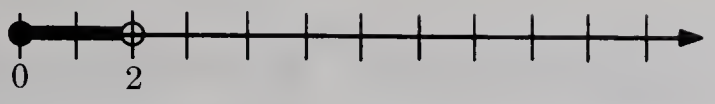
20.

21.

22.

23. $\emptyset$

24. $\emptyset$

25.


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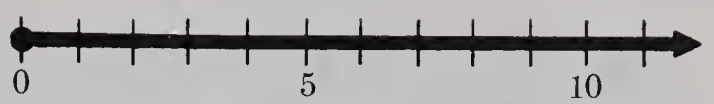

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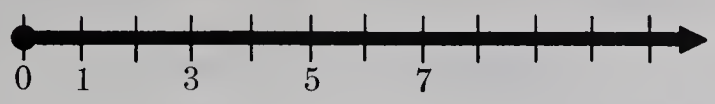

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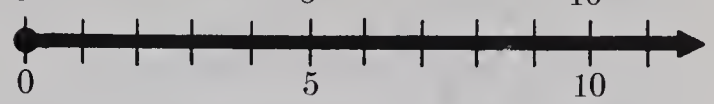
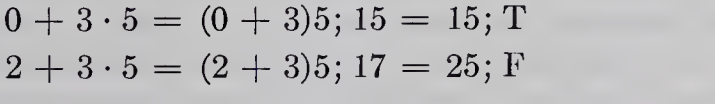


29. {8}

30. {5}

31.


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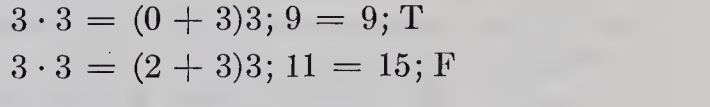

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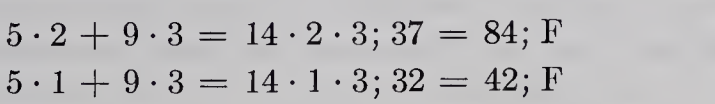
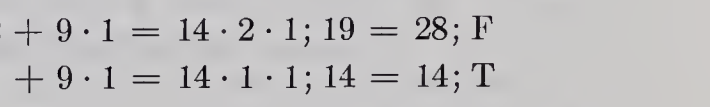
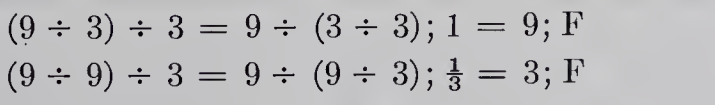


34.


35. {7}

36. {1}

37.


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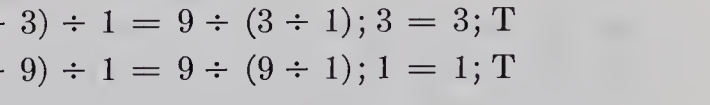
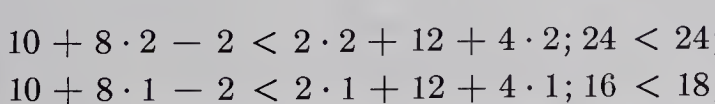


39.


40.

B

41. $9(3 + 2) = 9 \cdot 3 + 9 \cdot 2; 45 = 45;$ T
$9(5 + 2) = 9 \cdot 5 + 9 \cdot 2; 63 = 63;$ T
$9(3 + 4) = 9 \cdot 3 + 9 \cdot 4; 63 = 63;$ T
$9(5 + 4) = 9 \cdot 5 + 9 \cdot 4; 81 = 81;$ T

42. $12(1 \cdot 9) = (12 \cdot 1)9; 108 = 108;$ T
$12(3 \cdot 9) = (12 \cdot 3)9; 324 = 324;$ T
$12(1 \cdot 11) = (12 \cdot 1)11; 132 = 132;$ T
$12(3 \cdot 11) = (12 \cdot 3)11; 396 = 396;$ T

43. $(2 + 1)(10 + 3) > 3 \cdot 2 \cdot 10; 39 > 60;$ F
$(2 + 1)(21 + 3) > 3 \cdot 2 \cdot 21; 72 > 126;$ F
$(4 + 1)(10 + 3) > 3 \cdot 4 \cdot 10; 65 > 120;$ F
$(4 + 1)(21 + 3) > 3 \cdot 4 \cdot 21; 120 > 252;$ F

44. $12 \cdot 4 + 8 < 20 \cdot 4 + 8 \cdot 0; 56 < 80;$ T
$12 \cdot 7 + 8 < 20 \cdot 7 + 8 \cdot 0; 92 < 140;$ T
$12 \cdot 0 + 8 < 20 \cdot 0 + 8 \cdot 0; 8 < 0;$ F
$12 \cdot 4 + 8 < 20 \cdot 4 + 8 \cdot 1; 56 < 88;$ T
$12 \cdot 7 + 8 < 20 \cdot 7 + 8 \cdot 1; 92 < 148;$ T
$12 \cdot 0 + 8 < 20 \cdot 0 + 8 \cdot 1; 8 < 8;$ F

45. $(1)(1 + 1) = 1 \cdot 1 + 1 \cdot 1; 2 = 2;$ T
$(1)(2 + 1) = 1 \cdot 2 + 1 \cdot 1; 3 = 3;$ T
$(2)(1 + 1) = 2 \cdot 1 + 2 \cdot 1; 4 = 4;$ T
$(2)(2 + 1) = 2 \cdot 2 + 2 \cdot 1; 6 = 6;$ T
$(1)(1 + 2) = 1 \cdot 1 + 1 \cdot 2; 3 = 3;$ T
$(1)(2 + 2) = 1 \cdot 2 + 1 \cdot 2; 4 = 4;$ T
$(2)(1 + 2) = 2 \cdot 1 + 2 \cdot 2; 6 = 6;$ T
$(2)(2 + 2) = 2 \cdot 2 + 2 \cdot 2; 8 = 8;$ T

46. $0 + 3 \cdot 5 = (0 + 3)5; 15 = 15;$ T
$2 + 3 \cdot 5 = (2 + 3)5; 17 = 25;$ F
$0 + 3 \cdot 3 = (0 + 3)3; 9 = 9;$ T
$2 + 3 \cdot 3 = (2 + 3)3; 11 = 15;$ F

47. $5 \cdot 2 + 9 \cdot 3 = 14 \cdot 2 \cdot 3; 37 = 84;$ F
$5 \cdot 1 + 9 \cdot 3 = 14 \cdot 1 \cdot 3; 32 = 42;$ F
$5 \cdot 2 + 9 \cdot 1 = 14 \cdot 2 \cdot 1; 19 = 28;$ F
$5 \cdot 1 + 9 \cdot 1 = 14 \cdot 1 \cdot 1; 14 = 14;$ T

48. $12 \cdot 4 + 8 \cdot 4 < 20 \cdot 4; 80 < 80;$ F
$12 \cdot 7 + 8 \cdot 7 < 20 \cdot 7; 140 < 140;$ F

49. $(9 \div 3) \div 3 = 9 \div (3 \div 3); 1 = 9;$ F
$(9 \div 9) \div 3 = 9 \div (9 \div 3); \frac{1}{3} = 3;$ F
$(9 \div 3) \div 1 = 9 \div (3 \div 1); 3 = 3;$ T
$(9 \div 9) \div 1 = 9 \div (9 \div 1); 1 = 1;$ T

50. $10 + 8 \cdot 2 - 2 < 2 \cdot 2 + 12 + 4 \cdot 2; 24 < 24;$ F
$10 + 8 \cdot 1 - 2 < 2 \cdot 1 + 12 + 4 \cdot 1; 16 < 18;$ T

C

51. $10 - 4 \div 2 + 2 > 8 \div 4 + 8(2 \div 2); 10 > 10;$ F
$10 - 4 \div 4 + 2 > 8 \div 4 + 8(2 \div 4); 11 > 6;$ T
$10 - 8 \div 2 + 2 > 8 \div 8 + 8(2 \div 2); 8 > 9;$ F
$10 - 8 \div 4 + 2 > 8 \div 8 + 8(2 \div 4); 10 > 5;$ T

52. $\frac{6 \cdot 6 - 4 \cdot 4}{6 + 4} = 6 - 4; 2 = 2;$ T $\qquad \frac{6 \cdot 6 - 8 \cdot 8}{6 + 8} = 6 - 8;$ (36 − 64) not defined; undetermined

$\frac{8 \cdot 8 - 4 \cdot 4}{8 + 4} = 8 - 4; 4 = 4;$ T $\qquad \frac{8 \cdot 8 - 8 \cdot 8}{8 + 8} = 8 - 8; 0 = 0;$ T

53. a. $7 \cdot 3 + 5 \cdot 3 + 2 \cdot 0 > 12 \cdot 3 + 0 + 0; 36 > 36;$ F

b. $7 \cdot 4 + 5 \cdot 1 + 2 \cdot 2 > 12 \cdot 4 + 2 + 2; 37 > 52;$ F

54. a. $(5 + 2) + 7(2 + 3) + (5 + 3) = 19 + 2 + 8 \cdot 3 + 5; 50 = 50;$ T

b. $(5 + 5) + 7(2 + 2) + (3 + 2) = 19 + 5 + 8 \cdot 2 + 3; 43 = 43;$ T

55. a. $5(12 - 4) \neq 5 \cdot 12 - 4; 40 \neq 56;$ T

b. $1(6 - 0) \neq 1 \cdot 6 - 0; 6 \neq 6;$ F

56. a. $\frac{4 \cdot 15 + 5}{4} = 15 + 5; \frac{65}{4} = 20;$ F

b. $\frac{1 \cdot 12 + 2}{1} = 12 + 2; 14 = 14;$ T

Pages 50–51 · **WRITTEN EXERCISES**

A **1.** a. Seven times a given no. x; $x \geq 0$. b. The length of a rectangle is 7 times its width. If the width is x units, then the length is $7x$ units; $x > 0$. **2.** a. Nine times a given no. y; $y \geq 0$. b. Fred has 9 times as many dimes as Jon. Let y represent the no. of dimes belonging to Jon; then $9y$ represents Fred's dimes; $y \in \{1, 2, 3, 4, 5, \ldots, n\}$. **3.** a. The sum of a given no. x and 15; $x \geq 0$. b. Mae is 15 years older than Ruth. If x represents Ruth's age, then $x + 15$ represents Mae's age; $x > 0$. **4.** a. The difference of some no. y and 10; $y \geq 10$. b. In a turkey shoot Robin bagged 10 more fowl than John Little. If y represents the no. of turkeys shot by Robin, then John Little's catch is represented by $y - 10$; $y \in \{10, 11, 12, \ldots, n\}$. **5.** a. Twice a given no. a increased by 3; $a \geq 0$. b. Sam sold 3 more than twice as many tickets to the prom as Pam sold. If a stands for the no. of tickets sold by Pam, then $2a + 3$ represents the no. sold by Sam; $a \in \{0, 1, 2, 3, 4, \ldots, n\}$. **6.** a. Three times a given no. b diminished by 7; $b \geq \frac{7}{3}$. b. Find two nos. such that the second can be obtained by subtracting 7 from 3 times the first. If b is an expression for the first no., $3b - 7$ represents the second; $b \geq \frac{7}{3}$. **7.** a. Four times the sum of a given no. x and 1; $x \geq 0$. b. A side of one square exceeds a side of a second by one unit. If x represents the measure of a side of the second, then $4(x + 1)$ represents the perimeter of the first square; $x > 0$. **8.** a. The product of a given no. y diminished by 2 and the no. 5; $y \geq 2$. b. Joan has 2 less books than Kay and Caren has 5 times as many books as Joan. If y represents Kay's books, then $5(y - 2)$ represents Caren's books; $y \in \{2, 3, 4, \ldots, n\}$. **9.** One-fourth a given no. x diminished by 2; $x \geq 8$. b. There are two nos. such that subtracting 2 from $\frac{1}{4}$ the first number results in the second no. If x stands for the first no., then $\frac{x}{4} - 2$ represents the second; $x \geq 8$. **10.** a. One-third of a given no. n decreased by 3; $n \geq 9$. b. The Red Sox won 3 less than $\frac{1}{3}$ of the games won by the White Sox. Let n stand for the no. of White Sox wins; then $\frac{1}{3}n - 3$ represents the no. of Red Sox wins; $n \in \{9, 12, 15, \ldots, 168\}$. **11.** a. Twice the sum of twice some no. v and 1; $v \geq 0$. b. In a pie-eating contest Ed ate 1 more than twice as many pies as Bill, and Ken ate twice as many as Ed. If v stands for the no. of pies eaten by Bill, then $2(2v + 1)$ represents the number of pies eaten by Ken; $v \in \{0, 1, 2, \ldots, n\}$ assuming that a whole pie must be eaten in order to be counted. **12.** a. The product of a given no. u and another no. which is 3 times the given no. diminished by 1; $u \geq \frac{1}{3}$. b. Express the product of two nos. such that the first no. is 1 less than 3 times the second no. If u represents the second no. then $u(3u - 1)$ is an expression for the product of the two nos.; $u \geq \frac{1}{3}$. **13.** a. The sum of two nos. such that one no. is 5 more than twice the other no., where z represents the other no.; $z \geq 0$. b. Mr. Cella is 5 yr more than double his son's age. If z represents his son's age then the sum of their ages is represented by $z + (2z + 5)$; $z > 0$. **14.** a. The sum of two nos. such that one no. is twice the sum of 5 and the other no.; $m \geq 0$. b. Twice the result of adding 5 to Ned's age will give Tom's age. If m represents Ned's age, then $m + 2(m + 5)$ represents the sum of their ages; $m > 0$. **15.** a. The sum of two nos. such that the first is 3 less than $\frac{1}{5}$ the second; $s \geq 15$. b. If Sally is now s yr and Nan is 3 yr less than $\frac{1}{5}$ of Sally's age, then the sum of their ages can be expressed as $\left(\frac{s}{5} - 3\right) + s$; $s > 15$. **16.** a. The difference of two nos., where the larger no. is represented by r and the smaller no. is 3 less than $\frac{1}{2}$ the larger no.; $r \geq 6$. b. One car is moving at a rate which is 3 mph less than $\frac{1}{2}$ the rate of another. If r represents the rate of the faster car, then the difference of their rates is represented by $r - \left(\frac{r}{2} - 3\right)$; $r \geq 6$. **17.** a. The sum of two nos., where x stands for one no. and the second no. is the difference of 100 and the first no.; $0 \leq x \leq 100$. b. There is a pair of nos. whose sum is 100. If x represents the first such number then the sum of the two can be expressed as $x + (100 - x)$; $0 \leq x \leq 100$.

18. a. The sum of the ages of two brothers born exactly 1 yr apart, where the age of the younger boy is expressed as n; $n > 0$. b. If $(n + 1)$ stands for the larger of two consecutive whole nos., then the sum of the two nos. is $n + (n + 1)$; $n \in \{0, 1, 2, 3, \ldots\}$. **19.** a. The difference of the square of a no. s and 4; here $s \geq 2$. b. Two squares differ in area by 4. Let s stand for the measure of a side of the larger square; then the area of the smaller square is represented by $s^2 - 4$; $s > 2$. **20.** a. One is subtracted from the product of 4 and the square of some no. t; $t \geq \frac{1}{2}$. b. There are two nos. such that the first is 1 less than 4 times the square of the second. If t represents the second no., then $4t^2 - 1$ represents the first no.; $t \geq \frac{1}{2}$. **21.** a. The square of a no. subtracted from the no. w; $0 \leq w \leq 1$. b. For some squares the length of a side is greater than or equal to the area. Let w stand for the length of a side; then $(w - w^2)$ represents the difference between the length of a side and the area of the square; $0 < w \leq 1$. **22.** a. The square of some no. t minus the no.; here, $t = 0$ *and* the set of all t such that $t \geq 1$. b. For some squares the length of a side of the square is less than or equal to the area. Let t represent the length of a side; then the area of the square exceeds the measure of a side by $(t^2 - t)$; $t \geq 1$. **23.** a. The cube of a no. x subtracted from 8; $0 \leq x \leq 2$. b. Two cubes have edges of 2 and x, respectively, where $2 \geq x$. The difference in the volumes of the two cubes is $8 - x^3$; $0 < x \leq 2$. **24.** a. One less than $\frac{1}{8}$ the cube of a number y; $y \geq 2$. b. One no. is equal to 1 less than $\frac{1}{8}$ the cube of a second. If y stands for the second no., then $\frac{y^3}{8} - 1$ represents the first no.; $y \geq 2$.

Page 54 · WRITTEN EXERCISES

A **1.** $x + 5$ **2.** $d + c$ **3.** $2x + 4$ **4.** $5y + 2x$ **5.** $15 - n$
6. $3x - 2$ **7.** $z - 1$ **8.** $x - d$ **9.** $2x - 5$ **10.** $5k - 5$
11. $(3x)y$ **12.** $6a$ **13.** $\frac{r + 6}{3}$ **14.** $\frac{1}{3}(r - 6)$; $\frac{r - 6}{3}$ **15.** $\frac{1}{2}(x + y)$; $\frac{x + y}{2}$
16. $\frac{a - b}{c}$ **17.** $5(2 + y)$ **18.** $a(a - b)$ **19.** $x(x + 2) + 5$
20. $2u - \frac{v}{2}$

B **21.** $\frac{5}{4x + 3k}$ **22.** $(a + b) - ab$ **23.** $\frac{1}{2(a + b)}$ **24.** $\frac{mn}{3(m - n)}$
25. $\frac{3(r - s)}{2(r + s)}$

Pages 54–55 · PROBLEMS

A **1.** x, $2x$ **2.** l, $3l$ **3.** $90w$ **4.** $5i$ **5.** $3w$ **6.** $3n$ **7.** m, $3m$ **8.** $10s$ **9.** $400 - q$
10. $h + 10{,}000$ **11.** (a) w; (b) $2w + 6$ **12.** $3n - 17$

B **13.** (a) s; (b) $2s - 48$; (c) $(2s - 48) + s + s$ **14.** $57 - w$ **15.** $s + \frac{4}{5}s = 108$
16. $(3c + 7c) + 5$ **17.** $5c + 2(c + 28)$ **18.** (a) $2x$; (b) $2x + 3$

Pages 58–60 · PROBLEMS

A **1.** Let x = no. of games lost; then $3x$ = no. of games won; $3x = 84$; $x = 28$. **2.** Let n = no. of class members; $5n = 170$; $n = 34$. **3.** Let z = no. of days the roll lasted; $5z = 50$; $z = 10$. **4.** Let g = no. of girls in class; then $7g$ = no. of boys in class; $7g = 28$; $g = 4$. **5.** Let t = cost of lot; then $7t$ = cost of house; $7t = 18{,}200$; $t = \$2600$. **6.** Let s = man's monthly salary; $9s = 4050$; $s = \$450$. **7.** Let y = aunt's age; then $\frac{1}{7}y$ = Si's age; $\frac{1}{7}y = 8$; $y = 56$ yr. **8.** Let w = no. of girls in the city; then $\frac{1}{3}w$ = no. of blondes in the city; $\frac{1}{3}w = 10{,}195$; $w = 30{,}585$. **9.** Let s = length of a side of square; $P = 4s$; $50 = 4s$; $s = 12\frac{1}{2}$ in. **10.** Let l = length of rectangle; $A = lw$; $68 = 4l$; $l = 17$ ft. **11.** Let A = amount in Jack's account before deposit; $A + 55.25 = 1342.70$; $A = \$1287.45$. **12.** Let P = amount on deposit before withdrawal; $P - 15.75 = 672.39$; $P = \$688.14$. **13.** Let n = normal weight; $n + 11 = 109$; $n = 98$ lb. **14.** Let T = temperature (true reading); $T - 3.7 = 56$; $T = 59.7°$. **15.** Let E = Bill's weekly earnings; $E + 7.50$ = Fred's weekly earnings; $E + 7.50 = 115.00$; $E = \$107.50$. **16.** Let G = Mark's score; $G - 3$ = Dave's score; $G - 3 = 89$; $G = 92$.

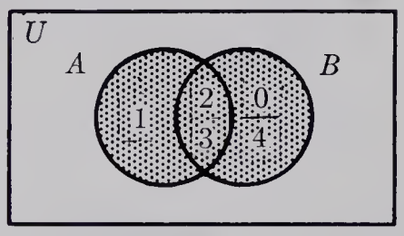
B **17.** Let m = no. miles by auto; then $9m$ = no. miles by airplane; $m + 9m = 500$; $m = 50$ mi.

18. Let s = no. freshmen who elect Spanish; then $\frac{s}{4}$ = no. freshmen who elect French; $s + \frac{s}{4} = 150$; $s = 120$. **19.** Let n = the number; $3(2n) = 84$; $n = 14$. **20.** Let x = max. no. of bottles milkman could have had originally; $x - 12 < 75$; $x < 87$; 86 (bottles). **21.** Let B = no. of books owned by John; then $2B + 1$ = no. of books owned by Sue; $2B + 1 = 59$; $B = 29$. **22.** Let N = machine B's daily output; then $4N - 600$ = machine A's daily output; $4N - 600 = 4800$; $N = 1350$. **23.** Let t = Jay's bowling score; then $\frac{t}{2} + 10$ = Mary's bowling score; $\frac{t}{2} + 10 = 100$; $t = 180$. **24.** Let n = the number; $\frac{n}{3} - 16 = 21$; $n = 111$.

C **25.** Let w = width of picture; then $2w - 4$ = length; $w + w + (2w - 4) + (2w - 4) = 76$; $6w - 8 = 76$; $w = 14$ ft; $2w - 4 = 24$ ft. **26.** Let p = avg. speed of auto; then $15p$ = avg. speed of jet; $d = rt$; $(p)(1)$ = distance traveled by auto; $(15p)(1)$ = distance traveled by jet; $(p)(1) + (15p)(1) = 640$; $16p = 640$; $p = 40$; $(p)(1) = 40$ mi. **27.** Let n = max. no. of tickets which Jo could have sold; then $2n + 3$ = no. of tickets Linda sold; $2n + 3 < 32$; $n < \frac{29}{2}$; 14 tickets. **28.** Let w = max. wt. of Helen; then $\frac{w}{2}$ = wt. of sister; $w + \frac{w}{2} \leq 165$; $\frac{3w}{2} \leq 165$; $w \leq 110$; 110 lb. **29.** Let n = the first no.; then $2n$ = the 2nd no.; and $3n - 1$ = the 3rd no.; $n + 2n + (3n - 1) = 35$; $6n - 1 = 35$; $n = 6$; $3n - 1 = 17$, *Ans.* **30.** Let l = length of 2nd side; then $3l - 13$ = length of 1st side; and $l + 18$ = length of 3rd side; $l + (3l - 13) + (l + 18) = 130$; $5l + 5 = 130$; $l = 25$ ft; $3l - 13 = 62$ ft; $l + 18 = 43$ ft.

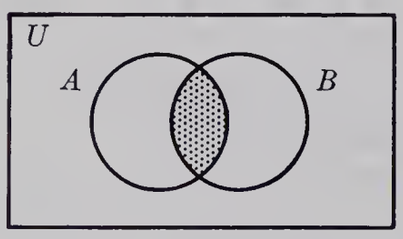
Page 61 · EXTRA FOR EXPERTS

1. $A \cup B = \{0, 1, 2, 3, 4\}$

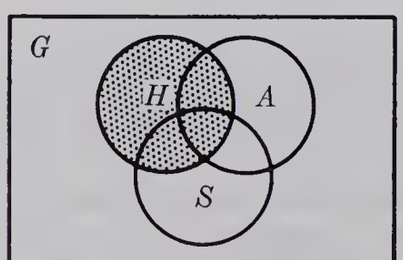


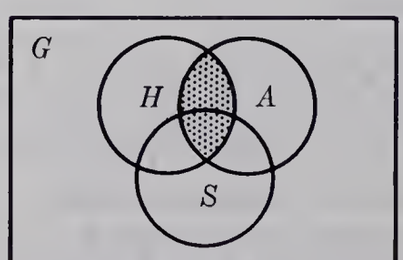
a. A and B are subsets of $A \cup B$; because every element of each A and B are elements of $A \cup B$.
b. $A \cup B = B$ when $A = \emptyset$ or when both A and B are empty or when $A = B$; e.g., $A = \emptyset$, $B = \{7, 8, 9\}$; $A \cup B = \{7, 8, 9\} = B$.

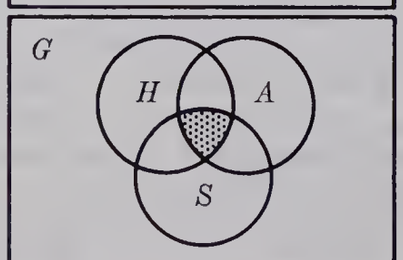
2. When $A = B$ (including case $A = B = \emptyset$). a. $A = \{5, 7, 6, 9\}$, $B = \{9, 5, 7, 6\}$, $A \cup B = \{5, 6, 7, 9\}$; $A \cap B = \{5, 6, 7, 9\}$

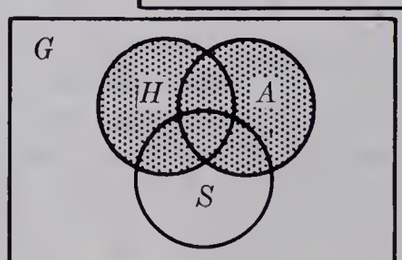
b. 

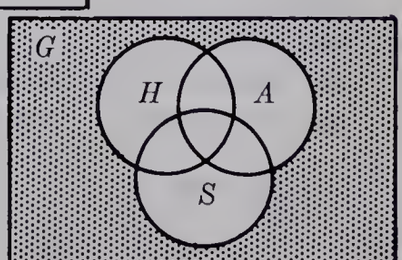
(for both ∪ and ∩)

3. a. 

b. 

c. 

d. 

e. 

Page 64 · CHAPTER TEST

1. $8 \cdot 3 = 24$ **2.** $6\ \frac{1}{3} + 3 = 2 + 3 = 5$

3. $6(3 + \frac{1}{3}) - 3 \cdot 3 \cdot \frac{1}{3} = 6(3\frac{1}{3}) - 3 = 20 - 3 = 17$ **4.** $(3 + \frac{1}{3})(3 - \frac{1}{3}) = (3\frac{1}{3})(2\frac{2}{3}) = \frac{80}{9}$

5. $2 \cdot 3^3 = 54$ **6.** $(2 \cdot 3)^2 = (6)^2 = 36$ **7.** $\{1, a\}$ **8.** $\{1, 2, 3, 4, 6, 12, m, n\}$

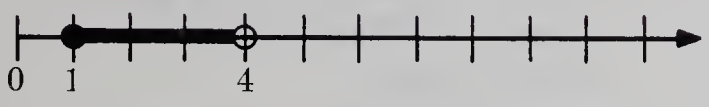
9. $\{\frac{1}{3}, p, q\}$ **10.** $8q$ **11.** $\frac{1}{5}$

12. num. coeff.: 3; base: $(x - 1)$; exp.: 2 **13.** $h \cdot h \cdot h$ *or* h^3 **14.** 6

15. 9, 12 **16.** $9 - 5 \cdot 1 \geq 6$; $4 \geq 6$; F

17. $9 - (1 + 8) = (9 - 1) + 8$; $0 = 16$; F. **18.** $\{1\}$ **19.** $\{6, 7, \ldots, 10\}$

20.



21.

22. a. The sum of *a* no. x and 5; $x \geq 0$. b. If your current age, in yr, is represented by x, then your age 5 yr from now will be $x + 5$; $x > 0$.

23. a. Five less than twice a no. x; $x \geq \frac{5}{2}$. b. The measure of angle A is 5 less than twice the measure of angle B. Let x equal the measure of angle B; then $2x - 5$ represents the measure of angle A; $x > \frac{5}{2}$.

24. Amount saved, in dollars $= 52s - 12e$.

25. Let w = width of pool; then $2w - 5$ = length of pool; $2w - 5 = 35$; $w = 20$ ft.

Pages 65–67 · CHAPTER REVIEW

1. symbol **2.** replacement, domain

3. $(5 \cdot 2 + 1)(5 \cdot 2 - 1) = (10 + 1)(10 - 1) = (11)(9) = 99$ **4.** product, quotient

5. coefficient **6.** n **7.** s, s **8.** exponent

9. c^5 **10.** 3, a, 2 **11.** 1 **12.** $(15)^3 = 3375$; $3 \cdot 15 = 45$ **13.** n

14. domain *or* replacement set **15.** left **16.** variable **17.** inequality

18.
$0 - 1 = 3$; F
$1 - 1 = 3$; F
$2 - 1 = 3$; F
$3 - 1 = 3$; F
$4 - 1 = 3$; T

19.
$0 < 5$; T
$1 < 5$; T
$2 < 5$; T
$3 < 5$; T
$4 < 5$; T

20.
$3 \cdot 0 \neq 9$; T
$3 \cdot 1 \neq 9$; T
$3 \cdot 2 \neq 9$; T
$3 \cdot 3 \neq 9$; F
$3 \cdot 4 \neq 9$; T

21.
$0 > 4$; F
$1 > 4$; F
$2 > 4$; F
$3 > 4$; F
$4 > 4$; F

22. solution **23.** root **24.** root **25.** (b) **26.** $x > 2$

27. Three times a no. x; $x \geq 0$

28. Three greater than the quotient of a no. x and 3; $x \geq 0$

29. The product of a no. x diminished by 1 and 3; $x \geq 1$

30. $a - 5$ **31.** $2b + 3$ **32.** $3c - 3$, $c \geq 1$ **33.** $\frac{d + e}{3}$

34. $3i$ **35.** $100 - 5n$, $n \in \{1, 2, 3, \ldots, 20\}$ **36.** symbol *or* variable **37.** check

38. $18n = 198$; $\{11\}$ **39.** $3P = 1440$; $\{480\}$ **40.** $300 \leq 25a \leq 500$; $12 \leq a \leq 20$

CHAPTER 3. Axioms, Equations, and Problem Solving

Pages 72–73 · WRITTEN EXERCISES

1. Add.: closed;
Subt.: closed;
Div.: not closed, $0 \div 0$ undefined;
Mult.: closed

2. Add.: not closed, $1 + 1$ not in set;
Subt.: not closed, $1 - 1$ not in set;
Div.: closed;
Mult.: closed

3. Add.: not closed, $2 + 2$ not in set;
Subt.: not closed, $2 - 2$ not in set;
Div.: not closed, $2 \div 2$ not in set;
Mult.: not closed, 2×2 not in set

4. Add.: not closed, $3 + 3$ not in set;
Subt.: not closed, $3 - 3$ not in set;
Div.: not closed, $3 \div 3$ not in set;
Mult.: not closed, 3×3 not in set

5. Add.: not closed, $2 + 3$ not in set;
Subt.: not closed, $1 - 3$ not in set;
Div.: not closed, $3 \div 2$ not in set;
Mult.: not closed, 2×3 not in set

6. Add.: not closed, $1 + 1$ not in set;
Subt.: not closed, $0 - 1$ not in set;
Div.: not closed, $1 \div 0$ undefined;
Mult.: closed

7. Add.: not closed, $1 + 2$ not in set;
Subt.: not closed, $1 - 2$ not in set;
Div.: not closed, $1 \div 2$ not in set;
Mult.: not closed, 2×2 not in set

8. Add.: closed;
Subt.: not closed, $15 - 20$ not in set;
Div.: not closed, $25 \div 10$ not in set;
Mult.: closed

9. Add.: not closed, $1 + 1$ not in set;
Subt.: not closed, $\frac{3}{2} - \frac{9}{5}$ not in set;
Div.: not closed, $\frac{1}{2} \div \frac{1}{8}$ not in set;
Mult.: not closed, $\frac{3}{2} \times \frac{3}{2}$ not in set

10. Add.: not closed, $\frac{1}{2} + \frac{1}{4}$ not in set;
Subt.: not closed, $\frac{1}{4} - \frac{1}{16}$ not in set;
Div.: not closed, $\frac{1}{2} \div \frac{1}{4}$ not in set;
Mult.: closed

11. Add.: closed;
Subt.: not closed, $2 - 2$ not in set;
Div.: closed;
Mult.: closed

12. Add.: not closed, $\frac{1}{4} + \frac{3}{4}$ not in set;
Subt.: not closed, $\frac{7}{3} - \frac{4}{3}$ not in set;
Div.: not closed, $\frac{1}{8} \div \frac{1}{16}$ not in set;
Mult.: not closed, $\frac{2}{3} \times \frac{3}{2}$ not in set

Page 79 · WRITTEN EXERCISES

A **1.** $56x$ **2.** $85y$ **3.** $65a$ **4.** $23b$ **5.** $30a$ **6.** $7x$ **7.** $12x + 6$ **8.** $15n + 3$
9. $3s + 4r$ **10.** $4x + 2y$ **11.** $2a + 3b + 4c - d$ **12.** $6u - 5v - 3w - 4$
13. $10n + 21$ **14.** $26z + 10$ **15.** $7b$ **16.** $4x + 30$ **17.** $16a + 4b + 1$
18. $s + 5t + 2$ **19.** $6a + 11ab$ **20.** $12x + 5xy$

B **21.** $3a + 3b + 7a + 7b = 10a + 10b$
22. $9m + 9n + 14m + 7n = 23m + 16n$
23. $24 + 8a + 12 + 4a = 36 + 12a$
24. $2a + 10 + 3a - 6 = 5a + 4$
25. $4d + 20 + 6d - 3 = 10d + 17$
26. $42e + 21 + 20e - 15 = 62e + 6$
27. $30w + 10v + 18w - 6v = 48w + 4v$
28. $18r + 72t + 8r - 32t = 26r + 40t$
29. $3a + 6b + 3 + 2a - 2 = 5a + 6b + 1$
30. $4r + 12s + 8 + 7s - 7 = 4r + 19s + 1$

C **31.** $3[8a + 15 - 5a] - 17 = 24a + 45 - 15a - 17 = 9a + 28$
32. $19 + 2[4b + 15b - 6] = 19 + 8b + 30b - 12 = 38b + 7$
33. $12r + 6s + 2[5r + 12s - 3r + 3] = 12r + 6s + 10r + 24s - 6r + 6 = 16r + 30s + 6$
34. $5[8m + 4n + 12 - 6m - 1] + 10m - 2n = 40m + 20n + 60 - 30m - 5 + 10m - 2n = 20m + 18n + 55$
35. $(28 + 22)37 = (50)37 = 1850$
36. $(77 + 23)19 = (100)19 = 1900$
37. $50(\frac{4}{5}) - 50(\frac{1}{2}) = 40 - 25 = 15$
38. $779(11 - 1) = 779(10) = 7790$
39. $4 \times (0) = 0$
40. $68 \times (32 \times 0) = 68 \times (0) = 0$
41. $7\frac{1}{2}(9 + 1) = 7\frac{1}{2}(10) = (7 + \frac{1}{2})10 = 7(10) + \frac{1}{2}(10) = 70 + 5 = 75$
42. $(\frac{1}{2} + 9\frac{1}{2})57 = (10)57 = 570$
43. $326(1000 + 1) = 326{,}000 + 326 = 326{,}326$
44. $30(4 + \frac{7}{10}) = 30(4) + 30(\frac{7}{10}) = 120 + 21 = 141$
45. $(3 + 4)29 + 3(29) = [(3 + 4) + 3]29 = [10]29 = 290$
46. $(7 - 5)46 - 2(46) = [(7 - 5) - 2]46 = (0)46 = 0$

Page 83 · WRITTEN EXERCISES

A **1.** $x + 32 - 32 = 81 - 32$; $x = 49$. $\{49\}$
2. $y + 17 - 17 = 45 - 17$; $y = 28$. $\{28\}$
3. $27 + R - 27 = 105 - 27$; $R = 78$. $\{78\}$
4. $91 + S - 91 = 91 - 91$; $S = 0$. $\{0\}$
5. $39 - 39 = t + 39 - 39$; $0 = t$. $\{0\}$
6. $73 - 73 = u + 73 - 73$; $0 = u$. $\{0\}$
7. $v - 47 + 47 = 47 + 47$; $v = 94$. $\{94\}$
8. $w - 37 + 37 = 73 + 37$; $w = 110$. $\{110\}$
9. $175 - 82 = z + 82 - 82$; $93 = z$. $\{93\}$
10. $314 - 165 = 165 + a - 165$; $149 = a$. $\{149\}$
11. $b - 17 + 17 = 0 + 17$; $b = 17$. $\{17\}$
12. $c - 51 + 51 = 0 + 51$; $c = 51$. $\{51\}$
13. $d + 3.2 - 3.2 = 7.8 - 3.2$; $d = 4.6$. $\{4.6\}$
14. $f + 2.7 - 2.7 = 3.1 - 2.7$; $f = .4$. $\{.4\}$
15. $h - .78 + .78 = 9.2 + .78$; $h = 9.98$. $\{9.98\}$
16. $k - .37 + .37 = 4.1 + .37$; $k = 4.47$. $\{4.47\}$

17. $\frac{3}{5} + m - \frac{3}{5} = \frac{18}{5} - \frac{3}{5}$; $m = 3$. {3}
18. $\frac{2}{7} + n - \frac{2}{7} = \frac{16}{7} - \frac{2}{7}$; $n = 2$. {2}
19. $\frac{2}{3} + \frac{5}{3} = Q - \frac{5}{3} + \frac{5}{3}$; $\frac{7}{3} = Q$. $\{2\frac{1}{3}\}$
20. $\frac{3}{10} + \frac{13}{10} = T - \frac{13}{10} + \frac{13}{10}$; $\frac{8}{5} = T$. $\{1\frac{3}{5}\}$
21. $m - 750 + 750 = 5 + 750$; $m = 755$. {755}
22. $0.04 + p - 0.04 = 1 - 0.04$; $p = .96$. {.96}
23. $a - 0.85 + 0.85 = 0.15 + 0.85$; $a = 1.00$. {1}
24. $b - 5 + 5 = \frac{3}{5} + 5$; $b = 5\frac{3}{5}$. $\{5\frac{3}{5}\}$
25. $h - \frac{2}{3} + \frac{2}{3} = 3 + \frac{2}{3}$; $h = 3\frac{2}{3}$. $\{3\frac{2}{3}\}$
26. $d - 2.5 + 2.5 = 3.95 + 2.5$; $d = 6.45$. {6.45}
27. $y - \frac{3}{8} + \frac{3}{8} = 2 + \frac{3}{8}$; $y = 2\frac{3}{8}$. $\{2\frac{3}{8}\}$
28. $p - \frac{5}{6} + \frac{5}{6} = 3\frac{1}{6} + \frac{5}{6}$; $p = 4$. {4}
29. $.8 + 75.7 = n - 75.7 + 75.7$; $76.5 = n$. {76.5}
30. $\frac{3}{4} - .75 = .75 + r - .75$; $0 = r$. {0}

Pages 84–85 · **WRITTEN EXERCISES**

A

1. $19 \cdot \frac{n}{19} = 19 \cdot 6$; $n = 114$. {114}
2. $27 \cdot \frac{r}{27} = 27 \cdot 8$; $r = 216$. {216}
3. $\frac{17s}{17} = \frac{187}{17}$; $s = 11$. {11}
4. $\frac{13t}{13} = \frac{169}{13}$; $t = 13$. {13}
5. $\frac{1.5n}{1.5} = \frac{3}{1.5}$; $n = 2$. {2}
6. $\frac{.7}{.7} = \frac{.7m}{.7}$; $1 = m$. {1}
7. $(1.2)\left(\frac{f}{1.2}\right) = (1.2)(15)$; $f = 18$. {18}
8. $(1.8)\left(\frac{x}{1.8}\right) = (1.8)(1.9)$; $x = 3.42$. {3.42}
9. $12 \cdot \frac{5}{3} = 12 \cdot \frac{1}{12}x$; $20 = x$. {20}
10. $24 \cdot \frac{3}{8} = 24 \cdot \frac{1}{24}y$; $9 = y$. {9}
11. $\frac{16}{12} = \frac{12t}{12}$; $1\frac{1}{3} = t$. $\{1\frac{1}{3}\}$
12. $\frac{20}{16} = \frac{16z}{16}$; $1\frac{1}{4} = z$. $\{1\frac{1}{4}\}$
13. $5(.8) = 5\left(\frac{y}{5}\right)$; $4 = y$. {4}
14. $(1.5)(1.6) = (1.5)\left(\frac{h}{1.5}\right)$; $2.4 = h$. {2.4}
15. $\frac{\frac{2}{3}}{6} = \frac{6c}{6}$; $\frac{1}{9} = c$. $\{\frac{1}{9}\}$
16. $\frac{12d}{12} = \frac{\frac{3}{4}}{12}$; $d = \frac{1}{16}$. $\{\frac{1}{16}\}$
17. $\frac{.36(y)}{.36} = \frac{4.8}{.36}$; $y = 13\frac{1}{3}$. $\{13\frac{1}{3}\}$
18. $\frac{.14(y)}{.14} = \frac{14}{.14}$; $y = 100$. {100}

B

19. $(3.2)\left(\frac{k}{3.2}\right) = (3.2)(0)$; $k = 0$. {0}
20. $(.25)\left(\frac{m}{.25}\right) = (.25)(0)$; $m = 0$. {0}
21. $(.6)\left(\frac{n}{.6}\right) = (.6)(.25)$; $n = .15$. {.15}
22. $(.03)\left(\frac{p}{.03}\right) = (.03)(.2)$; $p = .006$. {.006}
23. $\frac{4.5(y)}{4.5} = \frac{12.8}{4.5}$; $y = 2\frac{38}{45}$. $\{2\frac{38}{45}\}$
24. $\frac{3.2(t)}{3.2} = \frac{6.7}{3.2}$; $t = 2\frac{3}{32}$. $\{2\frac{3}{32}\}$
25. $10(3.4) = 10\left(\frac{y}{10}\right)$; $34 = y$. {34}
26. $15(.75) = 15\left(\frac{x}{15}\right)$; $11.25 = x$. {11.25}
27. $3 \cdot \frac{2}{3}y = 3 \cdot 6$; $2y = 18$; $\frac{2y}{2} = \frac{18}{2}$; $y = 9$. {9}
28. $4 \cdot \frac{5}{4}w = 4 \cdot 20$; $5w = 80$; $\frac{5w}{5} = \frac{80}{5}$; $w = 16$. {16}
29. $2 \cdot \frac{1}{2}r = 2 \cdot 3\frac{1}{2}$; $r = 7$. {7}
30. $3 \cdot \frac{1}{3}v = 3 \cdot 4\frac{2}{3}$; $v = 14$. {14}

C

31. $4 + 8y - 4 = 52 - 4$; $8y = 48$; $\frac{8y}{8} = \frac{48}{8}$; $y = 6$. {6}
32. $.5c + 8 - 8 = 23 - 8$; $.5c = 15$; $\frac{.5c}{.5} = \frac{15}{.5}$; $c = 30$. {30}
33. $9r - 9 + 9 = 27 + 9$; $9r = 36$; $\frac{9r}{9} = \frac{36}{9}$; $r = 4$. {4}
34. $6x - 3 + 3 = 27 + 3$; $6x = 30$; $\frac{6x}{6} = \frac{30}{6}$; $x = 5$. {5}
35. $2n - 5 + 5 = 8.5 + 5$; $2n = 13.5$; $\frac{2n}{2} = \frac{13.5}{2}$; $n = 6.75$. {6.75}
36. $29 - 7.79 = 7h + 7.79 - 7.79$; $21.21 = 7h$; $\frac{21.21}{7} = \frac{7h}{7}$; $h = 3.03$. {3.03}

37. a. Reflex. prop.
b. Given
c. Subst. principle

38. a. Reflex. prop.
b. Given
c. Subst. principle

39. a. Reflex. prop.
b. Given
c. Subst. principle

40. a. Reflex. prop.
b. Given
c. Subst. principle
d. Given
e. Subst. principle

41. a. Reflex. prop.
b. Given
c. Subst. principle
d. Given
e. Subst. principle

42. a. Reflex. prop.
b. Given
c. Subst. principle
d. Given
e. Subst. principle

Pages 88–89 · **WRITTEN EXERCISES**

A **1.** $8x = 24; \frac{8x}{8} = \frac{24}{8}; x = 3.$ {3}

2. $4n = 56; \frac{4n}{4} = \frac{56}{4}; n = 14.$ {14}

3. $28 = 18x + 10; 28 - 10 = 18x + 10 - 10; 18 = 18x; \frac{18}{18} = \frac{18x}{18}; 1 = x.$ {1}

4. $45 = 15x + 30; 45 - 30 = 15x + 30 - 30; 15 = 15x; \frac{15}{15} = \frac{15x}{15}; 1 = x.$ {1}

5. $\frac{1}{2}x - 10 = 0; \frac{1}{2}x - 10 + 10 = 0 + 10; \frac{1}{2}x = 10; 2 \cdot \frac{1}{2}x = 2 \cdot 10; x = 20.$ {20}

6. $\frac{1}{5}v - 5 = 0; \frac{1}{5}v - 5 + 5 = 0 + 5; \frac{1}{5}v = 5; 5 \cdot \frac{1}{5}v = 5 \cdot 5; v = 25.$ {25}

7. $20a + 85 = 95; 20a + 85 - 85 = 95 - 85; 20a = 10; \frac{20a}{20} = \frac{10}{20}; a = \frac{1}{2}.$ $\{\frac{1}{2}\}$

8. $17b + 173 = 221; 17b + 173 - 173 = 221 - 173; 17b = 48; \frac{17b}{17} = \frac{48}{17}; b = 2\frac{14}{17}.$ $\{2\frac{14}{17}\}$

9. $8x + 176 = 176; 8x + 176 - 176 = 176 - 176; 8x = 0; \frac{8x}{8} = \frac{0}{8}; x = 0.$ {0}

10. $11y + 154 = 154; 11y + 154 - 154 = 154 - 154; 11y = 0; \frac{11y}{11} = \frac{0}{11}; y = 0.$ {0}

11. $57 - 25 = 8y + 25 - 25; 32 = 8y; \frac{32}{8} = \frac{8y}{8}; 4 = y.$ {4}

12. $127 - 37 = 45t + 37 - 37; 90 = 45t; \frac{90}{45} = \frac{45t}{45}; 2 = t.$ {2}

13. $0 + 102 = 17h - 102 + 102; 102 = 17h; \frac{102}{17} = \frac{17h}{17}; 6 = h.$ {6}

14. $0 + 57 = 19n - 57 + 57; 57 = 19n; \frac{57}{19} = \frac{19n}{19}; 3 = n.$ {3}

15. $26 - 14 = 14 + 4n - 14; 12 = 4n; \frac{12}{4} = \frac{4n}{4}; 3 = n.$ {3}

16. $36 - 17 = 17 + 6r - 17; 19 = 6r; \frac{19}{6} = \frac{6r}{6}; 3\frac{1}{6} = r.$ $\{3\frac{1}{6}\}$

17. $\frac{3}{4}p + \frac{1}{8} - \frac{1}{8} = 2 - \frac{1}{8}; \frac{3}{4}p = 1\frac{7}{8} = \frac{15}{8}; 4 \cdot \frac{3}{4}p = 4 \cdot \frac{15}{8}; 3p = \frac{15}{2}; \frac{3p}{3} = \frac{\frac{15}{2}}{3}; p = \frac{5}{2} = 2\frac{1}{2}.$ $\{2\frac{1}{2}\}$

18. $\frac{5}{8}m + \frac{3}{4} - \frac{3}{4} = 7 - \frac{3}{4}; \frac{5}{8}m = 6\frac{1}{4} = \frac{25}{4}; 8 \cdot \frac{5}{8}m = 8 \cdot \frac{25}{4}; 5m = 50; \frac{5m}{5} = \frac{50}{5}; m = 10.$ {10}

19. $\frac{2}{3}z - \frac{2}{3} + \frac{2}{3} = 12 + \frac{2}{3}; \frac{2}{3}z = 12\frac{2}{3}; 3 \cdot \frac{2}{3}z = 3 \cdot 12\frac{2}{3}; 2z = 38; \frac{2z}{2} = \frac{38}{2}; z = 19.$ {19}

20. $\frac{3}{4}k - \frac{3}{4} + \frac{3}{4} = 24 + \frac{3}{4}; \frac{3}{4}k = 24\frac{3}{4}; 4 \cdot \frac{3}{4}k = 4 \cdot 24\frac{3}{4}; 3k = 99; \frac{3k}{3} = \frac{99}{3}; k = 33.$ {33}

21. $15.5n - 129.5 = 10; 15.5n - 129.5 + 129.5 = 10 + 129.5; 15.5n = 139.5; \frac{15.5n}{15.5} = \frac{139.5}{15.5};$
$n = 9.$ {9}

22. $16.3x - 130.4 = 16.3; 16.3x - 130.4 + 130.4 = 16.3 + 130.4; 16.3x = 146.7; \frac{16.3x}{16.3} = \frac{146.7}{16.3};$
$x = 9.$ {9}

23. $22.6x = 45.2; \frac{22.6x}{22.6} = \frac{45.2}{22.6}; x = 2.$ {2}

24. $3.5a = 70; \frac{3.5a}{3.5} = \frac{70}{3.5}; a = 20.$ {20}

25. $\frac{1}{2}z = 3.4; 2(\frac{1}{2}z) = 2(3.4); z = 6.8.$ {6.8}

26. $\frac{2}{6}k = 2.9; 3(\frac{1}{3}k) = 3(2.9); k = 8.7.$ {8.7}

27. $2m = 1; \frac{2m}{2} = \frac{1}{2}; m = \frac{1}{2}$. $\{\frac{1}{2}\}$

28. $3b = 8; \frac{3b}{3} = \frac{8}{3}; b = 2\frac{2}{3}$. $\{2\frac{2}{3}\}$

B **29.** $3x + 15 - 2x = 26; x + 15 = 26; x + 15 - 15 = 26 - 15; x = 11$. $\{11\}$

30. $19 = 7y - 21 - 6y; 19 = y - 21; 19 + 21 = y - 21 + 21; 40 = y$. $\{40\}$

31. $n - \frac{2}{3} + \frac{2}{3} = \frac{5}{6} + \frac{2}{3}; n = \frac{5}{6} + \frac{4}{6}; n = \frac{9}{6} = 1\frac{1}{2}$. $\{1\frac{1}{2}\}$

32. $k - \frac{3}{4} + \frac{3}{4} = \frac{17}{20} + \frac{3}{4}; k = \frac{17}{20} + \frac{15}{20}; k = \frac{32}{20} = 1\frac{3}{5}$. $\{1\frac{3}{5}\}$

33. $.3t + .2 - .2 = 1.7 - .2; .3t = 1.5; \frac{.3t}{.3} = \frac{1.5}{.3}.; t = 5$. $\{5\}$

34. $.7t - .5 + .5 = 2.3 + .5; .7t = 2.8; \frac{.7t}{.7} = \frac{2.8}{.7}; t = 4$. $\{4\}$

35. $11.4 + 6n = 11.4; 11.4 + 6n - 11.4 = 11.4 - 11.4; 6n = 0; \frac{6n}{6} = \frac{0}{6}; n = 0$. $\{0\}$

36. $4.4 + 6x = 10.4; 4.4 + 6x - 4.4 = 10.4 - 4.4; 6x = 6; \frac{6x}{6} = \frac{6}{6}; x = 1$. $\{1\}$

37. $.5r - 2.4 = 17.4; .5r - 2.4 + 2.4 = 17.4 + 2.4; .5r = 19.8; \frac{.5r}{.5} = \frac{19.8}{.5}; r = 39.6$. $\{39.6\}$

38. $2.5h - 7.2 = 32.8; 2.5h - 7.2 + 7.2 = 32.8 + 7.2; 2.5h = 40; \frac{2.5h}{2.5} = \frac{40}{2.5}; h = 16$. $\{16\}$

39. $3\frac{1}{2}a + 3 = 10; 3\frac{1}{2}a + 3 - 3 = 10 - 3; \frac{7}{2}a = 7; 2 \cdot \frac{7}{2}a = 2 \cdot 7; 7a = 14; \frac{7a}{7} = \frac{14}{7}; a = 2$. $\{2\}$

40. $3\frac{1}{3}z + \frac{1}{3} = 7; 3\frac{1}{3}z + \frac{1}{3} - \frac{1}{3} = 7 - \frac{1}{3}; \frac{10}{3}z = 6\frac{2}{3}; 3 \cdot \frac{10}{3}z = 3 \cdot 6\frac{2}{3}; 10z = 20; \frac{10z}{10} = \frac{20}{10}$; $z = 2$. $\{2\}$

C **41.** $7z + 9 = 51; 7z + 9 - 9 = 51 - 9; 7z = 42; \frac{7z}{7} = \frac{42}{7}; z = 6$. $\{6\}$

42. $2a + 8 = 56; 2a + 8 - 8 = 56 - 8; 2a = 48; \frac{2a}{2} = \frac{48}{2}; a = 24$. $\{24\}$

43. $11 + 2b = 67; 11 + 2b - 11 = 67 - 11; 2b = 56; \frac{2b}{2} = \frac{56}{2}; b = 28$. $\{28\}$

44. $32 + 7a = 32; 32 + 7a - 32 = 32 - 32; 7a = 0; \frac{7a}{7} = \frac{0}{7}; a = 0$. $\{0\}$

45. $6n + 2 = 8; 6n + 2 - 2 = 8 - 2; 6n = 6; \frac{6n}{6} = \frac{6}{6}; n = 1$. $\{1\}$

46. $20s - 15 = 0; 20s - 15 + 15 = 0 + 15; 20s = 15; \frac{20s}{20} = \frac{15}{20}; s = \frac{3}{4}$. $\{\frac{3}{4}\}$

47. $10 = 10$; {the nos. of arith.}

48. $6 = 6$; {the nos. of arith.}

49. $6 = 4; \emptyset$

50. $7 = 8; \emptyset$

Pages 89–91 · **PROBLEMS**

A **1.** Let n = the no.; $2n + 16 = 86; 2n + 16 - 16 = 86 - 16; 2n = 70; \frac{2n}{2} = \frac{70}{2}; n = 35$, *Ans.*

2. Let n = the no.; $3n + 17 = 98; 3n + 17 - 17 = 98 - 17; 3n = 81; \frac{3n}{3} = \frac{81}{3}; n = 27$, *Ans.*

3. Let y = the no.; $6y - 5 = 67; 6y - 5 + 5 = 67 + 5; 6y = 72; \frac{6y}{6} = \frac{72}{6}; y = 12$, *Ans.*

4. Let y = the no.; $2y - 17 = 109; 2y - 17 + 17 = 109 + 17; 2y = 126; \frac{2y}{2} = \frac{126}{2}; y = 63$, *Ans.*

5. Let w = width of tennis court; then $2w + 6$ = length; $2w + 6 = 78; 2w + 6 - 6 = 78 - 6; 2w = 72; \frac{2w}{2} = \frac{72}{2}; w = 36$ ft, *Ans.*

6. Let w = width of badminton court; then $2w + 4$ = length; $2w + 4 = 44; 2w + 4 - 4 = 44 - 4; 2w = 40; \frac{2w}{2} = \frac{40}{2}; w = 20$ ft, *Ans.*

7. Let c = price per lb of sirloin; then $c - 21$ = price per lb of round; $4(c - 21) + 2c = 450$; $4c - 84 + 2c = 450$; $6c - 84 = 450$; $6c - 84 + 84 = 450 + 84$; $6c = 534$; $\frac{6c}{6} = \frac{534}{6}$; $c = 89¢$, *Ans.*

8. Let z = no. of coats sold by Pat; then $\frac{z}{2} + 19$ = no. sold by Mary; $z + \left(\frac{z}{2} + 19\right) = 157$; $\frac{3z}{2} + 19 = 157$; $\frac{3z}{2} + 19 - 19 = 157 - 19$; $\frac{3z}{2} = 138$; $2 \cdot \frac{3z}{2} = 2 \cdot 138$; $3z = 276$; $\frac{3z}{3} = \frac{276}{3}$; $z = 92$, $\frac{z}{2} + 19 = 65$, *Ans.*

9. Let n = the no.; $4n - 3 = 325$; $4n - 3 + 3 = 325 + 3$; $4n = 328$; $\frac{4n}{4} = \frac{328}{4}$; $n = 82$, *Ans.*

10. Let n = the no.; $6n + 21 = 177$; $6n + 21 - 21 = 177 - 21$; $6n = 156$; $\frac{6n}{6} = \frac{156}{6}$; $n = 26$, *Ans.*

11. Let p = the no.; $2p + 17 = 27.6$; $2p + 17 - 17 = 27.6 - 17$; $2p = 10.6$; $\frac{2p}{2} = \frac{10.6}{2}$; $p = 5.3$, *Ans.*

12. Let w = the no.; $4w - 13 = 31.8$; $4w - 13 + 13 = 31.8 + 13$; $4w = 44.8$; $\frac{4w}{4} = \frac{44.8}{4}$; $w = 11.2$, *Ans.*

13. Let n = the no.; $(3n + 7n) + 5 = 385$; $10n + 5 = 385$; $10n + 5 - 5 = 385 - 5$; $10n = 380$; $\frac{10n}{10} = \frac{380}{10}$; $n = 38$, *Ans.*

14. Let n = the no.; $(3n + 2n) - 15 = 165$; $5n - 15 = 165$; $5n - 15 + 15 = 165 + 15$; $5n = 180$; $\frac{5n}{5} = \frac{180}{5}$; $n = 36$, *Ans.*

15. Let q = the smaller no.; then $78 - q$ = the larger no.; $3q + (78 - q) = 124$; $2q + 78 = 124$; $2q + 78 - 78 = 124 - 78$; $2q = 46$; $\frac{2q}{2} = \frac{46}{2}$; $q = 23$, *Ans.*

16. Let x = the smaller no.; then $121 - x$ = the larger no.; $4x + (121 - x) = 235$; $3x + 121 = 235$; $3x + 121 - 121 = 235 - 121$; $3x = 114$; $\frac{3x}{3} = \frac{114}{3}$; $x = 38$; $121 - x = 83$, *Ans.*

17. Let n = Emma's age; then $2n$ = Bob's age, and $n + 16$ = Kent's age; $n + 2n + (n + 16) = 60$; $4n + 16 = 60$; $4n + 16 - 16 = 60 - 16$; $4n = 44$; $\frac{4n}{4} = \frac{44}{4}$; $n = 11$ yr, $2n = 22$ yr, $n + 16 = 27$ yr, *Ans.*

18. Let w = width of lot; then $2w$ = length; $P = 2l + 2w$; $222 = 2(2w) + 2(w)$; $222 = 6w$; $\frac{222}{6} = \frac{6w}{6}$; $w = 37$ ft, $2w = 74$ ft, *Ans.*

19. Let w = width of house; then $3w$ = length; $P = 2l + 2w$; $192 = 2(3w) + 2(w)$; $192 = 8w$; $\frac{192}{8} = \frac{8w}{8}$; $w = 24$ ft, $3w = 72$ ft, *Ans.*

20. Let w = width of hen yard; then $3w$ = length; $P = 2l + 2w$; $72 = 2(3w) + 2(w)$; $72 = 8w$; $\frac{72}{8} = \frac{8w}{8}$; $w = 9$ ft, $3w = 27$ ft, *Ans.*

21. Let h = wt. of hydrogen; then $8h$ = wt. of oxygen; $h + 8h = 126$; $9h = 126$; $\frac{9h}{9} = \frac{126}{9}$; $h = 14$; $8h = 112$ g, *Ans.*

22. Let d = amount first boy to get; then $2d$ = amount owner of press to get; $d + 2d = 105$; $3d = 105$; $\frac{3d}{3} = \frac{105}{3}$; $d = 35¢$, $2d = 70¢$, *Ans.*

23. Let x = cost of skirt; then $1.5x$ = cost of jacket; $x + 1.5x = 15$; $2.5x = 15$; $\frac{2.5x}{2.5} = \frac{15}{2.5}$; $x = \$6$, $1.5x = \$9$, *Ans.*

24. Let w = wt. of 1 qt of water; then $.8w$ = wt. of 1 qt of kerosene; $w + .8w = 3.6$; $1.8w = 3.6$; $\frac{1.8w}{1.8} = \frac{3.6}{1.8}$; (a) $w = 2$ lb; (b) $.8w = 1.6$ lb.

25. Let c = no. of calories in milk; then $2c$ = no. of calories in sandwich; $c + 2c = 495$; $3c = 495$; $\frac{3c}{3} = \frac{495}{3}$; (a) $c = 165$ cal; (b) $2c = 330$ cal.

26. Let w = wt. of container; then $2w$ = wt. of regulating devices; and $\frac{w}{2}$ = wt. of recording equip.; $w + 2w + \frac{w}{2} = 840$; $\frac{7}{2}w = 840$; $2 \cdot \frac{7}{2}w = 2 \cdot 840$; $7w = 1680$; $\frac{7w}{7} = \frac{1680}{7}$; $w = 240$ lb, $2w = 480$ lb, $\frac{w}{2} = 120$ lb, *Ans.*

B **27.** Let c = word capacity of computer A; then $2c$ = capacity of computer B; and $4(2c)$ = capacity of computer C; $c + 2c + 4(2c) = 22{,}000$; $11c = 22{,}000$; $\frac{11c}{11} = \frac{22{,}000}{11}$; $c = 2000$, $2c = 4000$, $4(2c) = 16{,}000$, *Ans.*

28. Let X = no. of kg of cement in mixture; then $2X$ = no. of kg of sand in mixture, and $\frac{5}{2}(2X)$ = no. of kg of gravel in mixture; $X + 2X + \frac{5}{2}(2X) = 300$; $X + 2X + 5X = 300$; $8X = 300$; $\frac{8X}{8} = \frac{300}{8}$; $X = 37\frac{1}{2}$ kg, *Ans.*

29. Let l = length of shortest side; then $2l$ = length of longest side, and $2l - 1$ = length of third side; $l + 2l + 2l - 1 = 149$; $5l - 1 = 149$; $5l - 1 + 1 = 149 + 1$; $5l = 150$; $\frac{5l}{5} = \frac{150}{5}$; $l = 30$ in, $2l = 60$ in, $2l - 1 = 59$ in, *Ans.*

30. Let n = no. of cartons produced by machine B, then $\frac{3}{2}n$ = no. produced by machine A; and $\frac{3}{2}n + 100$ = no. produced by machine C; $n + \frac{3}{2}n + \frac{3}{2}n + 100 = 5020$; $4n + 100 = 5020$; $4n + 100 - 100 = 5020 - 100$; $4n = 4920$; $\frac{4n}{4} = \frac{4920}{4}$; $n = 1230$, $\frac{3}{2}n = 1845$, $\frac{3}{2}n + 100 = 1945$, *Ans.*

31. Let n = no. of nickels; value in cents = $5n$; then $4n$ = no. of dimes; value in cents = $40n$; and $2n$ = no. of pennies; value in cents = $2n$; $5n + 40n + 2n = 470$; $47n = 470$; $\frac{47n}{47} = \frac{470}{47}$; $n = 10$, $4n = 40$, $2n = 20$, *Ans.*

32. Let x = no. of watts used by radio; then $3x$ = no. of watts used by each lamp; $45x$ = no. of watts used by heater; $3(3x) + x + 45x = 1100$; $55x = 1100$; $\frac{55x}{55} = \frac{1100}{55}$; $x = 20$ watts, *Ans.*

Pages 93–94 · WRITTEN EXERCISES

A **1.** $7v - 2v = 45 + 2v - 2v$; $5v = 45$; $\frac{5v}{5} = \frac{45}{5}$; $v = 9$. {9}

2. $9u - 6u = 6u + 39 - 6u$; $3u = 39$; $\frac{3u}{3} = \frac{39}{3}$; $u = 13$. {13}

3. $6l + 3l = 18 - 3l + 3l$; $9l = 18$; $\frac{9l}{9} = \frac{18}{9}$; $l = 2$. {2}

4. $5b + 2b = 28 - 2b + 2b$; $7b = 28$; $\frac{7b}{7} = \frac{28}{7}$; $b = 4$. {4}

5. $11x - 5x = 8 + 5x - 5x$; $6x = 8$; $\frac{6x}{6} = \frac{8}{6}$; $x = 1\frac{1}{3}$. {$1\frac{1}{3}$}

6. $4y - 2y = 8 + 2y - 2y$; $2y = 8$; $\frac{2y}{2} = \frac{8}{2}$; $y = 4$. {4}

7. $12 - 3r + 3r = 3r + 3r$; $12 = 6r$; $\frac{12}{6} = \frac{6r}{6}$; $2 = r$. {2}

8. $7h - 35 + 35 = 0 + 35; 7h = 35; \frac{7h}{7} = \frac{35}{7}; h = 5.$ {5}

9. $9x - 24 + 24 - 3x = 3x + 24 - 3x; 6x = 24; \frac{6x}{6} = \frac{24}{6}; x = 4.$ {4}

10. $5 + 2b - 2b = 7b - 2b; 5 = 5b; \frac{5}{5} = \frac{5b}{5}; 1 = b.$ {1}

11. $8a - 5a = 5a + 18 - 5a; 3a = 18; \frac{3a}{3} = \frac{18}{3}; a = 6.$ {6}

12. $4x + 18 - 4x = 10x - 4x; 18 = 6x; \frac{18}{6} = \frac{6x}{6}; 3 = x.$ {3}

13. $7y - 9 + 9 - 2y = 2y + 9 - 2y; 5y = 9; \frac{5y}{5} = \frac{9}{5}; y = 1\frac{4}{5}.$ $\{1\frac{4}{5}\}$

14. $b + 28 - b = 6b - b; 28 = 5b; \frac{28}{5} = \frac{5b}{5}; 5\frac{3}{5} = b.$ $\{5\frac{3}{5}\}$

15. $5x - 6 + 6 - 3x = 3x + 6 - 3x; 2x = 6; \frac{2x}{2} = \frac{6}{2}; x = 3.$ {3}

16. $12 - 3h + 3h - 9 = 9 + 3h - 9; 3 = 3h; \frac{3}{3} = \frac{3h}{3}; 1 = h.$ {1}

17. $5x - 5 + 5 = 13 + 5; 5x = 18; \frac{5x}{5} = \frac{18}{5}; x = 3\frac{3}{5}.$ $\{3\frac{3}{5}\}$

18. $12 - y + y = 5y + y; 12 = 6y; \frac{12}{6} = \frac{6y}{6}; 2 = y.$ {2}

B **19.** $4z + 2 - 2 - 2z = 2z + 8 - 2 - 2z; 2z = 6; \frac{2z}{2} = \frac{6}{2}; z = 3.$ {3}

20. $5w + 2 - 2 - w = w + 7 - 2 - w; 4w = 5; \frac{4w}{4} = \frac{5}{4}; w = 1\frac{1}{4}.$ $\{1\frac{1}{4}\}$

21. $7c - 7 + 7 + c = 15 - c + 7 + c; 8c = 22; \frac{8c}{8} = \frac{22}{8}; c = 2\frac{3}{4}.$ $\{2\frac{3}{4}\}$

22. $12a - 3 + 3 + 2a = 4 - 2a + 3 + 2a; 14a = 7; \frac{14a}{14} = \frac{7}{14}; a = \frac{1}{2}.$ $\{\frac{1}{2}\}$

23. $4r + 2 - 2 - 2r = 2r + 2 - 2 - 2r; 2r = 0; \frac{2r}{2} = \frac{0}{2}; r = 0.$ {0}

24. $7f - 1 + 1 - f = 29 + f + 1 - f; 6f = 30; \frac{6f}{6} = \frac{30}{6}; f = 5.$ {5}

25. $8u + 2 - 2 + 3u = 13 - 3u - 2 + 3u; 11u = 11; \frac{11u}{11} = \frac{11}{11}; u = 1.$ {1}

26. $16 + 4y - 4y + 20 = 10y - 20 - 4y + 20; 36 = 6y; \frac{36}{6} = \frac{6y}{6}; 6 = y.$ {6}

27. $19r + 4 - 4 - 14r = 19 + 14r - 4 - 14r; 5r = 15; \frac{5r}{5} = \frac{15}{5}; r = 3.$ {3}

28. $5x + 1 - 1 - 4x = 4x + 2 - 1 - 4x; x = 1.$ {1}

29. $24x - 24 + 24 - 2x = 12 + 2x + 24 - 2x; 22x = 36; \frac{22x}{22} = \frac{36}{22}; x = 1\frac{7}{11}.$ $\{1\frac{7}{11}\}$

30. $12n + 8 - 8 + 6n = 18 - 6n - 8 + 6n; 18n = 10; \frac{18n}{18} = \frac{10}{18}; n = \frac{5}{9}.$ $\{\frac{5}{9}\}$

31. $5y + 10 - 5y - 6 = 6 + 6y - 5y - 6; 4 = y.$ {4}

32. $10w + 6 - 6 - 8w = 504 + 8w - 6 - 8w; 2w = 498; \frac{2w}{2} = \frac{498}{2}; w = 249.$ {249}

C **33.** $6 + 8 - 4t = 3t; 14 - 4t = 3t; 14 - 4t + 4t = 3t + 4t; 14 = 7t; \frac{14}{7} = \frac{7t}{7}; 2 = t.$ {2}

34. $7m + 15 - 5m = 19; 2m + 15 = 19; 2m + 15 - 15 = 19 - 15; 2m = 4; \frac{2m}{2} = \frac{4}{2}; m = 2.$ {2}

35. $4x - 6 = x + 21; 4x - 6 + 6 - x = x + 21 + 6 - x; 3x = 27; \frac{3x}{3} = \frac{27}{3}; x = 9.$ {9}

36. $\frac{4}{5}a + 8 = a; \frac{4}{5}a + 8 - \frac{4}{5}a = a - \frac{4}{5}a; 8 = \frac{5}{5}a - \frac{4}{5}a; 8 = \frac{1}{5}a; 5 \cdot 8 = 5 \cdot \frac{1}{5}a; 40 = a.$ {40}

37. $3t + 4 = 3t + 6; 3t + 4 - 4 - 3t = 3t + 6 - 4 - 3t; 0 = 2.$ $\emptyset$

38. $5 - b + b - 5 = b + 5 + b - 5; 0 = 2b; \frac{0}{2} = \frac{2b}{2}; 0 = b.$ {0}

39. $8 + 18x = 6x + 12$; $8 + 18x - 8 - 6x = 6x + 12 - 8 - 6x$; $12x = 4$; $\frac{12x}{12} = \frac{4}{12}$; $x = \frac{1}{3}$. $\{\frac{1}{3}\}$

40. $7y + 3 = 16 - 6y$; $7y + 3 - 3 + 6y = 16 - 6y - 3 + 6y$; $13y = 13$; $\frac{13y}{13} = \frac{13}{13}$; $y = 1$. $\{1\}$

41. $4b + 4 + 9 = 6b - 8 + b$; $4b + 13 = 7b - 8$; $4b + 13 - 4b + 8 = 7b - 8 - 4b + 8$;
$21 = 3b$; $\frac{21}{3} = \frac{3b}{3}$; $7 = b$. $\{7\}$

42. $10y + 15 - 4y = 6 + 3y + 39$; $6y + 15 = 3y + 45$; $6y + 15 - 15 - 3y = 3y + 45 - 15 - 3y$;
$3y = 30$; $\frac{3y}{3} = \frac{30}{3}$; $y = 10$. $\{10\}$

43. $\frac{2}{5}x + \frac{2}{5} + \frac{3}{5}x + \frac{3}{5} = 14$; $x + 1 = 14$; $x + 1 - 1 = 14 - 1$; $x = 13$. $\{13\}$

44. $2t + 8 - 3 = 5 + 2t$; $2t + 5 = 5 + 2t$; $2t + 5 - 5 - 2t = 5 + 2t - 5 - 2t$; $0 = 0$;
{nos. of arith.}

Pages 94–95 · **PROBLEMS**

A **1.** Let n = the no.; $n = 2n - 1$; $n - n + 1 = 2n - 1 - n + 1$; $1 = n$; $n = 1$, *Ans.*

2. Let n = the no.; $2n + 3 = 3n - 2$; $2n + 3 - 2n + 2 = 3n - 2 - 2n + 2$; $5 = n$; $n = 5$, *Ans.*

3. Let x = the no.; $\frac{x}{2} = 3x - 15$; $\frac{x}{2} - \frac{x}{2} + 15 = 3x - 15 - \frac{x}{2} + 15$; $15 = \frac{6x}{2} - \frac{x}{2}$; $15 = \frac{5x}{2}$;
$2 \cdot 15 = 2 \cdot \frac{5x}{2}$; $30 = 5x$; $\frac{30}{5} = \frac{5x}{5}$; $x = 6$, *Ans.*

4. Let w = wt. of each block; $3w = w + 7$; $3w - w = w + 7 - w$; $2w = 7$; $\frac{2w}{2} = \frac{7}{2}$; $w = 3\frac{1}{2}$ lb, *Ans.*

5. Let E = Tom's earnings per wk.; then $3E$ = Harry's earnings, and $E + 80$ = Dick's earnings;
$3E = E + 80$; $3E - E = E + 80 - E$; $2E = 80$; $\frac{2E}{2} = \frac{80}{2}$; $E = \$40$, $3E = \$120$, $E + 80 = \$120$, *Ans.*

6. Let z = smaller no.; then $46 - z$ = larger no.; $5z = 2(46 - z) + 6$; $5z = 92 - 2z + 6$; $5z = 98 - 2z$; $5z + 2z = 98 - 2z + 2z$; $7z = 98$; $\frac{7z}{7} = \frac{98}{7}$; $z = 14$, $46 - z = 32$, *Ans.*

7. Let a = Mary's age now; then $2a$ = Bill's age now; $(a - 1) + 10 = 2a$; $a + 9 = 2a$; $a + 9 - a = 2a - a$; $9 = a$; $a = 9$ yr, $2a = 18$ yr, *Ans.*

8. Let n = smaller no.; then $73 - n$ = larger no.; $2(73 - n) = 3n - 4$; $146 - 2n = 3n - 4$;
$146 - 2n + 2n + 4 = 3n - 4 + 2n + 4$; $150 = 5n$; $\frac{150}{5} = \frac{5n}{5}$; $n = 30$, $73 - n = 43$, *Ans.*

9. Let n = the no.; $2(n - 3) = \frac{n}{2} + 18$; $2n - 6 = \frac{n}{2} + 18$; $2n - 6 + 6 - \frac{n}{2} = \frac{n}{2} + 18 + 6 - \frac{n}{2}$;
$\frac{3n}{2} = 24$; $2 \cdot \frac{3n}{2} = 2 \cdot 24$; $3n = 48$; $\frac{3n}{3} = \frac{48}{3}$; $n = 16$, *Ans.*

10. Let c = cost of standard model; then $c + 1$ = cost of deluxe model; $3c + 5(c + 1) = 6.20$;
$3c + 5c + 5 = 6.20$; $8c + 5 = 6.20$; $8c + 5 - 5 = 6.20 - 5$; $8c = 1.20$; $\frac{8c}{8} = \frac{1.20}{8}$; $c = \$.15$,
$c + 1 = \$1.15$, *Ans.*

11. Let L = length of 2nd rectangle; then $L + 2$ = length of 1st rectangle; $3(L + 2) + 7(L) = 121$;
$3L + 6 + 7L = 121$; $10L + 6 = 121$; $10L + 6 - 6 = 121 - 6$; $10L = 115$; $\frac{10L}{10} = \frac{115}{10}$; $L =$
$11\frac{1}{2}$ ft, $L + 2 = 13\frac{1}{2}$ ft, *Ans.*

12. Let w = width; then $42 - w$ = length; $2(42 - w) = 3w - 1$; $84 - 2w = 3w - 1$; $84 - 2w +$
$2w + 1 = 3w - 1 + 2w + 1$; $85 = 5w$; $\frac{85}{5} = \frac{5w}{5}$; $w = 17$ in, $42 - w = 25$ in, *Ans.*

13. Let t = no. of 7¢ stamps; value in cents $= 7t$; then $50 - t$ = no. of 4¢ stamps; value in cents $=$
$4(50 - t)$; $7t + 4(50 - t) = 215$; $7t + 200 - 4t = 215$; $3t + 200 - 200 = 215 - 200$; $3t = 15$;
$\frac{3t}{3} = \frac{15}{3}$; $t = 5$, $50 - t = 45$, *Ans.*

14. Let s = measure of a side of square; then s = measure of 1st side of triangle, $s + 2$ = measure of 2nd side of triangle, and $s + 6$ = measure of 3rd side of triangle; $4s = s + (s + 2) + (s + 6)$; $4s$
$= 3s + 8$; $4s - 3s = 3s + 8 - 3s$; $s = 8$ in; $s + 6 = 14$ in, *Ans.*

15. Let y = no. of seats in a small bus; then $y + 12$ = no. of seats in a large bus; $5y + 4(y + 12) = 336$; $5y + 4y + 48 = 336$; $9y + 48 = 336$; $9y + 48 - 48 = 336 - 48$; $9y = 288$; $\frac{9y}{9} = \frac{288}{9}$; $y = 32$; $y + 12 = 44$, *Ans.*

16. Let n = the no.; $(4n + 2n) - 10 = 4n + 260$; $6n - 10 = 4n + 260$; $6n - 10 + 10 - 4n = 4n + 260 + 10 - 4n$; $2n = 270$; $\frac{2n}{2} = \frac{270}{2}$; $n = 135$, *Ans.*

B **17.** Let x = the no.; $2\left(\frac{x}{2} + 3x\right) = \frac{1}{2}(5x) + 27$; $x + 6x = \frac{5}{2}x + 27$; $7x = \frac{5}{2}x + 27$; $7x - \frac{5}{2}x = \frac{5}{2}x + 27 - \frac{5}{2}x$; $\frac{14}{2}x - \frac{5}{2}x = 27$; $\frac{9}{2}x = 27$; $2 \cdot \frac{9}{2}x = 2 \cdot 27$; $9x = 54$; $\frac{9x}{9} = \frac{54}{9}$; $x = 6$, *Ans.*

18. Let T = temp. in Juneau; then $2T$ = temp. in Omaha; $2T - 15 = T + 15$; $2T - 15 + 15 - T = T + 15 + 15 - T$; $T = 30°$, $2T = 60°$, *Ans.*

19. Let c = cost of bat; then $c - \frac{1}{2}$ = cost of ball; $5(c - \frac{1}{2}) + 2c = 11.50$; $5c - \frac{5}{2} + 2c = 11.50$; $7c - \frac{5}{2} = 11.50$; $7c - \frac{5}{2} + \frac{5}{2} = 11.50 + \frac{5}{2}$; $7c = 11.50 + 2.50$; $7c = 14$; $\frac{7c}{7} = \frac{14}{7}$; $c = \$2$, *Ans.*

20. Let c = cost of 1 lb of onions; then $2c$ = cost of 1 lb of potatoes; and $c + 8$ = cost of 1 lb of apples; $9(2c) = 6(c + 8)$; $18c = 6c + 48$; $18c - 6c = 6c + 48 - 6c$; $12c = 48$; $\frac{12c}{12} = \frac{48}{12}$; $c = 4¢$, $2c = 8¢$, $c + 8 = 12¢$, *Ans.*

21. Let J = Joe's age; then $J - 1$ = Dave's age, and $J + 15$ = Frank's age; $J + 15 = 3(J - 1)$; $J + 15 = 3J - 3$; $J + 15 - J + 3 = 3J - 3 - J + 3$; $18 = 2J$; $\frac{18}{2} = \frac{2J}{2}$; $9 = J$; $J = 9$ yr, $J - 1 = 8$ yr, $J + 15 = 24$ yr, *Ans.*

Pages 98–100 · CHAPTER TEST

1. Reflexive **2.** Symmetric **3.** No; 2×2 not in set **4.** Yes

5. (a) Comm. prop. for add.; (b) Assoc. prop. for add.

6. (a) Comm. prop. for mult.; (b) Assoc. prop. for mult.

7. (a) Distrib. prop. (b) Mult. prop. of 1 (c) Comm. prop. for add. (d) Distrib. prop. (e) Distrib. prop.

8. $3x + 4x + 8 - 5x - 8 = 2x$

9. $(\frac{3}{7} + 9\frac{4}{7})39 = (10)39 = 390$

10. $s - 73 + 73 = 91 + 73$; $s = 164$. {164}

11. $42 - 18 = t + 18 - 18$; $24 = t$. {24}

12. $\frac{7y}{7} = \frac{28}{7}$; $y = 4$. {4}

13. $2 \cdot \frac{3}{2}v = 2 \cdot 48$; $3v = 96$; $\frac{3v}{3} = \frac{96}{3}$; $v = 32$. {32}

14. $\frac{2}{5}(10) - 4 \stackrel{?}{=} 3 - .3(10)$; $4 - 4 = 3 - 3$. Yes

15. Let t = no. of dollars Amy invested; then $3t$ = no. of dollars Ben invested; $t + 3t = 46.28$; $4t = 46.28$; $\frac{4t}{4} = \frac{46.28}{4}$; $t = \$11.57$; $3t = \$34.71$, *Ans.*

16. $3d + 12d - 10 = 5$; $15d - 10 = 5$; $15d - 10 + 10 = 5 + 10$; $15d = 15$; $\frac{15d}{15} = \frac{15}{15}$; $d = 1$. {1}

17. Let p = cost of pencil in cents; then $p + 80$ = cost of pen in cents; $p + p + 80 = 278$; $2p + 80 = 278$; $2p + 80 - 80 = 278 - 80$; $2p = 198$; $\frac{2p}{2} = \frac{198}{2}$; $p = 99¢$, $p + 80 = 179¢ = \$1.79$, *Ans.*

18. $6h + 21 - 5 = 10h - 8 + 4h$; $6h + 16 = 14h - 8$; $6h + 16 - 6h + 8 = 14h - 8 - 6h + 8$; $24 = 8h$; $\frac{24}{8} = \frac{8h}{8}$; $3 = h$; {3}

19. Let l = length of smaller rectangle; then $l - 3$ = length of larger rectangle; $9(l - 3) = 4l + 48$; $9l - 27 = 4l + 48$; $9l - 27 - 4l + 27 = 4l + 48 - 4l + 27$; $5l = 75$; $\frac{5l}{5} = \frac{75}{5}$; $l = 15$ in; $l - 3 = 12$ in, *Ans.*

Pages 100–102 · CHAPTER REVIEW

1. reflexive **2.** symmetric **3.** transitive

4. number of arithmetic **5.** element **6.** addition, multiplication

7. No; $3 \div 6$ not in set **8.** No; $4 - 6$ not in set **9.** commutative, addition

10. nm, commutative **11.** commutative **12.** associative

13. associative **14.** associative **15.** distributive, subtraction

16. distributive **17.** multiplicative **18.** multiplicative, 0

19. 0 **20.** 0, divisor

21. (a) Distrib. (b) Subst. principle, Comm. for mult. (c) Mult. prop. of 0 (d) Assoc. for mult. (e) Subst. principle (f) Mult. prop. of 1 (g) Add. identity

22. $7r + 6r + 3 - 13r - 2 = 1$ **23.** $2k + 3[5 + 2k - 2] = 2k + 15 + 6k - 6 = 8k + 9$

24. proved **25.** equivalent **26.** member

27. $0 = h - 6; 0 + 6 = h - 6 + 6; 6 = h; \{6\}$ **28.** $n+8-8=8-8; n=0; \{0\}$

29. substituting **30.** member **31.** conditions

32. divide, 5 **33.** addition, multiplication **34.** equivalent

35. $\dfrac{12k}{12} = \dfrac{3}{12}; k = \frac{1}{4}; \{\frac{1}{4}\}$ **36.** $3 \cdot 18 = 3 \cdot \frac{2}{3}m; 54 = 2m; \dfrac{54}{2} = \dfrac{2m}{2}; 27 = m; \{27\}$

37. $6t = 12; \dfrac{6t}{6} = \dfrac{12}{6}; t = 2. \{2\}$ **38.** $y + 1 - 1 = \frac{4}{3} - 1; y = \frac{1}{3}. \{\frac{1}{3}\}$

39. $6n$, $7n$

40. Let n = original no.; $3n - \dfrac{n}{2} = 10; \dfrac{6n}{2} - \dfrac{n}{2} = 10; \dfrac{5n}{2} = 10; 2 \cdot \dfrac{5n}{2} = 2 \cdot 10; 5n = 20; \dfrac{5n}{5} = \dfrac{20}{5};$ $n = 4$, *Ans.*

41. additions, subtractions

42. $27 = 8a - 5; 27 + 5 = 8a - 5 + 5; 32 = 8a; \dfrac{32}{8} = \dfrac{8a}{8}; 4 = a. \{4\}$

43. $12 + 12b = 16\frac{4}{5}; 12 + 12b - 12 = 16\frac{4}{5} - 12; 12b = 4\frac{4}{5}; \dfrac{12b}{12} = \dfrac{4\frac{4}{5}}{12} = \dfrac{\frac{24}{5}}{12}; b = \frac{2}{5}. \{\frac{2}{5}\}$

44. Let a = daughter's age; then $3a - 2$ = father's age; $a + 3a - 2 = 50; 4a - 2 = 50; 4a - 2 + 2 = 50 + 2; 4a = 52; \dfrac{4a}{4} = \dfrac{52}{4}; a = 13$ yr, $3a - 2 = 37$ yr, *Ans.*

45. one

46. $2n + 7 = 6n - 3 + 6; 2n + 7 = 6n + 3; 2n + 7 - 2n - 3 = 6n + 3 - 2n - 3;$ $4 = 4n; \dfrac{4}{4} = \dfrac{4n}{4}; 1 = n. \{1\}$

47. $6r - \frac{8}{3} = r + 3r + 2; 6r - \frac{8}{3} = 4r + 2; 6r - \frac{8}{3} + \frac{8}{3} - 4r = 4r + 2 + \frac{8}{3} - 4r; 2r = \frac{6}{3} + \frac{8}{3};$ $2r = \frac{14}{3}; \dfrac{2r}{2} = \dfrac{\frac{14}{3}}{2}; r = \frac{7}{3}. \{\frac{7}{3}\}$

48. Let L = original length; then $L - 4$ = length after cutting; $6(L - 4) = \frac{3}{4}(6L); 6L - 24 = \frac{9}{2}L;$ $6L - 24 + 24 - \frac{9}{2}L = \frac{9}{2}L + 24 - \frac{9}{2}L; \frac{12}{2}L - \frac{9}{2}L = 24; \frac{3}{2}L = 24; 2 \cdot \frac{3}{2}L = 2 \cdot 24; 3L = 48; \dfrac{3L}{3} = \dfrac{48}{3}; L = 16$ in, original width $= 6$ in, *Ans.*

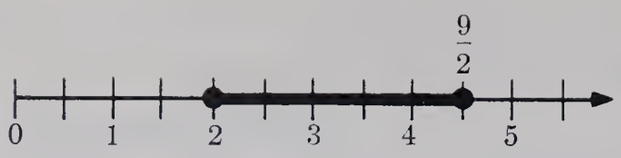
49. Let x = no. sold at 15¢ each; then $200 - x$ = no. sold at 10¢ each; $15x + 10(200 - x) = 2375;$ $15x + 2000 - 10x = 2375; 5x + 2000 = 2375; 5x + 2000 - 2000 = 2375 - 2000; 5x = 375; \dfrac{5x}{5} = \dfrac{375}{5}; x = 75, 200 - x = 125$, *Ans.*

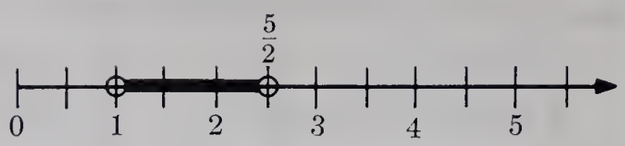
Pages 102–104 · CUMULATIVE REVIEW

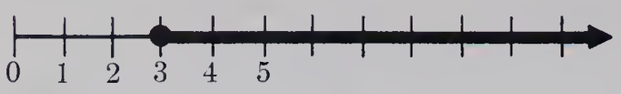
1. Inf. **2.** Finite **3.** Finite **4.** Inf. **5.** {Kansas, Kentucky}

6. {nos. between 0 and 48 which are each 1 less than a multiple of 6}

7. {The number 1 and whole nos. between 1 and 82 which are powers of 3}

8. 

9. 

10. 

11. $1 \times 2 = 2$

12. $3[9 - 2(2.2)] = 3[9 - 4.4] = 3[4.6] = 13.8$ **13.** $\{0, 1, 2, 3, 6, 9, 10, 12, 20\}$

14. $\{3, 9, 12\}$ **15.** $\{2\}$ **16.** Trans. *or* Subst. principle **17.** Distrib.

18. Symmetric **19.** Comm. for mult.

20. (a) Distrib. (b) Comm. for add. (c) Assoc. for add. (d) Distrib. (e) Subst. principle (f) Mult. prop. for 0 (g) Add. identity

21. $7n + 3[4n + 10 - 5n - 3] = 7n + 12n + 30 - 15n - 9 = 4n + 21$

22. $(\frac{11}{9} - \frac{2}{9})(d - 4) = (1)(d - 4) = d - 4$

23.

24.

25. Closed **26.** Not closed; $3 - 4$ not in set

27. $4(3\frac{1}{2} + 4) - 3 \cdot 0(1 + 3\frac{1}{2}) = 4(7\frac{1}{2}) - 3 \cdot 0(4\frac{1}{2}) = 30 - 0 = 30$

28. $\dfrac{2 \cdot 3\frac{1}{2} + 4}{2 \cdot 3\frac{1}{2} - 4} = \dfrac{7 + 4}{7 - 4} = \dfrac{11}{3} = 3\frac{2}{3}$ **29.** $\dfrac{3 \cdot 1^4 + 3 \cdot 4^2 - 2}{4(3\frac{1}{2})^2} = \dfrac{3 + 48 - 2}{4(\frac{49}{4})} = \dfrac{49}{49} = 1$

30. $16(4 - 3\frac{1}{2})^3 = 16(\frac{1}{2})^3 = 16 \cdot \frac{1}{8} = 2$ **31.** $2n - 3$ **32.** $\dfrac{a + b}{5}$ **33.** $s + 5$ **34.** $50 - x$

35. $(R + r)^2$ **36.** $2(R^2 + r^2)$

37.
$2 \cdot 0 - 1 = 5$; F
$2 \cdot 1 - 1 = 5$; F
$2 \cdot 2 - 1 = 5$; F
$2 \cdot 3 - 1 = 5$; T
$\{3\}$

38.
$2(0 - 1) = 5$; $0 - 1$ not defined; F
$2(1 - 1) = 5$; F
$2(2 - 1) = 5$; F
$2(3 - 1) = 5$; F
$\emptyset$

39.
$3 \cdot 0 > 0$; F
$3 \cdot 1 > 0$; T
$3 \cdot 2 > 0$; T
$3 \cdot 3 > 0$; T
$\{1, 2, 3\}$

40.
$4 \cdot 0 + 1 \geq 6$; F
$4 \cdot 1 + 1 \geq 6$; F
$4 \cdot 2 + 1 \geq 6$; T
$4 \cdot 3 + 1 \geq 6$; T
$\{2, 3\}$

41.
$0(5 \cdot 0) = 0 \cdot 5 \cdot 0 \cdot 0$; T
$1(5 \cdot 1) = 1 \cdot 5 \cdot 1 \cdot 1$; T
$2(5 \cdot 2) = 2 \cdot 5 \cdot 2 \cdot 2$; F
$3(5 \cdot 3) = 3 \cdot 5 \cdot 3 \cdot 3$; F
$\{0, 1\}$

42.
$0(0 - 1) = 0 \times 0 - 0$; T
$1(1 - 1) = 1 \times 1 - 1$; T
$2(2 - 1) = 2 \times 2 - 2$; T
$3(3 - 1) = 3 \times 3 - 3$; T
$\{0, 1, 2, 3\}$

43. a. $t + 5$; b. $t \in \{0, 1, 2, 3, \ldots\}$

44. First side $= a$; second side $= 2a$; sum of first and second sides $= 3a$; perimeter $= 27$; $\therefore$ third side $= 27 - 3a$

45. $8m - 13 + 13 - 3m = 67 + 3m + 13 - 3m$; $5m = 80$; $\dfrac{5m}{5} = \dfrac{80}{5}$; $m = 16$. $\{16\}$

46. $6s - 4 = 26 + 2s$; $6s - 4 + 4 - 2s = 26 + 2s + 4 - 2s$; $4s = 30$; $\dfrac{4s}{4} = \dfrac{30}{4}$; $s = 7\frac{1}{2}$. $\{7\frac{1}{2}\}$

47. $\frac{15}{4}y - 3 = 13 + \frac{6}{4}y + 2$; $\frac{15}{4}y - 3 = 15 + \frac{6}{4}y$; $\frac{15}{4}y - 3 + 3 - \frac{6}{4}y = 15 + \frac{6}{4}y + 3 - \frac{6}{4}y$;
$\frac{9}{4}y = 18$; $4 \cdot \frac{9}{4}y = 4 \cdot 18$; $9y = 72$; $\dfrac{9y}{9} = \dfrac{72}{9}$; $y = 8$. $\{8\}$

48. $\frac{7}{3}w + 1 = \frac{13}{3}w - 1$; $\frac{7}{3}w + 1 - \frac{7}{3}w + 1 = \frac{13}{3}w - 1 - \frac{7}{3}w + 1$; $2 = 2w$; $\dfrac{2}{2} = \dfrac{2w}{2}$; $w = 1$. $\{1\}$

49. $6t + 15 - 7 = 9t - 4$; $6t + 8 = 9t - 4$; $6t + 8 - 6t + 4 = 9t - 4 - 6t + 4$; $12 = 3t$;
$\dfrac{12}{3} = \dfrac{3t}{3}$; $4 = t$. $\{4\}$

50. $12 + 6k - 3 = 6k + 2$; $9 + 6k = 6k + 2$; $9 + 6k - 6k - 2 = 6k + 2 - 6k - 2$; $7 = 0$. $\emptyset$

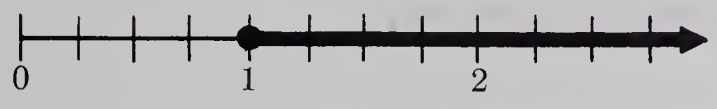
51. Let $n =$ the *no.*; $2n - \frac{2}{3}n = 68$; $\frac{6}{3}n - \frac{2}{3}n = 68$; $\frac{4}{3}n = 68$; $3 \cdot \frac{4}{3}n = 3 \cdot 68$; $4n = 204$; $\dfrac{4n}{4} = \dfrac{204}{4}$;
$n = 51$, *Ans.*

52. Let n = the smaller no.; then $20 - n$ = the larger no.; $4(20 - n) = 5n - 1$; $80 - 4n = 5n - 1$; $80 - 4n + 4n + 1 = 5n - 1 + 4n + 1$; $81 = 9n$; $\frac{81}{9} = \frac{9n}{9}$; $9 = n$, $20 - n = 11$, *Ans.*

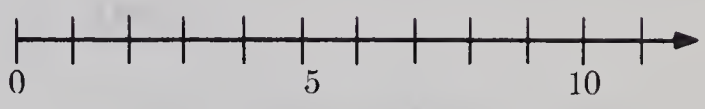
53. Let c = cost per kerchief; then $c + 50$ = cost per tie; $3c + 2(c + 50) = 595$; $3c + 2c + 100 = 595$; $5c + 100 = 595$; $5c + 100 - 100 = 595 - 100$; $5c = 495$; $\frac{5c}{5} = \frac{495}{5}$; $c = 99¢$; $c + 50 = 149¢ = \$1.49$, *Ans.*

Pages 106–107 · EXTRA FOR EXPERTS

1. {nos. of arith. ≥ 1};

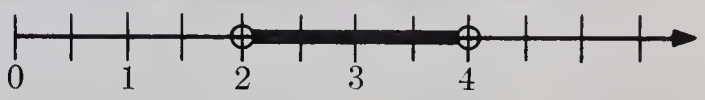


2. $\emptyset$;

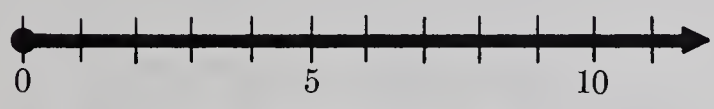


3. {the nos. of arith.};

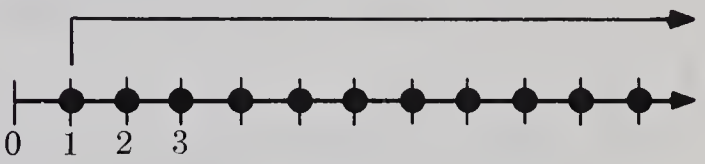
4. {the nos. of arith. between 2 and 4};



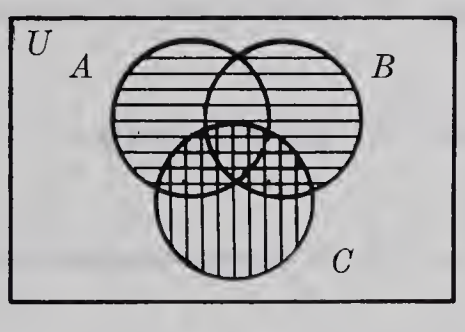
5. {the nos. of arith.};

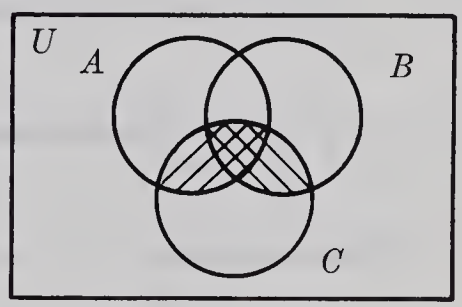


6. {1, 2, 3, . . .};

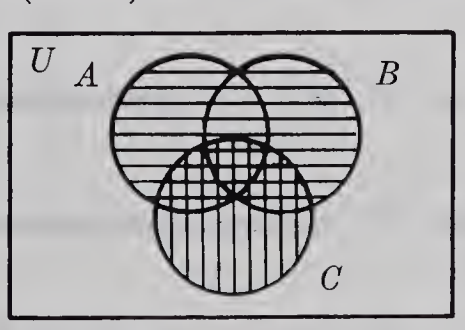


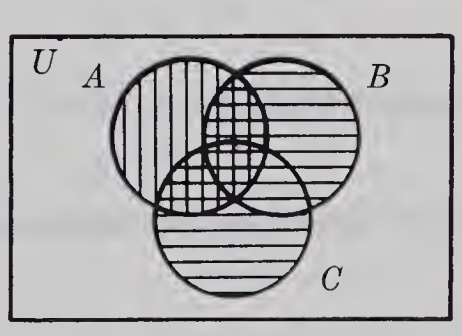
7. $C \cap (B \cup A) \stackrel{?}{=} (C \cap B) \cup (C \cap A)$



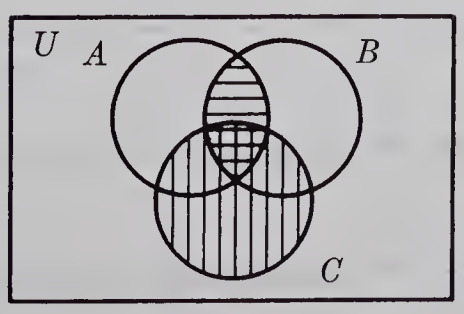


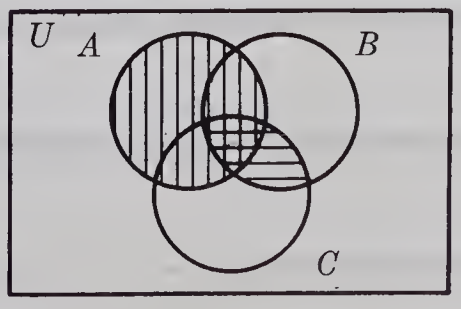
8. $C \cup (B \cup A) \stackrel{?}{=} (C \cup B) \cup A$



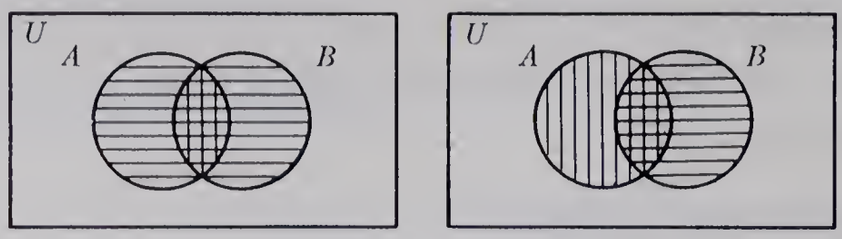


9. $C \cap (B \cap A) \stackrel{?}{=} (C \cap B) \cap A$





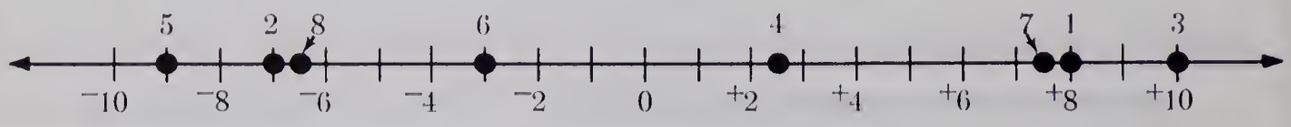
10. $(A \cup B) \cap (A \cap B) \stackrel{?}{=} A \cap B$



CHAPTER 4. The Negative Numbers

Page 113 · **WRITTEN EXERCISES**

1-8.



9. $^{+}10$ **10.** $^{+}5$ **11.** $^{-}3$ **12.** $^{-}5$

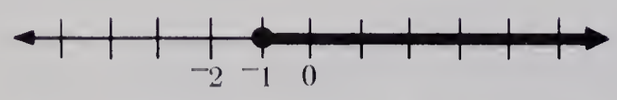
Pages 114–115 · **WRITTEN EXERCISES**

A

1. $\{^{-}4, ^{-}6\}$ **2.** $\{^{-}4, ^{-}1, 0, ^{+}2, ^{+}4\}$ **3.** $\{^{-}1, 0, ^{+}2, ^{+}4\}$ **4.** $\{^{-}4, ^{-}6\}$
5. $\{^{-}6, ^{-}4, ^{-}1, ^{+}2, ^{+}4\}$ **6.** $\{0, ^{+}2\}$ **7.** $\{^{-}6, ^{-}4\}$ **8.** $\{^{+}2, ^{+}4\}$
9. $\{^{-}6, ^{-}4, ^{-}1, ^{+}2, ^{+}4\}$ **10.** $\emptyset$ **11.** $\{0, ^{+}2, ^{+}4\}$ **12.** $\{^{-}6, ^{-}4, ^{-}1\}$

B

13.



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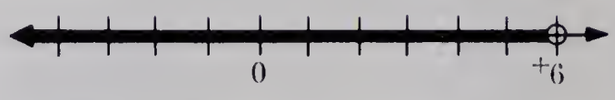

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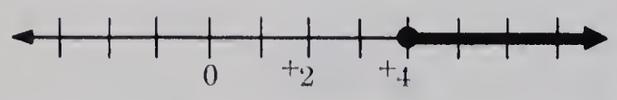

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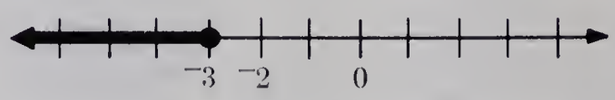

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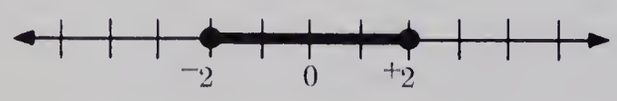

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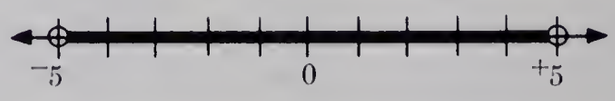
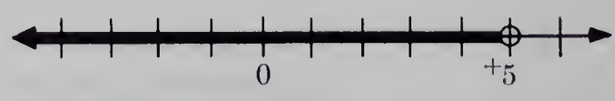


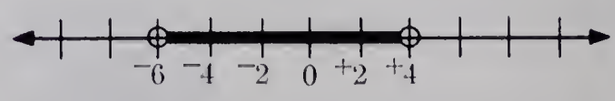
19.

20.



21.

22.



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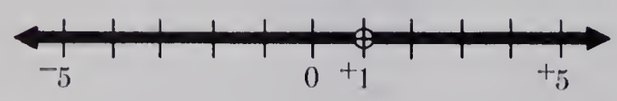

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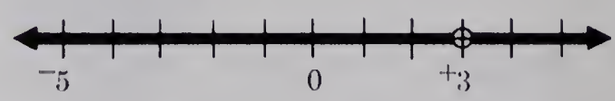


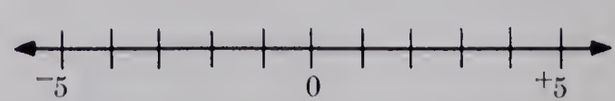
25.

26.

C

27.

28.



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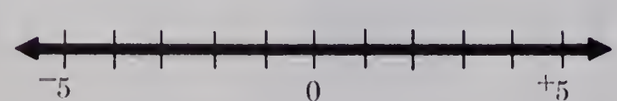

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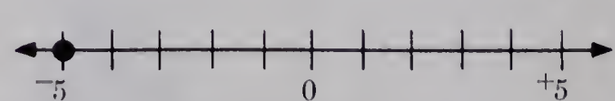
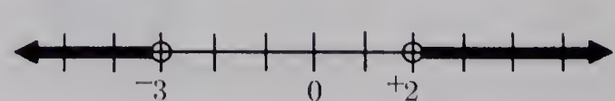


31.

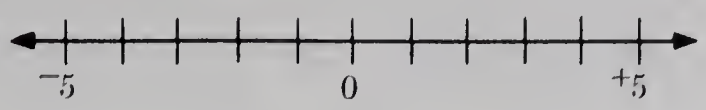
32.



33.

34.

35.



Pages 115–116 · **PROBLEMS**

1. Let a = Phil's assets in dollars; $a > {}^{+}10{,}000$; $a \in \{\text{fractional nos.} > {}^{+}10{,}000\}$
2. Let x = no. of dollars in Mary's bank account (let us agree to *whole* dollars); $0 \leq x < {}^{+}50$; $x \in \{0, {}^{+}1, {}^{+}2, \ldots, {}^{+}49\}$
3. Let R = no. of Ron's home runs; $R > {}^{+}10$; $R \in \{{}^{+}11, {}^{+}12, {}^{+}13, \ldots\}$
4. Let T = winter temp. at North Pole (F°); $T < -10$; $T \in \{\text{negative nos.} < -10\}$
5. Let A = altitude of earth's land surface; ${}^{-}1292 \leq A \leq {}^{+}29{,}028$; $A \in \{\text{nos.} \geq {}^{-}1292 \text{ and } \leq {}^{+}29{,}028\}$
6. Let L = latitude in the equatorial zone; ${}^{-}23\frac{1}{2} < L < {}^{+}23\frac{1}{2}$; $L \in \{\text{nos.} > {}^{-}23\frac{1}{2} \text{ and } < {}^{+}23\frac{1}{2}\}$
7. Let c = Marjorie's change in wt.; ${}^{-}4 \leq c \leq {}^{+}4$; $c \in \{\text{nos.} \geq {}^{-}4 \text{ and } \leq {}^{+}4\}$
8. Let c = change in price of stock to nearest $\frac{1}{8}$ dollar; ${}^{-}10 \leq c \leq {}^{+}10$; $c \in \{{}^{-}10, {}^{-}9\frac{7}{8}, {}^{-}9\frac{3}{4}, \ldots, {}^{+}9\frac{7}{8}, {}^{+}10\}$
9. Let T = temp. in Death Valley to nearest $\frac{1}{10}$ degree (C); ${}^{-}9.4 \leq T \leq {}^{+}56.7$; $T \in \{{}^{-}9.4, {}^{-}9.3, {}^{-}9.2, \ldots, {}^{+}56.7\}$
10. Let R = Roger's score in whole points; ${}^{-}10 \leq R \leq {}^{+}15$; $R \in \{{}^{-}10, {}^{-}9, {}^{-}8, \ldots, {}^{+}15\}$

Pages 118–119 · **WRITTEN EXERCISES**

A

1. ${}^{+}58 + {}^{+}22 = {}^{+}80$
2. ${}^{+}25 + {}^{+}16 = {}^{+}41$
3. ${}^{-}43 + {}^{-}46 = {}^{-}89$
4. ${}^{-}34 + {}^{-}86 = {}^{-}120$
5. ${}^{+}1° + 0° = {}^{+}1°$
6. ${}^{+}5° + {}^{-}15° = {}^{-}10°$
7. ${}^{-}1.4 + {}^{+}1.4 = 0$
8. ${}^{-}44 + {}^{+}44 = 0$
9. ${}^{-}3 + {}^{-}20 = {}^{-}23$
10. ${}^{+}8 + {}^{-}58 = {}^{-}50$
11. ${}^{+}\frac{5}{2} + {}^{-}\frac{5}{2} = 0$
12. ${}^{+}6 + {}^{-}6 = 0$
13. ${}^{-}7 + 0 = {}^{-}7$
14. ${}^{+}5 + 0 = {}^{+}5$
15. ${}^{+}4 + ({}^{+}54 + {}^{+}22) = {}^{+}4 + {}^{+}76 = {}^{+}80$
16. ${}^{+}3 + ({}^{+}22 + {}^{+}16) = {}^{+}3 + {}^{+}38 = {}^{+}41$
17. $({}^{-}43 + {}^{-}25) + {}^{-}21 = {}^{-}68 + {}^{-}21 = {}^{-}89$
18. $({}^{-}34 + {}^{-}24) + {}^{-}62 = {}^{-}58 + {}^{-}62 = {}^{-}120$
19. ${}^{-}7° + ({}^{+}8° + 0°) = {}^{-}7° + {}^{+}8° = {}^{+}1°$
20. $({}^{+}5° + 0°) + {}^{-}15° = {}^{+}5° + {}^{-}15° = {}^{-}10°$
21. $({}^{-}1.4 + {}^{-}3.0) + {}^{+}4.4 = {}^{-}4.4 + {}^{+}4.4 = 0$
22. ${}^{-}14 + ({}^{-}30 + {}^{+}44) = {}^{-}14 + {}^{+}14 = 0$
23. ${}^{+}.8 + ({}^{-}3.8 + {}^{-}20) = {}^{+}.8 + {}^{-}23.8 = {}^{-}23$
24. $({}^{+}8 + {}^{-}38) + {}^{-}20 = {}^{-}30 + {}^{-}20 = {}^{-}50$
25. $({}^{+}\frac{5}{2} + {}^{+}\frac{1}{2}) + {}^{-}3 = {}^{+}3 + {}^{-}3 = 0$
26. $({}^{+}6 + {}^{-}\frac{14}{3}) + {}^{-}\frac{4}{3} = {}^{+}\frac{4}{3} + {}^{-}\frac{4}{3} = 0$
27. $(({}^{-}3 + {}^{-}4) + {}^{+}6) + {}^{-}6 = ({}^{-}7 + {}^{+}6) + {}^{-}6 = {}^{-}1 + {}^{-}6 = {}^{-}7$
28. $(({}^{+}8 + {}^{-}3) + {}^{-}14) + {}^{+}14 = ({}^{+}5 + {}^{-}14) + {}^{+}14 = {}^{-}9 + {}^{+}14 = {}^{+}5$

B

29. ${}^{+}1 + {}^{+}8 = {}^{+}9$. $\{{}^{+}8\}$
30. ${}^{+}4 + {}^{+}3 = {}^{+}7$. $\{{}^{+}3\}$
31. ${}^{-}2 + {}^{-}3 = {}^{-}5$. $\{{}^{-}3\}$
32. ${}^{-}5 + {}^{-}1 = {}^{-}6$. $\{{}^{-}5\}$
33. ${}^{-}13 + {}^{+}3 = {}^{-}10$. $\{{}^{-}13\}$
34. ${}^{-}6 + {}^{+}14 = {}^{+}8$. $\{{}^{+}14\}$
35. ${}^{-}9 + {}^{-}5 = {}^{-}14$. $\{{}^{-}9\}$
36. ${}^{+}12 = {}^{+}18 + {}^{-}6$. $\{{}^{+}18\}$
37. ${}^{-}100 = {}^{+}9 + {}^{-}109$. $\{{}^{-}109\}$
38. ${}^{+}29 + {}^{-}4 = {}^{+}25$. $\{{}^{-}4\}$
39. ${}^{+}3 + 0 = {}^{+}3$. $\{0\}$
40. $0 + {}^{-}5 = {}^{-}5$. $\{0\}$
41. ${}^{+}6 + {}^{+}5 = {}^{+}11$. $\{{}^{+}6\}$
42. ${}^{+}3 + {}^{+}5 = {}^{+}8$. $\{{}^{+}5\}$
43. ${}^{-}6 + {}^{+}5 = {}^{-}1$. $\{{}^{+}5\}$
44. ${}^{+}1 + {}^{-}4 = {}^{-}3$. $\{{}^{+}1\}$
45. $0 + 0 = 0$. $\{0\}$
46. ${}^{+}3 + {}^{+}3 = {}^{+}6$. $\{{}^{+}3\}$

Pages 119–120 · **PROBLEMS**

A

1. $({}^{+}200) + ({}^{+}45) = {}^{+}245$ mph
2. $({}^{+}8000) + ({}^{-}2000) = {}^{+}6000$ ft
3. (a) ${}^{-}7, {}^{-}3, {}^{-}5$; (b) ${}^{-}7$ to ${}^{-}3 = {}^{+}4$ fathoms; ${}^{-}3$ to ${}^{-}5 = {}^{-}2$ fathoms
4. $({}^{-}24000) + ({}^{-}7000) + ({}^{-}8000) = {}^{-}39000$ ft
5. $({}^{-}3) + ({}^{-}2) + ({}^{-}3) + ({}^{+}\frac{1}{2}) = {}^{-}7\frac{1}{2}°$
6. $({}^{-}3.56) + ({}^{-}2.34) + ({}^{-}34.43) + ({}^{-}23.25) + ({}^{+}25.00) + ({}^{+}35.00) = {}^{-}\$3.58$
7. $({}^{-}2) + ({}^{-}3) + ({}^{-}10) = {}^{-}15$ *or* 15 floors down
8. $({}^{+}2) + ({}^{+}\frac{1}{8}) + ({}^{+}\frac{1}{8}) + ({}^{+}\frac{1}{8}) + (1\frac{1}{2}) = {}^{+}\$3\frac{7}{8}$
9. $({}^{+}\frac{1}{8}) + ({}^{+}\frac{1}{4}) + ({}^{+}\frac{1}{8}) + ({}^{+}1\frac{1}{8}) + ({}^{+}\frac{3}{4}) = {}^{+}\$2\frac{3}{8}$
10. $({}^{-}\frac{1}{2}) + ({}^{-}\frac{1}{8}) + ({}^{-}\frac{1}{8}) = {}^{-}\$\frac{3}{4}$
11. $({}^{-}1\frac{1}{8}) + ({}^{-}\frac{3}{8}) + (0) + ({}^{-}1\frac{3}{4}) + ({}^{+}2\frac{1}{8}) = {}^{-}\$1\frac{1}{8}$
12. $({}^{-}20) + ({}^{-}15) + ({}^{-}15) + ({}^{-}5) + ({}^{-}5) + ({}^{-}5) + ({}^{-}3) + ({}^{-}3) + ({}^{-}3) + ({}^{-}3) + ({}^{-}3) + ({}^{-}3) + ({}^{-}3) + ({}^{-}3) = {}^{-}89$ M

Pages 122–123 · **WRITTEN EXERCISES**

A **1.** $4 + (-6) = -2$ **2.** $3 + (6) = 9$ **3.** $-5 + (-4) = -9$ **4.** $3 + (-8) = -5$
5. $-.7 + (-5) = -5.7$ **6.** $-.9 + (5) = 4.1$ **7.** $5 + (-1) = 4$
8. $-4 + 6 = 2$ **9.** -4.4 **10.** -6.4 **11.** $5\frac{3}{8}$
12. $-5\frac{20}{30} + 7\frac{18}{30} + (-2\frac{25}{30}) = -\frac{27}{30} = -\frac{9}{10}$

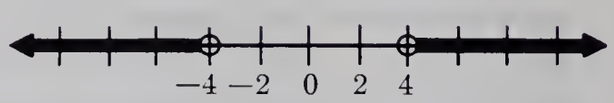
B **13.** $-(-2) = 2$. $\{-2\}$ **14.** $-3 = -(3)$. $\{3\}$ **15.** $-(0) = 0$. $\{0\}$ **16.** $-(-1) = 1$. $\{-1\}$
17. $-2 = -(2)$. $\{2\}$ **18.** $-(-3) = 3$. $\{-3\}$ **19.** $(3) + (-2) = 1$. $\{3\}$
20. $(-2) + 1 = -1$. $\{-2\}$ **21.** $-(0) < 1; -(1) < 1; -(2) < 1; -(3) < 1$. $\{0, 1, 2, 3\}$
22. $-(-3) > 1; -(-2) > 1$. $\{-3, -2\}$ **23.** $-(2) \leq -2; -(3) \leq -2$. $\{2, 3\}$
24. $-(1) \leq -1; -(2) \leq -1; -(3) \leq -1$. $\{1, 2, 3\}$
25. $0 < -(-2) < 3; 0 < -(-1) < 3$. $\{-2, -1\}$ **26.** $\emptyset$
27. $-2 \leq -(-2) \leq 2; -2 \leq -(-1) \leq 2; -2 \leq -(0) \leq 2; -2 \leq -(1) \leq 2; -2 \leq -(2) \leq 2$. $\{-2, -1, 0, 1, 2\}$
28. $\frac{5}{3} + (-\frac{1}{5}) + .25 = \frac{5}{3} + (-\frac{1}{5}) + \frac{1}{4} = \frac{100}{60} + (-\frac{12}{60}) + \frac{15}{60} = \frac{103}{60}$
29. $-\frac{5}{3} + \frac{1}{5} + (-.25) = -\frac{5}{3} + \frac{1}{5} + (-\frac{1}{4}) = -\frac{100}{60} + \frac{12}{60} + (-\frac{15}{60}) = -\frac{103}{60}$
30. $-[\frac{1}{5} + .25 + (-1.5)] = -[\frac{1}{5} + \frac{1}{4} + (-\frac{3}{2})] = -[\frac{4}{20} + \frac{5}{20} + (-\frac{30}{20})] = -(-\frac{21}{20}) = \frac{21}{20}$
31. $-\frac{1}{5} + (-.25) + 1.5 = -\frac{1}{5} + (-\frac{1}{4}) + \frac{3}{2} = -\frac{4}{20} + (-\frac{5}{20}) + \frac{30}{20} = \frac{21}{20}$
32. $\frac{5}{3} + \frac{1}{5} + (-.25) + (-1.5) = \frac{5}{3} + \frac{1}{5} + (-\frac{1}{4}) + (-\frac{3}{2}) = \frac{100}{60} + \frac{12}{60} + (-\frac{15}{60}) + (-\frac{90}{60}) = \frac{7}{60}$
33. $-\frac{5}{3} + \frac{1}{5} + .25 + 1.5 = -\frac{5}{3} + \frac{1}{5} + \frac{1}{4} + \frac{3}{2} = -\frac{100}{60} + \frac{12}{60} + \frac{15}{60} + \frac{90}{60} = \frac{17}{60}$
34. $-[-\frac{5}{3} + .25 + (-1.5)] = -[-\frac{5}{3} + \frac{1}{4} + (-\frac{3}{2})] = -[-\frac{20}{12} + \frac{3}{12} + (-\frac{18}{12})] = -(-\frac{35}{12}) = \frac{35}{12}$
35. $\frac{5}{3} + -[(-.25) + 1.5] = \frac{5}{3} + -[(-\frac{1}{4}) + \frac{3}{2}] = \frac{20}{12} + -[(-\frac{3}{12}) + \frac{18}{12}] = \frac{20}{12} + -(\frac{15}{12}) = \frac{5}{12}$
36. $\frac{1}{5} + (-\frac{5}{3}) + (-1.5) = \frac{1}{5} + (-\frac{5}{3}) + (-\frac{3}{2}) = \frac{6}{30} + (-\frac{50}{30}) + (-\frac{45}{30}) = -\frac{89}{30}$
37. $\frac{1}{5} + [-(\frac{5}{3} + 1.5)] = \frac{1}{5} + [-(\frac{5}{3} + \frac{3}{2})] = \frac{6}{30} + [-(\frac{50}{30} + \frac{45}{30})] = \frac{6}{30} + (-\frac{95}{30}) = -\frac{89}{30}$
38. $[\frac{5}{3} + \frac{1}{5}] + [-(.25 + 1.5)] = [\frac{5}{3} + \frac{1}{5}] + [-(\frac{1}{4} + \frac{3}{2})] = [\frac{100}{60} + \frac{12}{60}] + [-(\frac{15}{60} + \frac{90}{60})] = (\frac{112}{60}) + [-(\frac{105}{60})] = \frac{7}{60}$
39. $[-\frac{5}{3} + \frac{1}{5}] + [.25 + 1.5] = [-\frac{5}{3} + \frac{1}{5}] + [\frac{1}{4} + \frac{3}{2}] = [-\frac{100}{60} + \frac{12}{60}] + [\frac{15}{60} + \frac{90}{60}] = [-\frac{88}{60}] + [\frac{105}{60}] = \frac{17}{60}$

Page 124 · **WRITTEN EXERCISES**

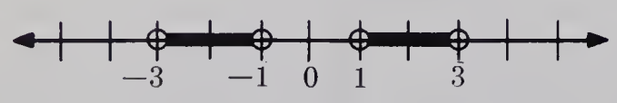
A **1.** $\{0\}$ **2.** $\{-4, 4\}$ **3.** $\{-5, 5\}$ **4.** $\emptyset$

5.

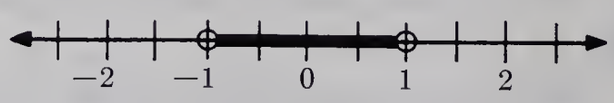
6.



7.



8.



9. $\emptyset$ **10.** $\{0\}$ **11.** $\emptyset$ **12.** $\emptyset$
13. {the nonnegative nos.} **14.** {nos. ≤ 0} **15.** {all nos. other than 0} **16.** {the directed nos.}

B **17.** $|2.3| + |-16| + |-5\frac{1}{2}| = 2.3 + 16 + 5.5 = 23.8$
18. $-(2.3 + |-5\frac{1}{2}|) + 3\frac{2}{5} = -(2.3 + 5.5) + 3.4 = -(7.8) + 3.4 = -4.4$
19. $-2.3 + (-16) + |-5\frac{1}{2} + 3\frac{2}{5}| = -2.3 + (-16) + |-5.5 + 3.4| = -2.3 + (-16) + 2.1 = -16.2$
20. $2.3 + (-[-16]) + |-5\frac{1}{2}| + 3\frac{2}{5} = 2.3 + 16 + 5.5 + 3.4 = 27.2$
21. $|2.3|\,|-16|\,|-5\frac{1}{2}| = (2.3)(16)(5.5) = 202.4$ **22.** $|-16|\,|-5\frac{1}{2}|\,|3\frac{2}{5}| = (16)(5.5)(3.4) = 299.2$
23. $2.3 + 3\frac{2}{5} + |-5\frac{1}{2}| = 2.3 + 3.4 + 5.5 = 11.2$ **24.** $|-16 + (-5\frac{1}{2})| + 3\frac{2}{5} = 21.5 + 3.4 = 24.9$
25. $2.3 + |-16 + (-5\frac{1}{2})| = 2.3 + 21.5 = 23.8$

Page 127 · **WRITTEN EXERCISES**

A **1.** $(6 + 10) + (-3 - 9) = 16 + (-12) = 4$ **2.** $(8 + 9) + (-7 - 1) = 17 + (-8) = 9$
3. $(54\frac{1}{2} + 16) + (-11\frac{3}{4} - 4\frac{1}{3}) = 70\frac{1}{2} + (-11\frac{9}{12} - 4\frac{4}{12}) = 70\frac{6}{12} + (-16\frac{1}{12}) = 54\frac{5}{12}$

4. $43\frac{1}{3} + (-45\frac{3}{5} - 54\frac{2}{3} - 12\frac{2}{5}) = 43\frac{1}{3} + (-45\frac{9}{15} - 54\frac{10}{15} - 12\frac{6}{15}) = 43\frac{1}{3} + (-112\frac{10}{15}) = 43\frac{1}{3} + (-112\frac{2}{3}) = -69\frac{1}{3}$

5. $(24 + 17) + (-18 - 32) = 41 + (-50) = -9$

6. $(22 + 35) + (-14 - 11) = 57 + (-25) = 32$

7. $13 + (-12) = 1$ **8.** $9 + (-11) = -2$ **9.** $24 + (-60) = -36$ **10.** $66 + (-66) = 0$

11. $5 + (-5) = 0$ **12.** $26 + (-21) = 5$ **13.** $16 + (-22) = -6$ **14.** $43 + (-8) = 35$

15. $11 + (-3\frac{1}{2} - 3\frac{2}{3}) = 11 + (-3\frac{3}{6} - 3\frac{4}{6}) = 11 + (-7\frac{1}{6}) = 3\frac{5}{6}$

16. $\left(\frac{10}{30} + \frac{24}{30} + \frac{5}{30} + 2 + 1\right) + \left(-\frac{20}{30} - \frac{25}{30} - \frac{9}{30} - 3\frac{15}{30} - 1\right) = 4\frac{9}{30} + (-6\frac{9}{30}) = -2$

17. $17.5 + (-15.5) = 2$ **18.** $5.4 + (-16.9) = -11.5$

B **19.** $[b + (-4)] + (-b)$

$= (-b) + [b + (-4)]$	Comm. prop. for add.
$= [(-b) + b] + (-4)$	Assoc. prop. for add.
$= 0 + (-4)$	Prop. of opposites
$= -4$	Prop. of 0

20. $[(-60) + c] + (60)$

$= (60) + [(-60) + c]$	Comm. prop. for add.
$= [(60) + (-60)] + c$	Assoc. prop. for add.
$= 0 + c$	Prop. of opposites
$= c$	Prop. of 0

21. $a + (b + c)$

$= a + (c + b)$	Comm. prop. for add.
$= (a + c) + b$	Assoc. prop. for add.

22. $[a + (-b)] + [b + (-a)]$

$= \{[a + (-b)] + b\} + (-a)$	Assoc. prop. for add.
$= \{a + [(-b) + b]\} + (-a)$	Assoc. prop. for add.
$= \{a + 0\} + (-a)$	Prop. of opposites
$= a + (-a)$	Prop. of 0
$= 0$	Prop. of opposites

23. $a + (b + c)$

$= (b + c) + a$	Comm. prop. for add.
$= b + (c + a)$	Assoc. prop. for add.

24. $(-m) + (k + m)$

$= (-m) + (m + k)$	Comm. prop. for add.
$= [(-m) + m] + k$	Assoc. prop. for add.
$= 0 + k$	Prop. of opposites
$= k$	Prop. of 0

25. $(-5) + [(-s) + (s + 5)]$

$= (-5) + [\{(-s) + s\} + 5]$	Assoc. prop. for add.
$= (-5) + [0 + 5]$	Prop. of opposites
$= (-5) + [5]$	Prop. of 0
$= 0$	Prop. of opposites

26. $[a + c] + [(-a) + (-c)]$

$= [a + c] + [-(a + c)]$	Prop. of opposite of sum
$= 0$	Prop. of opposites

27. $(-2) + 17 = 15$ **28.** $10 + (-3) = 7$ **29.** $4 + (-5) = -1$ **30.** $-5 + (1) = -4$

31. $9 + (-26) = -17$ **32.** $(-1) + 6 = 5$ **33.** $(-11) + 3 = -8$ **34.** $-5 + (0) = -5$

35. $\frac{2}{3} + (\frac{1}{3}) = 1$

Page 128 · **PROBLEMS**

1.
$\$35.00$
$\$14.75 \quad -\$26.10$
$\$18.15 \quad -\$\ 7.50$
$\$67.90 + (-\$33.60) = \$34.30$

2.
$\$13.00 \quad -\$\ 7.50$
$\$\ 6.50 \quad -\$\ 1.25$
$\$19.50 + (-\$10.75) = \$8.75$

3.
$-\$13.68$
$-\$\ 1.09$
$-\$\ 4.17$
$-\$\ \ .75$
$-\$19.69 + (\$.37) = -\$19.32$

4. $(73) + (-25) + (-18) + (-15) + (185) + (64) = 322 + (-58) = \$264$

5. $(-375) + (650) = 275$ ft **6.** $15 + (-3) + 8 + (-9) + (-12) = 23 + (-24) = -1$ yd

7. $(-4) + (-1) = -5$; $(-4) + 0 = -4$; $(-4) + 1 = -3$; $(-4) + 3 = -1$; $-1 + 0 = -1$; $-1 + 1 = 0$; $-1 + 3 = 2$; $0 + 1 = 1$; $0 + 3 = 3$; $1 + 3 = 4$. $\{-5, -4, -3, -1, 0, 1, 2, 3, 4\}$; G not closed under add.

8. $(-3) + (\frac{1}{2}) + (\frac{1}{2}) + (-4) = 1 + (-7) = -6$ lb

9. Let x = net change in temp.; $49 + x = 21.5$; $x = -27.5°$F

Pages 131–132 · **WRITTEN EXERCISES**

A **1.** $29 + (30) = 59$ **2.** $-69 + (72) = 3$ **3.** $-25 + (25) = 0$

4. $-80 + (-20) = -100$ **5.** $t + 2 + (-t - 5) = -3$

6. $s + (-7) + (-s) + (3) = -4$ **7.** $a + b - 12 + (-a - b + 4) = -8$

8. $a - k + 14 + (-a + k + 9) = 23$ **9.** $1.5 + (-.5) = 1$ **10.** $1.7 + (-1.9) = -.2$

11. $.6 + (.3) = .9$ **12.** $-5.7 + (.7) = -5$ **13.** $0 + (\frac{3}{4}) = \frac{3}{4}$ **14.** $0 + (-\frac{2}{3}) = -\frac{2}{3}$

15. $[300 - 450] + [-230 + 1066] = 1366 + (-680) = 686$

16. $[1492 - 1678] + [44 - 12] = 1536 + (-1690) = -154$

17. $x + y + (-x + y) - y = y$ **18.** $u - v + (-u - v) + v = -v$

19. $(h + 9) + (7 - h) = 16$ **20.** $(-10 + k) + (-12 - k) = -22$

21. $-2 - (7) = -2 + (-7) = -9$ **22.** $-7 - (18) = -7 + (-18) = -25$

23. $b - (b + 4) = b + (-b - 4) = -4$ **24.** $c - (8 + c) = c + (-8 - c) = -8$

25. $(23\pi - 1) - (23\pi + 1) = (23\pi - 1) + (-23\pi - 1) = -2$

26. $(-8\pi + 5) - (8\pi - 5) = (-8\pi + 5) + (-8\pi + 5) = -16\pi + 10$

27. $[37 + (-12)] - 49 = 25 + (-49) = -24$ **28.** $[42 + (-51)] - 27 = -9 + (-27) = -36$

29. $(-32 + a) - (-97 + a) = (-32 + a) + (97 - a) = 65$

30. $(19 - r) - (18 - r) = (19 - r) + (-18 + r) = 1$

B **31.** $-(a - b)$
$= -(a + [-b])$ Meaning of subt.
$= (-a) + (b)$ Prop. of opposite of sum
$= -a + b$

32. $-(a + b + c)$
$= (-[a + b]) + (-c)$ Prop. of opposite of sum
$= (-a) + (-b) + (-c)$
Prop. of opposite of sum

33. Not closed; difference of two odd integers is an even no.

34. Closed; difference of two even integers is an even no.

35. Same as **33** **36.** Not closed; e.g., $\frac{3}{2} - (-\frac{1}{2}) = \frac{4}{2}$ which is not a member of the set.

Page 132 · **PROBLEMS**

A **1.** $-244 - (-276) = -244 + 276 = 32$ ft **2.** $-11 - (-3) = -11 + 3 = -8°$

3. $-5 - (8) = -5 + (-8) = -13°$ **4.** $-113 - (161) = -113 + (-161) = -274$ ft

5. $-404 - (-431) = -404 + (431) = 27$ yr **6.** $-212 - (-287) = -212 + (287) = 75$ yr

7. $1.25 - (-4.38) = 1.25 + (4.38) = \$5.63$ **8.** $250 - (-175) = 250 + (175) = 425$

9. (current year) $-$ (-146) = (current year + 146) yr

10. $-37{,}800 - (29{,}012) = -37{,}800 + (-29{,}012) = -66{,}812$ ft

11. $-\$9.47 - (\$317.25) = -\$9.47 + (-\$317.25) = -\$326.72$; spent \$326.72

12. $41 - (-23) = 41 + 23 = 64°$

Pages 137–138 · **WRITTEN EXERCISES**

A **1.** a. $(-4)(21) = -84$;
b. $(-4)(10) + (-4)(11) = -40 - 44 = -84$

2. a. $(-3)(-7) = 21$;
b. $(-3)(-2) + (-3)(-5) = 6 + 15 = 21$

3. a. $(-1)(4) = -4$;
b. $(20)(4) - (21)(4) = 80 - 84 = -4$

4. a. $(5)(-9) = -45$;
b. $(5)(-6) - (5)(3) = -30 - 15 = -45$

5. a. $0(-3) = 0$;
b. $0(-8) + 0(5) = 0 + 0 = 0$

6. a. $(-3 + 7)0 = (4)0 = 0$;
b. $(-3)0 - (-7)0 = 0 - 0 = 0$

7. a. $12 - 52 = -40$;
b. $4(3 - 13) = 4(-10) = -40$

8. a. $-18 - 2 = -20$;
b. $(-2)(9 + 1) = (-2)(10) = -20$

9. a. $-1188 - 12 = -1200$;
b. $(-99)(12) + (-1)(12) = (-99 - 1)12 = (-100)12 = -1200$

10. a. $3960 - 3960 = 0$;
b. $40(99 - 99) = 40(0) = 0$

11. a. $91 - 91 = 0$;
b. $-7(-13 + 13) = -7(0) = 0$

12. a. $-12(4\frac{1}{3}) = -52$;
b. $(-12)(4) + (-12)(\frac{1}{3}) = -48 - 4 = -52$

13. a. $14(-5\frac{1}{7}) = -72$;
b. $14(-5) - 14(\frac{1}{7}) = -70 - 2 = -72$

14. a. 148; b. $-48(-3 - \frac{1}{12}) = (-48)(-3) - (-48)(\frac{1}{12}) = 144 + 4 = 148$

15. a. $-(5) = -5$; b. $-1(-7 + 12) = (-1)(-7) + (-1)(12) = 7 - 12 = -5$
16. a. $-(-54) = 54$; b. $-1(-34 - 20) = (-1)(-34) - (-1)(20) = 34 + 20 = 54$
17. a. $.8 - (2.18) = -1.38$;
b. $.8 - 1(2.5 - .32) = .8 + (-1)(2.5) - (-1)(.32) = .8 - 2.5 + .32 = -1.38$
18. a. $6 - (-1.63) = 6 + 1.63 = 7.63$;
b. $6 - 1(-1.9 + .27) = 6 + (-1)(-1.9) + (-1)(.27) = 6 + 1.9 - .27 = 7.63$
19. a. $-.3 - .2(-1) = -.3 + .2 = -.1$;
b. $-.3 + (-.2)(-.7) - (-.2)(.3) = -.3 + .14 + .06 = -.1$
20. a. $-.7 - .2(-.5) = -.7 + .10 = -.6$
b. $-.7 + (-.2)(-.1) - (-.2)(.4) = -.7 + .02 + .08 = -.6$
21. $7a - 10a + 5 = -3a + 5$ **22.** $7r - 8r + 2 = -r + 2$
23. $-8x + x - 7 = -7x - 7$ **24.** $-15y + 22$
25. $-5nt + 3nt + 2 = -2nt + 2$ **26.** $-9hk + 6hk - 3 = -3hk - 3$
27. $4r - 7r + 6s = -3r + 6s$ **28.** $15a + b - 16b = 15a - 15b$
29. $-2n - 3$ **30.** $\frac{4}{3}y - \frac{2}{3}y - \frac{1}{2} = \frac{2}{3}y - \frac{1}{2}$ **31.** $-3a + 5a = 2a$
32. $11u - 7u - 1 = 4u - 1$ **33.** $37k^3 - 41k^3 = -4k^3$ **34.** $6xyz - 10xyz = -4xyz$
35. $-1.1d^2 + 2.3d^2 = 1.2d^2$ **36.** $5(r + s) - 14(r + s) = -9(r + s) = -9r - 9s$
37. $-2x^2 + 2x$ **38.** $-7y - 2y^2$ **39.** $2m - 3$ **40.** $u^3 + u^2$

B **41.** $-2r - 6s - 5r - 5s = -7r - 11s$ **42.** $-7a + 21b - 9b + 9a = 2a + 12b$
43. $3p - 6q - 5q + 2p = 5p - 11q$ **44.** $28v - 4t + t + 3v = 31v - 3t$
45. $2[3a - 3 + 5] - 4 = 2[3a + 2] - 4 = 6a + 4 - 4 = 6a$
46. $3[-6 + 2a - 8] + 26 = 3[-14 + 2a] + 26 = -42 + 6a + 26 = -16 + 6a$
47. $-7n - 5[2 + 4n - 3] = -7n - 5[-1 + 4n] = -7n + 5 - 20n = -27n + 5$
48. $-10h - 4[-1 - 9h + 6] = -10h - 4[5 - 9h] = -10h - 20 + 36h = 26h - 20$
49. $.7(.3)(-3) - .5(.2) = -.63 - .10 = -.73$ **50.** $.3(-2) - .2(-1)(.2) = -.6 + .04 = -.56$
51. $-\frac{1}{2}(.2) + .4(-3) - \frac{1}{3}(.3) = -.1 - 1.2 - .1 = -1.4$ **52.** $.2(-1 + 1)^2 = .2(0)^2 = 0$
53. $(-1)^2 + .2(-1) + .1 = 1 - .2 + .1 = 1.1 - .2 = .9$
54. $1.3(-3 + 1)^3 = 1.3(-2)^3 = 1.3(-8) = -10.4$
55. $(-3)^3 - 3(-3)^2 + 3(-3) - 1 = -27 - 3(9) - 9 - 1 = -27 - 27 - 9 - 1 = -64$
56. $.04(.3)^3 + .05(-3)^3 = .04(.027) + .05(-27) = .00108 - 1.35 = -1.34892$
57. $.01(-2)^4 - .01(-1)^4 = .01(16) - .01(1) = .16 - .01 = .15$

C **58.** $(-1)(.2 + 3(-2))^3 = (-1)(.2 - 6)^3 = (-1)(-5.8)^3 = (-1)(-195.112) = 195.112$
59. $(-2)(2(.2) - (-3))^2 = (-2)(3.4)^2 = (-2)(11.56) = -23.12$
60. $(.3)^2(-3 - (-1)^3) = (.09)(-3 + 1) = (.09)(-2) = -.18$
61. $(-3)^3((.2)^2 + (-1)^5) = (-27)(.04 - 1) = (-27)(-.96) = 25.92$
62. $1.1(2(-1)^2 - .2)(2(-1)^2 + .2) = 1.1(2 - .2)(2 + .2) = 1.1(1.8)(2.2) = 4.356$
63. $1.01(3(-3) + (.3)^2)(3(-3) - (.3)^2) = 1.01(-9 + .09)(-9 - .09) = 1.01(-8.91)(-9.09) = 81.801819$

Pages 141–142 · **WRITTEN EXERCISES**

A **1.** $(-\frac{3}{7})(\frac{1}{3}) = -\frac{1}{7}$ **2.** $(-.25)(-\frac{1}{5}) = .05$ **3.** $(-12)(-3) = 36$ **4.** $(-\frac{4}{9})(\frac{1}{4}) = -\frac{1}{9}$
5. $-.36t(\frac{1}{6}) = -.06t$ **6.** $(-16)(-4) = 64$ **7.** $(36)(-2) = -72$ **8.** $24.5(-\frac{1}{5}) = -4.9$
9. $(3b)(-3) = -9b$ **10.** $(8)(-4) = -32$ **11.** $2.52(-\frac{1}{7}) = -.36$ **12.** $(7c)(-7) = -49c$
13. $(-15.6)(\frac{1}{12}) = -1.3$ **14.** $1.82(-\frac{1}{13}) = -.14$ **15.** $(-.195)(-\frac{1}{13}) = .015$
16. $(-39.1)(\frac{1}{17}) = -2.3$ **17.** $4.56(-\frac{1}{24}) = -.19$ **18.** $(-.528)(-\frac{1}{24}) = .022$

B **19.** $\frac{(10)(-6)}{(-2)(-1)(3)} = \frac{-60}{6} = -10$ **20.** $\frac{(-2)(3)(10)}{(-1)(-6)} = \frac{-60}{6} = -10$
21. $\frac{(-2)^2}{(-1)^3} = \frac{4}{-1} = -4$ **22.** $\frac{(-2)(3)^2}{(-6)} = \frac{(-2)(9)}{-6} = \frac{-18}{-6} = 3$
23. $\frac{(-6)(10)^2}{(-2)^3} = \frac{(-6)(100)}{-8} = \frac{-600}{-8} = 75$ **24.** $\frac{(-1)(-6)^2}{(3)^2} = \frac{(-1)(36)}{9} = \frac{-36}{9} = -4$

25. $\dfrac{(-1)^5}{(-2)(3)} = \dfrac{-1}{-6} = \dfrac{1}{6}$

26. $\dfrac{(-1)^6}{(-6)(10)} = \dfrac{1}{-60} = -\dfrac{1}{60}$

27. $\dfrac{(3)^3}{(-2)^2(-6)} = \dfrac{27}{(4)(-6)} = \dfrac{27}{-24} = -\dfrac{9}{8}$

28. $\dfrac{(-2)^3}{(3)^2(10)} = \dfrac{-8}{(9)(10)} = \dfrac{-8}{90} = -\dfrac{4}{45}$

29. $\dfrac{(-2+2)^4}{(-6)} = \dfrac{(0)^4}{-6} = 0$

30. $\dfrac{(-6+6)^3}{(-2)} = \dfrac{(0)^3}{-2} = 0$

C **31.** $\dfrac{(-2\cdot 3)^2 - (-6)^2}{(10)^2} = \dfrac{(-6)^2 - (36)}{100} = \dfrac{36-36}{100} = \dfrac{0}{100} = 0$

32. $\left(\dfrac{-6}{-2}\right)^3 - (3)^3 = (3)^3 - (3)^3 = 27 - 27 = 0$

33. $\dfrac{(-2+1)^3 + (-1)^7}{(-6+4)^2} = \dfrac{-1+(-1)}{(-2)^2} = \dfrac{-2}{4} = -\dfrac{1}{2}$

34. $\dfrac{(10-1)^2 - (3)^3}{((-2)(-6))^2} = \dfrac{81-27}{(12)^2} = \dfrac{54}{144} = \dfrac{3}{8}$

35. $(-6)^2 + \left(\dfrac{10}{-2}\right)^3 = 36 + (-5)^3 = 36 - 125 = -89$

36. $\dfrac{(-1)^8 + (-6-1)^2}{(3+2)^2} = \dfrac{1+49}{25} = \dfrac{50}{25} = 2$

Pages 143–144 · **PROBLEMS**

1. Assumed avg.: 2000; sum of deviations = $200 + (-20) + 50 + (-120) + 100 + (-70) = 140$; avg. dev. = $\frac{140}{6} = 23\frac{1}{3}$; true avg. = $2000 + 23\frac{1}{3} = \$2023\frac{1}{3}$

2. Assumed avg.: 8000; sum of deviations = $(-350) + 630 + (-2400) + 1550 + 550 + (-450) + 520 = 50$; avg. dev. = $\frac{50}{7} = 7\frac{1}{7}$; true avg. = $8000 + 7\frac{1}{7} = 8007\frac{1}{7}$ M

3. Assumed avg.: −5800; sum of dev. = $600 + 200 + (-100) + (-300) + (-700) = -300$; avg. dev. = $\dfrac{-300}{5} = -60$; true avg. = $-5800 - 60 = -5860$ ft

4. Assumed avg.: 17.0; sum of dev. = $(.4) + (-.4) + (-.3) + (.9) + (-1.1) + (.3) + (.5) = .3$; avg. dev. = $\dfrac{.3}{7} = \dfrac{3}{70}$; true avg. = $17.0 + \frac{3}{70} = 17\frac{3}{70}$ g

5. Assumed avg.: 0; sum of dev. = $(13) + (5) + (-6) + (-9) + (-14) + (-4) + (2) = -13$; avg. dev. = $\dfrac{-13}{7} = -\dfrac{13}{7}$; true avg. = $0 - \frac{13}{7} = -1\frac{6}{7}°$

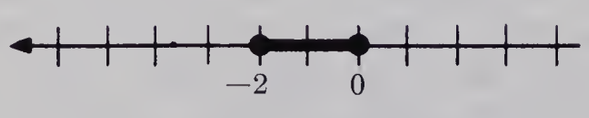
6. Assumed avg.: 0; sum of dev. = $(-\frac{3}{2}) + (-\frac{1}{2}) + (-\frac{1}{2}) + (\frac{5}{2}) + (\frac{3}{2}) + (-\frac{1}{2}) + (\frac{7}{2}) + (0) + (-\frac{1}{4}) + (\frac{9}{2}) = \frac{35}{4}$; avg. dev. = $\dfrac{\frac{35}{4}}{10} = \dfrac{35}{40} = \dfrac{7}{8}$; true avg. = $0 + \frac{7}{8} = \frac{7}{8}$

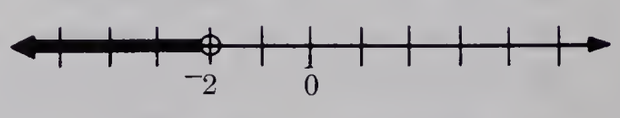
7. Assumed avg.: 4.185; sum of dev. = $(-.008) + (.003) + (.011) + (.011) + (.001) + (.003) + (-.001) + (-.004) = .016$; avg. dev. = $\dfrac{.016}{8} = .002$; true avg. = $4.185 + .002 = 4.187$

8. Assumed avg.: 1650.0; sum of dev. = $(4.8) + (5.6) + (4.8) + (-6.5) + (-4.9) + (-4.0) + (-.4) + (-4.2) = -4.8$; avg. dev. = $\dfrac{-4.8}{8} = -.6$; true avg. = $1650.0 - .6 = 1649.4$

Pages 145–146 · **CHAPTER TEST**

1. a. −3; b. 0 **2.** 3

3. a. 

b. 

4. $.01(3) = .03$; $(3 - .03) \le d \le 3 + .03$; $2.97 \le d \le 3.03$; $d \in$ {directed nos. between 2.97 and 3.03 inclusive}

5. $(47) + (-85) = -38$ **6.** $(-125) + (80) = -45$ ft

7. a. $-(-3) = 3$; $\{-3\}$; b. $-(-3) > 0$; $-(-1) > 0$. $\{-3, -1\}$

8. 2, −2 **9.** $153 + (-215) = -62$

10. $(15) + (-44) + (-1) + (7) + (-12) = (22) + (-57) = -35$

11. $[19 - (-5)] - [19 + (-5)] = 24 - 14 = 10$

12. $47.6 - 61.3 = -13.7°$

13. $59(-18) - 32(-1)(59) = 59[(-18) - 32(-1)] = 59[-18 + 32] = 59(14) = 826$

14. $3a - 3b - 7a + 3b = -4a$

15. $-5(-3)^4 \cdot \frac{1}{9}(-1)^{17} = -5(81) \cdot \frac{1}{9}(-1) = -405(-\frac{1}{9}) = 45$

16. $5ab^4 - 40b^5 - 3a^3b + 9a^2b^2$

17. a. $\frac{-1.56}{13} = \frac{13t}{13}$; $-.12 = t$; $\{-.12\}$. b. $\frac{-16h}{-16} = \frac{-12}{-16}$; $h = \frac{3}{4}$; $\{\frac{3}{4}\}$

18. a. -8; b. $\frac{4}{7}$ **19.** $36x^9y^8(-\frac{1}{24}) = -\frac{3}{2}x^9y^8$

20. Sum of dev. $= (-2) + (-2) + 0 + (2) + (3) + (6) + (13) + (16) = 36$; avg. dev. $= \frac{36}{8} = 4\frac{1}{2}$; true avg. $= -3 + 4\frac{1}{2} = 1\frac{1}{2}°$

Pages 146–150 · CHAPTER REVIEW

1. right **2.** minus **3.** positive, negative **4.** -4.5 **5.** directed, signed **6.** C

7. D **8.** Q **9.** M **10.** greater **11.** right **12.** negative **13.** less **14.** -2

15. 2 **16.** 0 **17.** -3 **18.** $\{1, 4\}$ **19.** $\{-2, -4, -5\}$ **20.** $\{-2, 0\}$ **21.** right

22. left **23.** -7 **24.** $(-61) + 45 = -16$ **25.** $(-23) + (23) = 0$ **26.** $\{9\}$ **27.** $\{0\}$

28. negative, same **29.** 0 **30.** 0 **31.** 0, additive inverse **32.** opposites **33.** $\{-1\}$

34. $\{0, 1, 4\}$ **35.** $\{-4, -2, -1, 0, 1\}$ **36.** distance **37.** magnitude

38. $|-3|, 3$ **39.** -3.2 **40.** 12 **41.** $-3\frac{1}{5}$ **42.** .085 **43.** $\{4, -4\}$ **44.** $\emptyset$

45. $\{-5, -4, -2, 0, 1, 4\}$ **46.** $\{-5, -4, -2, 4\}$ **47.** positive **48.** negative

49. a. difference; b. negative; c. negative **50.** $30 + (-60) = -30$

51. $94 + (-94) = 0$ **52.** $(382 + 291) + (-425 - 36) = 673 + (-461) = 212$

53. $(112 + 259) + (-78 - 407) = 371 + (-485) = -114$

54. $n - 58 + 91 = n + 33$ **55.** $-39 + 77 - t = 38 - t$ **56.** opposite *or* inverse **57.** 5

58. $26 + (-59) + (12) = 38 + (-59) = -21$ **59.** $(-17) + (-83) + (62) = (-100) + 62 = -38$

60. $[38 + (-83)] - (-29) = (-45) + 29 = -16$ **61.** $-12 - (-47 + 74) = -12 - (27) = -39$

62. $r + 28 + 81 = r + 109$ **63.** $-32 - s + 47 = -s + 15$

64. $-8 - (-1) = -8 + 1 = -7°$ **65.** $-373 - (-421) = -373 + 421 = 48$ yr

66. positive **67.** negative **68.** opposite **69.** -266

70. $-14(19) = -266$

71. $(6\frac{1}{2})(-1)(17) - 6\frac{1}{2}(-15) = 6\frac{1}{2}\big(-17 - (-15)\big) = 6\frac{1}{2}(-17 + 15) = 6\frac{1}{2}(-2) = -13$

72. $\frac{h}{4} + 8$ **73.** $-3 + \frac{1}{6}k$ **74.** $10a - 31b$ **75.** $31m - 47mn$ **76.** 0

77. $(-43)(-52)(10)(1) = 22{,}360$ **78.** $-5(-2)(3)^2 = (10)(9) = 90$

79. $-5(-1)\big(2^3 - (-2)^3\big) = 5\big(8 - (-8)\big) = 5(16) = 80$ **80.** positive **81.** negative

82. divisor **83.** 0 **84.** $(-30)(-\frac{1}{45}) = \frac{2}{3}$

85. $(-18) \div [3(-6)] = (-18) \div (-18) = 1$ **86.** $\frac{-14v}{-14} = \frac{126}{-14}$; $v = -9$; $\{-9\}$

87. $\frac{-36w}{-36} = \frac{-\frac{9}{4}}{-36}$; $w = -\frac{9}{4}(-\frac{1}{36})$; $w = \frac{1}{16}$; $\{\frac{1}{16}\}$ **88.** $\frac{1}{-\frac{3}{2}} = \frac{-\frac{3}{2}x}{-\frac{3}{2}}$; $-\frac{2}{3} = x$; $\{-\frac{2}{3}\}$

89. 1 **90.** 0, multiplicative inverse **91.** reciprocal **92.** 1 **93.** $-\frac{1}{2}$ **94.** $\frac{5}{11}$ **95.** $\frac{3}{2}$

96. $\frac{a - b}{2}$ **97.** No; $2 \div \frac{1}{2} = 2(2) = 4$, not in set. **98.** Yes, except for 0

99. deviation, directed

100. Sum of dev. $= (-17) + (-8) + (-5) + (-3) + (2) + (8) + (9) = -14$; avg. dev. $= \frac{-14}{7} = -2$; true avg. $= 5 - 2 = 3$

Pages 152–153 · **EXTRA FOR EXPERTS**

1. a. $(3 + 3) + 4$ $3 + (3 + 4)$ b. $(2 + 3) + 1$ $2 + (3 + 1)$ c. Assoc. prop. for add.
$= 1 + 4$ $= 3 + 2$ $= 0 + 1$ $= 2 + 4$
$= 0$ $= 0$ $= 1$ $= 1$

2. Comm. prop. for add. holds: $2 + 4 = 1$ and $4 + 2 = 1$; and $3 + 4 = 2$ and $4 + 3 = 2$.
Comm. prop. for mult. holds: $2 \cdot 4 = 3$ and $4 \cdot 2 = 3$; and $3 \cdot 4 = 2$ and $4 \cdot 3 = 2$.

3. System is closed under add., subt., and mult.

4. The identity element for add. is 0 and for mult. is 1.

5. a. $x = 4$ (by use of the table);
b. $y = 3$
c. $s = 4$

6. a. $x = 4$
b. $w = 4$
c. $2t = 3; t = 4$

7. Because the system is closed under subtraction. To subtract in clock arithmetic, start at the minuend and count counterclockwise the number of spaces named by the subtrahend.

8. a. 0: $4 + 0 = 4; 5 + 0 = 5; 9 + 0 = 9; 0 + 6 = 0$
b. Yes: $4 \cdot 0 = 0; 5 \cdot 0 = 0; 9 \cdot 0 = 0; 0 \cdot 6 = 0$

CHAPTER 5. Equations, Inequalities, and Problem Solving

Page 159 · **WRITTEN EXERCISES**

A **1.** $29y = 203; y = 7$. $\{7\}$

2. $24x = 144; x = 6$. $\{6\}$

3. $8 = \frac{n}{2}; (2)(8) = (2)\left(\frac{n}{2}\right); 16 = n$. $\{16\}$

4. $10 = \frac{x}{5}; (5)(10) = (5)\left(\frac{x}{5}\right); 50 = x$. $\{50\}$

5. $8 = -\frac{n}{5}; (-5)(8) = (-5)\left(-\frac{n}{5}\right); -40 = n$. $\{-40\}$

6. $19 = -\frac{7s}{2}; (-2)(19) = (-2)\left(-\frac{7s}{2}\right); -38 = 7s; -\frac{38}{7} = s$. $\{-\frac{38}{7}\}$

7. $360 = -6z; -60 = z$. $\{-60\}$

8. $714 = -17r; -42 = r$. $\{-42\}$

9. $38b = 171; b = \frac{9}{2}$. $\{\frac{9}{2}\}$

10. $72x = 540; x = \frac{15}{2}$. $\{\frac{15}{2}\}$

11. $-26y = 221; y = -\frac{17}{2}$. $\{-\frac{17}{2}\}$

12. $-60w = 40; w = -\frac{2}{3}$. $\{-\frac{2}{3}\}$

13. $x = \frac{4rs}{rs}; x = 4$. $\{4\}$, $rs \neq 0$

14. $x = \frac{mn}{mn}; x = 1$. $\{1\}$, $mn \neq 0$

15. $x = a$. $\{a\}$

16. $x = 5a$. $\{5a\}$

17. $\frac{2y}{3} = -2; (3)\left(\frac{2y}{3}\right) = (3)(-2); 2y = -6; y = -3$. $\{-3\}$

18. $\frac{3w}{4} = -6; (4)\left(\frac{3w}{4}\right) = (4)(-6); 3w = -24; w = -8$. $\{-8\}$

19. $(.2)\left(\frac{5y}{.2}\right) = (.2)(.3); 5y = .06; y = .012$. $\{.012\}$ *or* $\{\frac{3}{250}\}$

20. $(.5)\left(\frac{3x}{.5}\right) = (.5)(.6); 3x = .3; x = .1$. $\{.1\}$ *or* $\{\frac{1}{10}\}$

21. $2w = 2l + 3; w = \frac{2l + 3}{2} \cdot \left\{\frac{2l + 3}{2} \cdot\right\}$

22. $4t = -5h; t = \frac{-5h}{4} \cdot \left\{\frac{-5h}{4}\right\}$

23. $b = \frac{y - a}{2} \cdot \left\{\frac{y - a}{2}\right\}$

24. $p = 3a - 2c$. $\{3a - 2c\}$

25. $p - a = -prt; \frac{p - a}{-pr} = t. \left\{\frac{p - a}{-pr}\right\}, pr \neq 0$ *or* $prt = a - p; t = \frac{a - p}{pr} \cdot \left\{\frac{a - p}{pr}\right\}, pr \neq 0$

26. $p - a = -prt; \frac{p - a}{-pt} = r. \left\{\frac{p - a}{-pt}\right\}, pt \neq 0$ *or* $prt = a - p; r = \frac{a - p}{pt} \cdot \left\{\frac{a - p}{pt}\right\}, pt \neq 0$

27. $2x = 7b; x = \frac{7b}{2} \cdot \left\{\frac{7b}{2}\right\}$

28. $2y = -3m; y = \frac{-3m}{2} \cdot \left\{\frac{-3m}{2}\right\}$

29. $b = p - a - c$. $\{p - a - c\}$

30. $c = s - o - p$. $\{s - o - p\}$

31. $W = Pt$. $\{Pt\}$, $t \neq 0$

32. $w = pA$. $\{pA\}$, $A \neq 0$

33. $bh = 2A; b = \frac{2A}{h} \cdot \left\{\frac{2A}{h}\right\}, h \neq 0$

34. $Bh = 3V; h = \frac{3V}{B} \cdot \left\{\frac{3V}{B}\right\}, B \neq 0$

35. $\pi = \frac{A}{4r^2} \cdot \left\{\frac{A}{4r^2}\right\}, r \neq 0$

36. $r = \frac{C}{2\pi} \cdot \left\{\frac{C}{2\pi}\right\}$

37. $h = \frac{3V}{\pi r^2} \cdot \left\{\frac{3V}{\pi r^2}\right\}, r \neq 0$

38. $r^3 = \frac{3V}{4\pi} \cdot \left\{\frac{3V}{4\pi}\right\}$

39. $h = \frac{3V}{B} \cdot \left\{\frac{3V}{B}\right\}, B \neq 0$

B **40.** $9 - \frac{3n}{2} + 3 = n + 2; 2\left[12 - \frac{3n}{2}\right] = 2[n + 2]; 24 - 3n = 2n + 4; 20 = 5n; 4 = n.$ $\{4\}$

41. $2n - \frac{8n}{3} - 8 = n + 7; 3\left[2n - \frac{8n}{3} - 8\right] = 3[n + 7]; 6n - 8n - 24 = 3n + 21;$
$-2n - 24 = 3n + 21; -5n = 45; n = -9.$ $\{-9\}$

42. $5x + 5 = 4x + 8; x = 3.$ $\{3\}$

43. $24x - 6 = 12 + 6x; 18x = 18; x = 1.$ $\{1\}$

44. $12n + 8 = 18 - 6n + 8; 12n + 8 = 26 - 6n; 18n = 18; n = 1.$ $\{1\}$

45. $5x + 5 = 3x + 6; 2x = 1; x = \frac{1}{2}.$ $\{\frac{1}{2}\}$

46. $5r + r + 15 - 3r - 6 = 0; 3r + 9 = 0; 3r = -9; r = -3.$ $\{-3\}$

47. $69 = 4h + 20 - h + 1; 69 = 3h + 21; 48 = 3h; 16 = h.$ $\{16\}$

48. $5y + 10 = 13 + 8y - 4; 5y + 10 = 9 + 8y; 1 = 3y; \frac{1}{3} = y.$ $\{\frac{1}{3}\}$

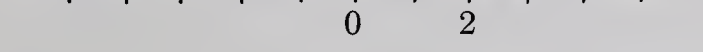
49. $5y + 10 = 6 + 6y - 3; 5y + 10 = 3 + 6y; 7 = y.$ $\{7\}$

50. $3x - 27 = 74 - 122 - 4x; 3x - 27 = -48 - 4x; 7x = -21; x = -3.$ $\{-3\}$

51. $2w - 4w - 8 = 5w + 20; -2w - 8 = 5w + 20; -28 = 7w; w = -4.$ $\{-4\}$

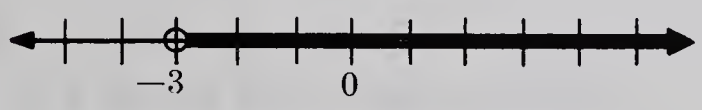
Page 163 · WRITTEN EXERCISES

A **1.** $5a \geq 10; a \geq 2$

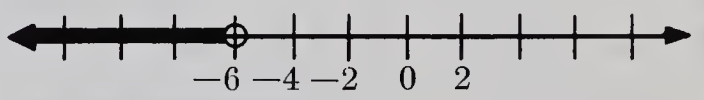


2. $6b \leq 24; b \leq 4$

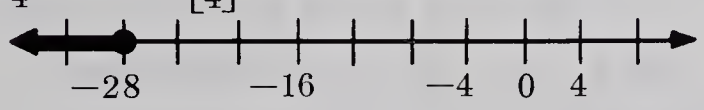
3. $-3s < 9; s > -3$



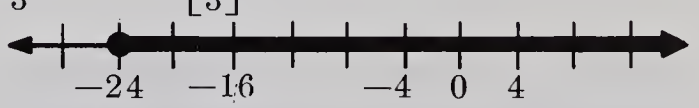
4. $-2t > 12; t < -6$



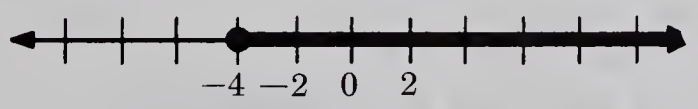
5. $\frac{z}{4} \leq -7; 4\left[\frac{z}{4}\right] \leq 4[-7]; z \leq -28$



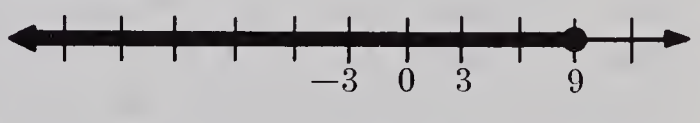
6. $\frac{w}{3} \geq -8; 3\left[\frac{w}{3}\right] \geq 3[-8]; w \geq -24$



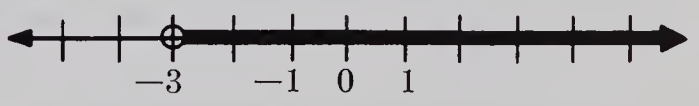
7. $-12 \leq 3t; -4 \leq t$



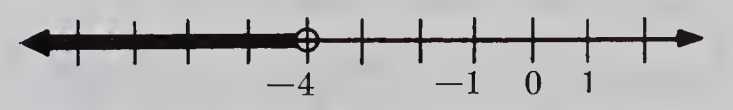
8. $36 \geq 4n; 9 \geq n$



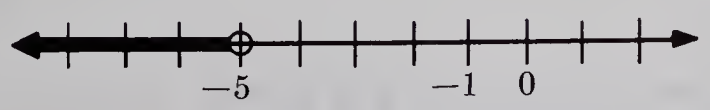
9. $\frac{2}{3}p > -2; 3[\frac{2}{3}p] > 3[-2]; 2p > -6; p > -3$



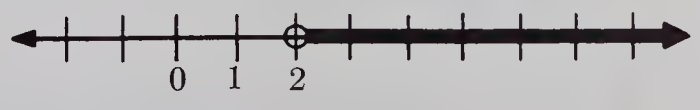
10. $\frac{3}{4}q < -3; 4[\frac{3}{4}q] < 4[-3]; 3q < -12; q < -4$



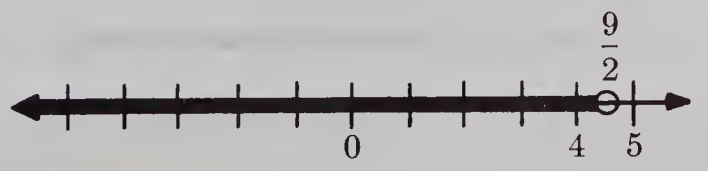
11. $2x < -10; x < -5$



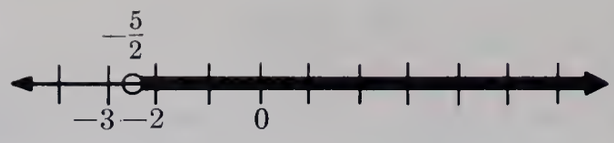
12. $5y > 10; y > 2$



13. $-5w + 5 > -13 - w; 18 > 4w; \frac{9}{2} > w$

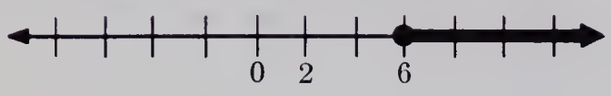


14. $-5v - 2 < 13 + v;\ -6v < 15;\ v > -\frac{5}{2}$

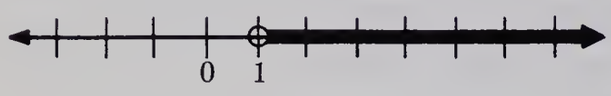


15. $6x - 12 + 6x \geq 0;\ 12x \geq 12;\ x \geq 1$

16. $-y + 18 - 2y \leq 0;\ -3y + 18 \leq 0;\ -3y \leq -18;\ y \geq 6$



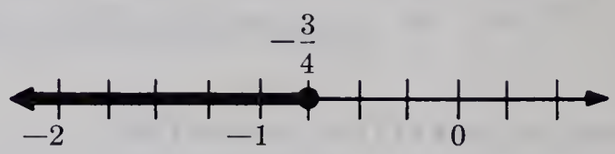
17. $-2t + 4 < 2t;\ 4 < 4t;\ 1 < t$



18. $3 - 5w > -2w;\ 3 > 3w;\ 1 > w$

19. $0 \geq -102r + 34;\ 102r \geq 34;\ r \geq \frac{1}{3}$

20. $0 \leq -204v - 153;\ 204v \leq -153;\ v \leq -\frac{3}{4}$



21. $-6m + 12 < 12 + 6m;\ 0 < 12m;\ 0 < m$

22. $-60 - 15z > -60 + 15z;\ 0 > 30z;\ 0 > z$

B **23.** $10\left[\frac{3}{10}(2t - 20) + 5\right] \leq 10\left[\frac{t - 60}{10}\right];\ 3(2t - 20) + 50 \leq t - 60;\ 6t - 60 + 50 \leq t - 60;$
$6t + 50 \leq t;\ 5t \leq -50;\ t \leq -10.\ \{t \leq -10\}$

24. $6\left[\frac{5}{6}(18 - g) - 1\right] \geq 6\left[\frac{g + 90}{6}\right];\ 5(18 - g) - 6 \geq g + 90;\ 90 - 5g - 6 \geq g + 90;$
$-5g - 6 \geq g;\ -6g \geq 6;\ g \leq -1.\ \{g \leq -1\}$

25. $2h - 2 < 2h;\ -2 < 0.$ {directed nos.}

26. $3 + 3y > 3y;\ 3 > 0.$ {directed nos.}

27. $3|x| + 3 \leq 3 - |x|;\ 4|x| \leq 0;\ |x| \leq 0.$ By def., $|x| \geq 0;\ \therefore x = 0.\ \{0\}$

28. $-4|n| + 2 \geq 2 - 3|n|;\ 0 \geq |n|.$ By def., $|n| \geq 0;\ \therefore n = 0.\ \{0\}$

29. $-2 + 14b < 14b - 35 - 17;\ -2 + 14b < 14b - 52;\ 0 \not< -50;\ \therefore \emptyset$

30. $-6 - 15c > 1 - 15c + 5;\ -6 - 15c > 6 - 15c;\ -6 \not> 6;\ \therefore \emptyset$

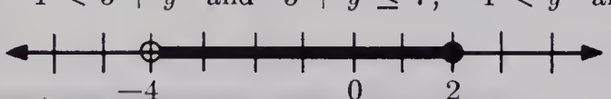
31. $7d + 21 - 5d + 15 \geq 2d + 40 - 4;\ 2d + 36 \geq 2d + 36;\ 0 \geq 0.$ {directed nos.}

32. $3k - 12 - k - 8 \leq -2 + 2k - 18;\ 2k - 20 \leq 2k - 20;\ 0 \leq 0.$ {directed nos.}

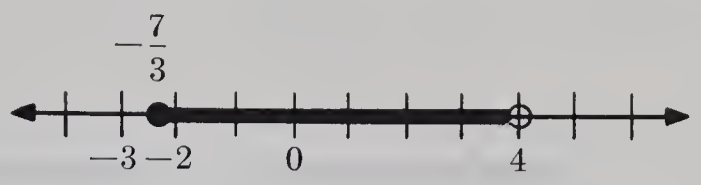
Pages 165–166 · **WRITTEN EXERCISES**

A **1.** $-3 \leq x + 4$ and $x + 4 \leq 0;\ -7 \leq x$ and $x \leq -4;\ -7 \leq x \leq -4,$ *Ans.*

2. $1 < 5 + y$ and $5 + y \leq 7;\ -4 < y$ and $y \leq 2;\ -4 < y \leq 2,$ *Ans.*

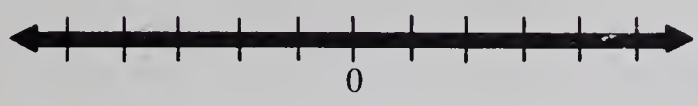


3. $-8 \leq -1 + 3a$ and $-1 + 3a < 11$;
$-7 \leq 3a$ and $3a < 12$;
$-\frac{7}{3} \leq a$ and $a < 4$. $-\frac{7}{3} \leq a < 4$, *Ans.*

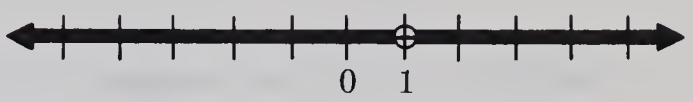


4. $-7 \leq 4b - 5$ and $4b - 5 \leq 19$;
$-2 \leq 4b$ and $4b \leq 24$;
$-\frac{1}{2} \leq b$ and $b \leq 6$. $-\frac{1}{2} \leq b \leq 6$, *Ans.*

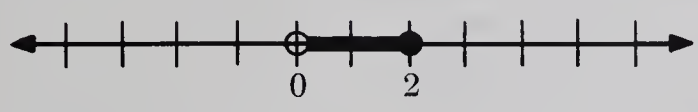
5. $x \geq 0$ or $-(x) \geq 0$;
$x \geq 0$ or $x \leq 0$. {directed nos.}



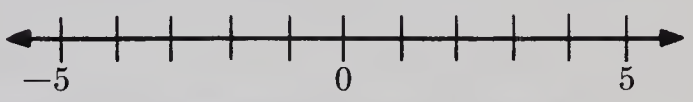
6. $(b - 1) > 0$ or $-(b - 1) > 0$;
$b > 1$ or $-b + 1 > 0$; $-b > -1$;
$b > 1$ or $b < 1$, *Ans.*



7. $m > 0$ and $m \leq 2$. $0 < m \leq 2$, *Ans.*



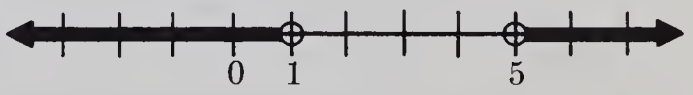
8. $n < -7$ and $n \geq -7$. $\emptyset$



9. $4 > -2r$ and $r > 1$;
$-2 < r$ and $r > 1$. $r > 1$, *Ans.*

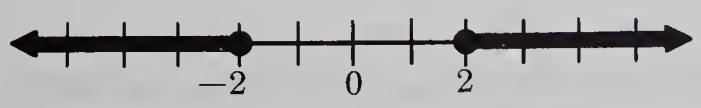
10. $-v < 4$ and $-3v \geq -3$;
$v > -4$ and $v \leq 1$. $-4 < v \leq 1$, *Ans.*

11. $(3 - w) > 2$ or $-(3 - w) > 2$;
$-w > -1$ or $-3 + w > 2$;
$w < 1$ or $w > 5$, *Ans.*

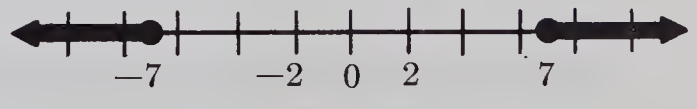


12. $1 < (4 - k)$ or $1 < [-(4 - k)]$;
$-3 < -k$ or $1 < -4 + k$;
$3 > k$ or $5 < k$. $k < 3$ or $k > 5$, *Ans.*

13. $(r) - 3 \geq -1$ or $(-r) - 3 \geq -1$;
$r \geq 2$ or $-r \geq 2$;
$r \geq 2$ or $r \leq -2$, *Ans.*



14. $(u) - 4 \geq 3$ or $(-u) - 4 \geq 3$;
$u \geq 7$ or $-u \geq 7$;
$u \geq 7$ or $u \leq -7$, *Ans.*

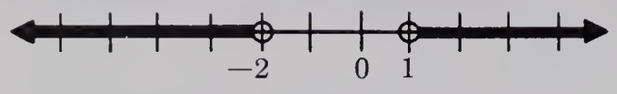


B

15. $-2v > 2$ or $3v < 9$;
$v < -1$ or $v < 3$. $v < 3$, *Ans.*

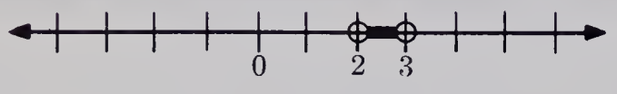
16. $6p \leq -12$ or $-4p > 12$;
$p \leq -2$ or $p < -3$. $p \leq -2$, *Ans.*

17. $(x - 1) > 0$ and $(x + 2) > 0$ or $(x - 1) < 0$ and $(x + 2) < 0$;
$x > 1$ and $x > -2$ or $x < 1$ and $x < -2$;
$\therefore\ x > 1$ or $x < -2$, *Ans.*

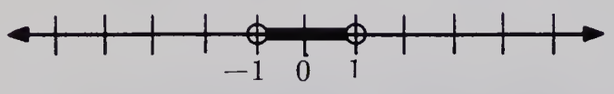


18. $(y + 4) > 0$ and $(y + 3) > 0$ or $(y + 4) < 0$ and $(y + 3) < 0$;
$y > -4$ and $y > -3$ or $y < -4$ and $y < -3$;
$\therefore\ y > -3$ or $y < -4$, *Ans.*

19. $(r - 2) < 0$ and $(r - 3) > 0$ or $(r - 2) > 0$ and $(r - 3) < 0$;
$r < 2$ and $r > 3$ or $r > 2$ and $r < 3$;
$\emptyset$ or $2 < r < 3$. $2 < r < 3$, *Ans.*



20. $(s + 1) > 0$ and $(s - 1) < 0$ or $(s + 1) < 0$ and $(s - 1) > 0$;
$s > -1$ and $s < 1$ or $s < -1$ and $s > 1$;
$-1 < s < 1$ or $\emptyset$. $-1 < s < 1$, *Ans.*



Pages 168–169 · **PROBLEMS**

A

1. Let x = no. fish for Robert; then $x + 5$ = no. for Michael; $x + (x + 5) = 19$; $2x + 5 = 19$; $2x = 14$; $x = 7$; $x + 5 = 12$. Robert, 7; Michael, 12, *Ans.*

2. Let n = no. knit by Jr. Red Cross; then $n + 24$ = no. knit by Sr. Red Cross; $n + (n + 24) = 50$; $2n + 24 = 50$; $2n = 26$; $n = 13$; $n + 24 = 37$. 13, *Ans.*

3. Let t = amt. spent on food; then $150 - t$ = remainder of income;
$15 \le \frac{1}{2}(150 - t) - t \le 30$;
$15 \le \frac{1}{2}(150 - t) - t$ and $\frac{1}{2}(150 - t) - t \le 30$;
$15 \le 75 - \frac{1}{2}t - t$ and $75 - \frac{1}{2}t - t \le 30$;
$15 \le 75 - \frac{3}{2}t$ and $75 - \frac{3}{2}t \le 30$;
$\frac{3}{2}t \le 60$ and $-\frac{3}{2}t \le -45$;
$t \le 40$ and $t \ge 30$. $\$30 \le t \le \$40$, *Ans.*

4. Let n = no. qt picked by Otto; then $\frac{n}{2} + 3$ = no. picked by Tom; $n + \left(\frac{n}{2} + 3\right) = 36$;
$2\left[n + \left(\frac{n}{2} + 3\right)\right] = 2\,[36]$; $2n + n + 6 = 72$; $3n + 6 = 72$; $3n = 66$; $n = 22$;
$\frac{n}{2} + 3 = \frac{22}{2} + 3 = 14$. Otto, 22; Tom, 14, *Ans.*

5. Let A = amt. could reduce saving; then $(12)(5) = 60$ = amt. saved 1st 12 wk.; and $40(5 - A)$ = amt. could save last 40 wk.; $60 + 40(5 - A) \ge 150$; $60 + 200 - 40A \ge 150$; $260 - 40A \ge 150$; $-40A \ge -110$; $A \le \$2\frac{11}{4}$ *or* $A \le \$2.75$, *Ans.*

6. Let w = width; then $2w + 25$ = length; $2(w) + 2(2w + 25) = 650$; $2w + 4w + 50 = 650$; $6w + 50 = 650$; $6w = 600$; $w = 100$; $2w + 25 = 225$. 100 ft, 225 ft, *Ans.*

7. Let w = width; then $3w + 6$ = length; $2(w) + 2(3w + 6) = 188$; $2w + 6w + 12 = 188$; $8w + 12 = 188$; $8w = 176$; $w = 22$; $3w + 6 = 3(22) + 6 = 72$. 22 ft, 72 ft, *Ans.*

8. Let n = smaller no.; then $3n + 5$ = larger no; $(3n + 5) - n \geq 31$; $2n + 5 \geq 31$; $2n \geq 26$; $n \geq 13$. 13, *Ans.*

9. Let s = speed of Local; then $2s + 5$ = speed of Limited; $2s + 5 = 63$; $2s = 58$; $s = 29$ mph, *Ans.*

10. Let p = no. pennies; then $2p$ = no. nickels and $\frac{4}{3}p$ = no. dimes; $1(p) + 5(2p) + 10(\frac{4}{3}p) \geq 365$; $p + 10p + \frac{40}{3}p \geq 365$; $11p + \frac{40}{3}p \geq 365$; $3[11p + \frac{40}{3}p] \geq 3[365]$; $33p + 40p \geq 1095$; $73p \geq 1095$, $p \geq 15$; $2p \geq 30$; $\frac{4}{3}p \geq 20$. No. coins $\geq (15 + 30 + 20)$. 65 coins, *Ans.*

11. Let N = Sally's phone no.; $18 + 90 = 6N - 6060$; $108 = 6N - 6060$; $6168 = 6N$; $N = 1028$, *Ans.*

12. Let l = length of rectangle; $9l = 144$; $l = 16$ ft, *Ans.*

13. Let w = width of 2nd rectangle; $3(12w) = (9)(8)$; $36w = 72$; $w = 2$ ft, *Ans.*

14. Let h = height; $(30)(24)(h) = (270)(36)$; $720h = 9720$; $h = 13\frac{1}{2}$ ft, *Ans.*

15. $(15)(10)(9) = 1350$ = vol. of bin; $\dfrac{(10.5)(2000)}{28} = 750$ = vol. of egg coal. Let h = height of coal in bin; $h = (\frac{750}{1350})9$; $h = 5$ ft, *Ans.*

16. Let R = no. rev.; $(\frac{22}{7})(2)$ = circum. of wheel in ft; $R = \dfrac{5500}{(\frac{22}{7})(2)}$; $R = \dfrac{5500}{\frac{44}{7}}$; $R = 875$, *Ans.*

B **17.** Let w = width; then $2w$ = length and $w + 1$ = height; $(4w) + 4(2w) + 4(w + 1) = 52$; $4w + 8w + 4w + 4 = 52$; $16w + 4 = 52$; $16w = 48$; $w = 3$. 3 in, 6 in, 4 in, *Ans.*

18. Let x = measure of each equal side; then $2x - 4$ = measure of base; $0 < [x + x + (2x - 4)] < 75$;

$0 < [x + x + (2x - 4)]$ and $[x + x + (2x - 4)] < 75$;
$0 < [2x + 2x - 4]$ and $[2x + 2x - 4] < 75$;
$0 < 4x - 4$ and $4x - 4 < 75$;
$4 < 4x$ and $4x < 79$;
$1 < x$ and $x < \frac{79}{4}$.

Since the base $2x - 4 > 0$, $x > 2$;
therefore, $x \in \{2\frac{1}{2}, 3, 3\frac{1}{2}, \ldots, 19\frac{1}{2}$ ft$\}$ and $(2x - 4) \in (1, 2, 3, \ldots, 35$ ft$\}$, *Ans.*

19. Let m = Mr. Fry's wt.; then $m - 50$ = Mrs. Fry's wt. and $\dfrac{m}{2}$ = daughter's wt.; $m + (m - 50) - \dfrac{m}{2} \geq 220$;

$2m - \dfrac{m}{2} \geq 270$; $2\left[2m - \dfrac{m}{2}\right] \geq 2[270]$; $4m - m \geq 540$; $3m \geq 540$; $m \geq 180$;

$m - 50 \geq 130$. 130 lb, *Ans.*

20. Let A = amt. in dollars earned in regular job; then $\dfrac{A}{3}$ = amt. earned in writing and $\dfrac{A}{2}$ = amt. earned by sister; $A + \dfrac{A}{3} \geq \dfrac{A}{2} + 14{,}000$; $6\left[A + \dfrac{A}{3}\right] \geq 6\left[\dfrac{A}{2} + 14{,}000\right]$; $6A + 2A \geq 3A + 84{,}000$; $5A \geq 84{,}000$; $A \geq \$16{,}800$. \$16,800, *Ans.*

21. Let x = no. bu wheat; then $2x$ = no. bu corn; $240 \leq (x)(.06) + (2x)(.03) \leq 480$;

$240 \leq .06x + .06x$ and $.06x + .06x \leq 480$;
$240 \leq .12x$ and $.12x \leq 480$;
$2000 \leq x$ and $x \leq 4000$;

Max. amt. wheat = 4000 bu; max. amt. corn = 8000 bu; min. amt. wheat = 2000 bu; min. amt. corn = 4000 bu. a. max. amt. corn = 8000 bu, b. min. amt. corn = 4000 bu, *Ans.*

22. Let n = no. chairs; then $\dfrac{n}{4}$ = no. tables; $(n)\left(\dfrac{1}{2}\right) = \dfrac{n}{2}$ = hr to produce n chairs; $\left(\dfrac{n}{4}\right)\left(\dfrac{1}{3}\right) = \dfrac{n}{12}$ = hr to produce $\dfrac{n}{4}$ tables; $126 \leq \dfrac{n}{2} + \dfrac{n}{12} \leq 140$; $126 \leq \dfrac{n}{2} + \dfrac{n}{12}$ and $\dfrac{n}{2} + \dfrac{n}{12} \leq 140$; $12[126] \leq 12\left[\dfrac{n}{2} + \dfrac{n}{12}\right]$ and $12\left[\dfrac{n}{2} + \dfrac{n}{12}\right] \leq 12[140]$; $1512 \leq 6n + n$ and $6n + n \leq 1680$; $1512 \leq 7n$ and $7n \leq 1680$; $216 \leq n$ and $n \leq 240$. 216, 240, *Ans.*

Pages 171–172 · **PROBLEMS**

A

1. Let n = 1st integer; then $n + 1$ = 2nd integer; $n + (n + 1) = 57$; $2n + 1 = 57$; $2n = 56$; $n = 28$, $n + 1 = 29$, *Ans.*

2. Let I = 1st integer; then $I + 1$ = 2nd integer; $I + (I + 1) = 75$; $2I + 1 = 75$; $2I = 74$; $I = 37$, $I + 1 = 38$, *Ans.*

3. Let x = 1st odd integer; then $x + 2$ = 2nd odd integer and $x + 4$ = 3rd odd integer; $x + (x + 2) + (x + 4) = 57$; $3x + 6 = 57$; $3x = 51$; $x = 17$, $x + 2 = 19$, $x + 4 = 21$, *Ans.*

4. Let y = 1st odd integer; then $y + 2$ = 2nd odd integer and $y + 4$ = 3rd odd integer; $y + (y + 2) + (y + 4) = 111$; $3y + 6 = 111$; $3y = 105$; $y = 35$, $y + 2 = 37$, $y + 4 = 39$, *Ans.*

5. Let N = 1st even no.; then $n + 2$ = 2nd even no., $n + 4$ = 3rd even no., and $n + 6$ = 4th even no.; $n + (n + 2) + (n + 4) + (n + 6) = 100$; $4n + 12 = 100$; $4n = 88$; $n = 22$, $n + 2 = 24$, $n + 4 = 26$, $n + 6 = 28$, *Ans.*

6. Let t = 1st even no.; then $t + 2$ = 2nd even no., $t + 4$ = 3rd even no., and $t + 6$ = 4th even no.; $t + (t + 2) + (t + 4) + (t + 6) = 164$; $4t + 12 = 164$; $4t = 152$; $t = 38$, $t + 2 = 40$, $t + 4 = 42$, $t + 6 = 44$, *Ans.*

7. Let p = 1st integer; then $p + 1$ = 2nd integer and $p + 2$ = 3rd integer; $p + (p + 2) = 128$; $2p + 2 = 128$; $2p = 126$; $p = 63$, $p + 1 = 64$, $p + 2 = 65$, *Ans.*

8. Let q = 1st integer; then $q + 1$ = 2nd integer, $q + 2$ = 3rd integer, and $q + 3$ = 4th integer; $(q + 2) + (q + 3) = 63$; $2q + 5 = 63$; $2q = 58$; $q = 29$, $q + 1 = 30$, $q + 2 = 31$, $q + 3 = 32$, *Ans.*

9. Let n = length, in inches, of smallest shelf; then $n + 2$ = length of 2nd shelf, $n + 4$ = length of 3rd shelf, and $n + 6$ = length of largest shelf; $n + (n + 2) + (n + 4) + (n + 6) = 60$; $4n + 12 = 60$; $4n = 48$; $n = 12$, $n + 2 = 14$, $n + 4 = 16$, $n + 6 = 18$, *Ans.*

10. Let s = length, in ft, of smallest side; then $s + 1$ = length of 2nd side and $s + 2$ = length of longest side; $s + (s + 1) + (s + 2) = 240$; $3s + 3 = 240$; $3s = 237$; $s = 79$, $s + 1 = 80$, $s + 2 = 81$, *Ans.*

11. Let x = smaller even integer; then $x + 2$ = larger even integer; $x = 2(x + 2) + 2$; $x = 2x + 4 + 2$; $-x = 6$; $x = -6$, $x + 2 = -4$, *Ans.*

12. Let y = smaller odd integer; then $y + 2$ = larger odd integer; $y + 2 = \frac{1}{3}y - 4$; $3[y + 2] = 3[\frac{1}{3}y - 4]$; $3y + 6 = y - 12$; $2y = -18$; $y = -9$, $y + 2 = -7$, *Ans.*

B

13. Let x = 1st integer; then $x + 1$ = 2nd integer, $x + 2$ = 3rd integer, and $x + 3$ = 4th integer; $5(x + 3) - 2(x + 1) = 7$; $5x + 15 - 2x - 2 = 7$; $3x + 13 = 7$; $3x = -6$; $x = -2$, $x + 1 = -1$, $x + 2 = 0$, $x + 3 = 1$, *Ans.*

14. Let n = 1st even integer; then $n + 2$ = 2nd even integer, $n + 4$ = 3rd even integer, and $n + 6$ = 4th even integer; $4(n + 6) - \frac{1}{2}(n + 2) = 9$; $2[4(n + 6) - \frac{1}{2}(n + 2)] = 2[9]$; $8(n + 6) - 1(n + 2) = 18$; $8n + 48 - n - 2 = 18$; $7n + 46 = 18$; $7n = -28$; $n = -4$, $n + 2 = -2$, $n + 4 = 0$, $n + 6 = 2$, *Ans.*

15. Let x = smaller odd integer; then $x + 2$ = larger odd integer; $3x < 2(x + 2)$; $3x < 2x + 4$; $x < 4$; $x + 2 < 6$. 3, 5, *Ans.*

16. Let y = 1st even integer; then $y + 2$ = 2nd even integer and $y + 4$ = 3rd even integer; $y + (y + 2) + (y + 4) > 24 - 2(y + 4)$; $3y + 6 > 24 - 2y - 8$; $3y + 6 > 16 - 2y$; $5y > 10$; $y > 2$; $y + 2 > 4$; $y + 4 > 6$. 4, 6, 8, *Ans.*

17. Let x = smaller integer; then $x + 1$ = larger integer; $(x + 1) > \frac{x}{2} + 4$; $2(x + 1) > 2\left(\frac{x}{2} + 4\right)$; $2x + 2 > x + 8$; $x > 6$; $x + 1 > 7$. 7, 8, *Ans.*

18. Let n = 1st integer; then $n + 1$ = 2nd integer and $n + 2$ = 3rd integer; $n + (n + 2) < 18 + \frac{(n + 1)}{2}$; $2n + 2 < 18 + \frac{(n + 1)}{2}$; $2[2n + 2] < 2\left[18 + \frac{(n + 1)}{2}\right]$; $4n + 4 < 36 + (n + 1)$; $4n + 4 < 36 + n + 1$; $4n + 4 < 37 + n$; $3n < 33$; $n < 11$; $n + 1 < 12$; $n + 2 < 13$. 10, 11, 12, *Ans.*

Pages 175–176 · **WRITTEN EXERCISES**

1. 90° **2.** 70° **3.** 60° **4.** 30° **5.** 20° **6.** 28° **7.** $(180 - 3n)°$ **8.** $(180 - n)°$
9. $(180 - m - n)°$ **10.** $(90 - x)°$ **11.** $(150 - 2n)°$ **12.** $(200 - 3n)°$ **13.** $-150°$
14. $-75°$ **15.** 0° **16.** $-90°$ **17.** 15° **18.** 0°

19. $(2n + 30)° = (4n - 10)°$; $40° = 2n$; $n = 20°$, *Ans.* **20.** $2m° = (m + 10)°$; $m = 10°$, *Ans.*
21. 30° **22.** 155° **23.** 250° **24.** 125° **25.** 220° **26.** 330°

Page 177 · **PROBLEMS**

A **1.** Let n = no. degrees in comp.; then $n + 12$ = no. degrees in angle; $n + (n + 12) = 90$; $2n + 12 = 90$; $2n = 78°$; $n = 39°$, *Ans.*

2. Let x = no. degrees in smaller angle; then $x + 28$ = no. degrees in larger angle; $x + (x + 28) = 90$; $2x + 28 = 90$; $2x = 62$; $x = 31°$, $x + 28 = 59°$, *Ans.*

3. Let x = no. degrees in comp.; then $2x - 15$ = no. degrees in angle; $x + (2x - 15) = 90$; $3x - 15 = 90$; $3x = 105$; $x = 35$; $2x - 15 = 55°$, *Ans.*

4. Let x = measure, in degrees, of 1st angle; then $3x - 18$ = measure, in degrees, of 2nd angle; $x + (3x - 18) = 90$; $4x - 18 = 90$; $4x = 108$; $x = 27°$, $3x - 18 = 63°$, *Ans.*

5. Let n = degree measure of smaller angle; then $4n$ = degree meas. of larger angle; $n + 4n = 180$; $5n = 180$; $n = 36°$, $4n = 144°$, *Ans.*

6. Let x = degree meas. of smaller angle; then $5x$ = degree meas. of larger angle; $x + 5x = 180$; $6x = 180$; $x = 30°$, $5x = 150°$, *Ans.*

7. Let y = degree meas. of smallest angle; then $2y$ = degree meas. of 2nd angle and $2y + 5$ = degree meas. of largest angle; $y + (2y) + (2y + 5) = 180$; $5y + 5 = 180$; $5y = 175$; $y = 35°$, $2y = 70°$, $2y + 5 = 75°$, *Ans.*

8. Let n = degrees in smallest angle, then $3n$ = degrees in 2nd angle and $4n - 20$ = degrees in 3rd angle; $n + (3n) + (4n - 20) = 180$; $8n - 20 = 180$; $8n = 200$; $n = 25°$, $3n = 75°$, $4n - 20 = 80°$, *Ans.*

9. Let x = degree meas. of each of the equal angles; then $2x - 36$ = degree meas. of 3rd angle; $x + x + (2x - 36) = 180$; $4x - 36 = 180$; $4x = 216$; $x = 54°$, $x = 54°$, $2x - 36 = 72°$, *Ans.*

10. Let t = degree meas. of each of the equal angles; then $\frac{5}{2}(t + t) - 23$ = meas. of 3rd angle; $t + t + \frac{5}{2}(t + t) - 23 = 180$; $2t + \frac{5}{2}(2t) - 23 = 180$; $2t + 5t - 23 = 180$; $7t - 23 = 180$; $7t = 203$; $t = 29°$, $t = 29°$, $\frac{5}{2}(t + t) - 23 = 122°$, *Ans.*

11. Let x = degree meas. of smallest angle; then $x + 23$ = degree meas. of 2nd angle and $(2x + 23) - 6$ = degree meas. of 3rd angle; $x + (x + 23) + (2x + 23 - 6) = 180$; $x + x + 23 + 2x + 17 = 180$; $4x + 40 = 180$; $4x = 140$; $x = 35°$, $x + 23 = 58°$, $(2x + 23) - 6 = 87°$, *Ans.*

12. Let y = degree meas. of smallest angle; then $y + 18$ = degree meas. of 2nd angle; and $2y + 18 - 12$ = degree meas. of 3rd angle; $y + (y + 18) + (2y + 18 - 12) = 180$; $2y + 18 + 2y + 6 = 180$; $4y + 24 = 180$; $4y = 156$; $y = 39°$, $y + 18 = 57°$, $(2y + 18 - 12) = 84°$, *Ans.*

13. Let n = degree meas. of angle; then $90 - n$ = degree meas. of comp. and $180 - n$ = degree meas. of supp.; then $90 - n = \frac{180 - n}{2} + 5$; $2[90 - n] = 2\left[\frac{180 - n}{2} + 5\right]$; $180 - 2n = 180 - n + 10$; $-n = 10°$; $n = -10°$, *Ans.*

14. Let z = degree meas. of angle; then $90 - z$ = degree meas. of comp. and $180 - z$ = degree meas. of supp.; then $180 - z = 2(90 - z) - 12$; $180 - z = 180 - 2z - 12$; $z = -12°$, *Ans.*

Pages 180–182 · **PROBLEMS**

A **1.** Let h = no. hr; $625h + 325h = 1900$; $950h = 1900$; $h = 2$ hr, *Ans.*

	r	· t	= d
West.	625	h	$625h$
East.	325	h	$325h$

2. Let h = no. hr; $350h + 325h = 2025$; $675h = 2025$; $h = 3$ hr, *Ans.*

	r	· t	= d
1st plane	350	h	$350h$
2nd plane	325	h	$325h$

3. Let p = rate of train from Grand Cy.; $4p + 188 = 388$; $4p = 200$; $p = 50$ mph, *Ans.*

	r	$\cdot\ t$	$= d$
Tr. fr. G.C.	p	4	$4p$
Tr. fr. Bell.	47	4	188

4. Let p = rate of southbound boat; $6p + 54 = 120$; $6p = 66$; $p = 11$ mph, *Ans.*

	r	$\cdot\ t$	$= d$
North. boat	9	6	54
South. boat	p	6	$6p$

5. Let h = train's time in hr; then $h - 4$ = plane's time in hr; $50h = 300(h - 4)$; $50h = 300h - 1200$; $-250h = -1200$; $h = \frac{24}{5}$; since $4\frac{4}{5}$ hr = 4 hr 48 min, Time = 9:00 A.M. + 4 hr 48 min = 1:48 P.M., *Ans.*

	r	$\cdot\ t$	$= d$
Train	50	h	$50h$
Plane	300	$h - 4$	$300(h - 4)$

6. Let h = time for freight; then $h - 2$ = time for express; $30h = 50(h - 2)$; $30h = 50h - 100$; $-20h = -100$; $h = 5$. Time = 5:00 A.M. + 5 hr = 10:00 A.M., *Ans.*

	r	$\cdot\ t$	$= d$
Freight	30	h	$30h$
Express	50	$h - 2$	$50(h - 2)$

7. Let p = rate of ship; $15p = 420$; $p = 28$ mph, *Ans.*

	r	$\cdot\ t$	$= d$
Plane	280	$\frac{3}{2}$	420
Ship	p	15	$15p$

8. Let p = Mr. Gomez' rate; $p(\frac{3}{4}) + 213 = \frac{3}{4}(320)$; $\frac{3}{4}p + 213 = 240$; $\frac{3}{4}p = 27$; $p = 36$ mph, *Ans.*

9. Let h = hiking time in hr; then $10 - h$ = riding time in hr; $2h = 18(10 - h)$; $2h = 180 - 18h$; $20h = 180$; $h = 9$; $2h = 18$ mi, *Ans.*

	r	$\cdot\ t$	$= d$
Hiking	2	h	$2h$
Riding	18	$10 - h$	$18(10 - h)$

10. Let h = time walking in hr; then $3 - h$ = time riding in hr; $2h = 10(3 - h)$; $2h = 30 - 10h$; $12h = 30$; $h = \frac{5}{2}$; $2h = 5$ mi, *Ans.*

	r	$\cdot\ t$	$= d$
Riding	10	$3 - h$	$10(3 - h)$
Walking	2	h	$2h$

11. Let h = time walked; then $1 - h$ = time ridden; $5h = 20(1 - h)$; $5h = 20 - 20h$; $25h = 20$; $h = \frac{4}{5}$; $5h = 4$ mi, *Ans.*

	r	$\cdot\ t$	$= d$
Walking	5	h	$5h$
Riding	20	$1 - h$	$20(1 - h)$

12. Let h = time motoring; then $5 - h$ = time rowing; $8h = 2(5 - h)$; $8h = 10 - 2h$; $10h = 10$; $h = 1$; $2(5 - h) = 8$ mi, *Ans.*

	r	$\cdot\ t$	$= d$
Motoring	8	h	$8h$
Rowing	2	$5 - h$	$2(5 - h)$

13. Let h = time by each; $55h + 285h = 1020$; $340h = 1020$; $h = 3$; $3 + 3 = 6$ hr, *Ans.*

	r	$\cdot\ t$	$= d$
Train	55	h	$55h$
Plane	285	h	$285h$

14. Let h = time by each; $45h + 50h = 665$; $95h = 665$; $h = 7$; $7 + 7 = 14$ hr, *Ans.*

	r	$\cdot\ t$	$= d$
Auto	45	h	$45h$
Train	50	h	$50h$

15. Let $h =$ time for each; $39h - 31h = 24$; $8h = 24$; $h = 3$ hr, *Ans.*

	r	$\cdot\ t$	$= d$
Bus	31	h	$31h$
Train	39	h	$39h$

16. Let h = jet's time; then $h + 33$ = train's time; $600h = 50(h + 33)$; $600h = 50h + 1650$; $550h = 1650$; $h = 3$; $h + 33 = 36$ hr, *Ans.*

	r	$\cdot\ t$	$= d$
Jet	600	h	$600h$
Train	50	$h + 33$	$50(h + 33)$

17. Let m = time for each; $325m = 300m + 150$; $25m = 150$; $m = 6$ min, *Ans.*

	r	$\cdot\ t$	$= d$
Stan	325	m	$325m$
Walter	300	m	$300m$

18. Let p = Ned's rate; then $10p - 50 = 200$; $10p = 250$; $p = 25$ ft/sec, *Ans.*

	r	$\cdot\ t$	$= d$
Ned	p	10	$10p$
Jed	20	10	200

B **19.** Let p = rate 1st hr; then $2p$ = rate remainder of time; $p + 3p = 240$; $4p = 240$; $p = 60$ mph, *Ans.*

	r	$: t$	$= d$
1st hr	p	1	p
Remainder	$2p$	$\frac{3}{2}$	$3p$

20. Let p = rate for rest of trip; $4(16) + 6p = 10(22)$; $64 + 6p = 220$; $6p = 156$; $p = 26$ mph, *Ans.*

	r	$\cdot\ t$	$= d$
1st part	16	4	64
2nd part	p	6	$6p$

21. Let h = time on bus; then $\frac{3}{2} - h$ = time walking; $20h \leq 4(\frac{3}{2} - h)$; $20h \leq 6 - 4h$; $24h \leq 6$; $h \leq \frac{1}{4}$; $\frac{1}{4}(20) = 5$ mi, *Ans.*

	r	$\cdot\ t$	$= d$
On bus	20	h	$20h$
Walking	4	$\frac{3}{2} - h$	$4(\frac{3}{2} - h)$

22. Let h = time, in hr, can spend installing parts. Time going $= \dfrac{d}{r} = \dfrac{6}{8} = \dfrac{3}{4}$ hr; time returning at fastest speed $= \dfrac{d}{r} = \dfrac{6}{12} = \dfrac{1}{2}$ hr and $(\frac{3}{4} + \frac{1}{2}) = \frac{5}{4}$ hr = total travel time. Since total time away allowed = 2 hr, max. time, h, which can be spent installing parts is $(2 - \frac{5}{4})$ hr $= \frac{3}{4}$ hr, *Ans.*

Pages 183–184 · **PROBLEMS**

A

1. Let n = no. 25¢ souvenirs; then $(1000 - n)$ = no. 20¢ souvenirs; $.25n + .20(1000 - n) = 220$; $.25n + 200 - .20n = 220$; $.05n = 20$; $n = 400$; $1000 - n = 600$. 400 @ 25¢, 600 @ 20¢, *Ans.*

2. Let n = no. sold at 38¢; then $348 - n$ = no. sold at 25¢; $.38n + .25(348 - n) = 100.91$; $100[.38n + .25(348 - n)] = 100[100.91]$; $38n + 25(348 - n) = 10091$; $38n + 8700 - 25n = 10091$; $13n = 1391$; $n = 107$; $348 - n = 241$. 107 @ 38¢, 241 @ 25¢, *Ans.*

3. Let x = no. lb \$1.65 candy; then $100 - x$ = no. lb \$1.90 candy; $(1.65)x + (1.90)(100 - x) = 1.75(100)$; $1.65x + 190 - 1.90x = 175$; $-.25x = -15$; $-100[-.25x] = -100[-15]$; $25x = 1500$; $x = 60$; $100 - x = 40$. 60 lb @ \$1.65, 40 lb @ \$1.90, *Ans.*

4. Let n = no. lb 90¢ tea; then $20 - n$ = no. lb \$1.50 tea; $.90n + 1.50(20 - n) = 1.20(20)$; $100[.90n + 1.50(20 - n)] = 100[24]$; $90n + 150(20 - n) = 2400$; $90n + 3000 - 150n = 2400$; $-60n = -600$; n = 10; $20 - n = 10$. 10 lb @ \$.90, 10 lb @ \$1.50, *Ans.*

5. Let z = no. lb 84¢ coffee; then $24 + z$ = no. lb in total mix. (@ 68¢ lb); $.84z + .60(24) = .68(24 + z)$; $100[.84z + .60(24)] = 100\ [.68(24 + z)]$; $84z + 60(24) = 68(24 + z)$; $84z + 1440 = 1632 + 68z$; $16z = 192$; $z = 12$ lb, *Ans.*

6. Let x = no. adults; then $328 - x$ = no. children; $.42x + .18(328 - x) = 69.12$; $100[.42x + .18(328 - x)] = 100[69.12]$; $42x + 18(328 - x) = 6912$; $42x + 5904 - 18x = 6912$; $24x = 1008$; $x = 42$ adults, *Ans.*

7. Let n = no. nickels; then $48 - n$ = no. pennies; $5n + 1(48 - n) = 100$; $5n + 48 - n = 100$; $4n + 48 = 100$; $4n = 52$; $n = 13$ nickels, $48 - n = 35$ pennies, *Ans.*

8. Let n = no. 1¢ stamps; then $n + 19$ = no. 3¢ stamps; $1(n) + 3(n + 19) = 305$; $n + 3n + .57 = 305$; $4n + 57 = 305$; $4n = 248$; $n = 62$; $n + 19 = 81$. 62 @ 1¢, 81 @ 3¢, *Ans.*

B

9. Let w = no. lb walnuts; then $2w$ = no. lb peanuts and $21 - 3w$ = no. lb almonds;

$1.30w + .65(2w) + 1.80(21 - 3w) = 1.00(21)$; $1.30w + 1.30w + 37.80 - 5.40w = 21.00$; $-2.80w = -16.80$; $w = 6$; $2w = 12$; $21 - 3w = 3$. 6 lb walnuts, 12 lb peanuts, 3 lb almonds, *Ans.*

10. Let n = no. 6¢ stamps; then $3n$ = no. 3¢ stamps and $6n$ = no. $1\frac{1}{2}$¢ stamps and no. 1¢ stamps; $6(n) + 3(3n) + \frac{3}{2}(6n) + 1(6n) = 300$; $6n + 9n + 9n + 6n = 300$; $30n = 300$; $n = 10$; $3n = 30$; $6n = 60$. 10 @ 6¢, 30 @ 3¢, 60 @ $1\frac{1}{2}$¢, 60 @ 1¢, *Ans.*

11. Let N = no. half-dollars; then $2N$ = no. quarters and $\dfrac{N}{2}$ = no. dollar bills.

To be possible: $\frac{1}{2}(N) + \frac{1}{4}(2N) + 1\left(\dfrac{N}{2}\right) = 20$, N a positive integer;

$4\left[\dfrac{N}{2} + \dfrac{2N}{4} + \dfrac{N}{2}\right] = 4[20]$, $2N + 2N + 2N = 80$; $6N = 80$; $N = \frac{40}{3}$; but $\frac{40}{3}$ is not an integer; $\therefore$ *impossible.*

12. Let y = no. pencils @ 10¢; then $9y$ = no. erasers @ 5¢ and $100 - 10y$ = no. clips @ $\frac{1}{2}$¢; $10(y) + 5(9y) + \frac{1}{2}(100 - 10y) = 100$; $10y + 45y + 50 - 5y = 100$; $50y = 50$; $y = 1$; $9y = 9$; $100 - 10y = 90$. 1 pencil, 9 erasers, 90 clips, *Ans.*

C

13. Let n = no. g 60% alloy; then $(10 - n)$ = no. g 30% alloy. $.6n$ = no. g gold in 60% alloy and $.3(10 - n)$ = no. g gold in 30% alloy. $.6n + .3(10 - n) = 5$; $.6n + 3 - .3n = 5$; $.3n + 3 = 5$; $.3n = 2$; $10[.3n] = 10[2]$; $3n = 20$; $n = \frac{20}{3}$g *or* $6\frac{2}{3}$ g, *Ans.*

14. Let n = no. liters 50% soln.; then $(10 - n)$ = no. liters 75% soln. $.50(n)$ = no. liters acid in 50% soln., and $.75(10 - n)$ = no. liters acid in 75% soln. $.50n + .75(10 - n) = \frac{2}{3}(10)$; $300[.50n + .75(10 - n)] = 300[\frac{2}{3}(10)]$; $150n + 225(10 - n) = 200(10)$; $150n + 2250 - 225n = 2000$; $-75n = -250$; $n = \frac{10}{3}$; $10 - n = \frac{20}{3}$. $\frac{10}{3}$ liters 50% soln., $\frac{20}{3}$ liters 75% soln., *Ans.*

Pages 187–188 · **CHAPTER TEST**

1. $\dfrac{3y}{5} = -36$; $5\left(\dfrac{3y}{5}\right) = 5(-36)$; $3y = -180$; $y = -60$. $\{-60\}$

2. $5s - 12s - 21 = 9 - s$; $-7s - 21 = 9 - s$; $-6s = 30$; $s = -5$. $\{-5\}$

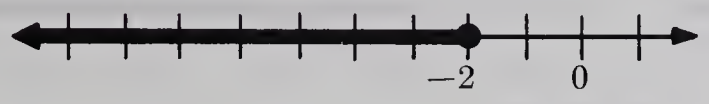
3. $a\left(\frac{x}{a} - c\right) = a(b)$; $x - ac = ab$; $x = ab + ac$. $\{ab + ac\}$, $a \neq 0$

4. $p = 2l + 2w$; $p - 2w = 2l$; $\frac{p - 2w}{2} = l$. $\left\{\frac{p - 2w}{2}\right\}$

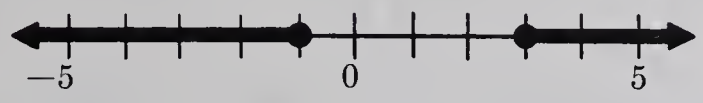
5. a. $x < -5$ b. $-3 < v$

6. $3x > -6$; $x > -2$.

7. $47 \leq 7 - 15t + 10$; $47 \leq 17 - 15t$; $30 \leq -15t$; $-2 \geq t$.



8. (Optional) a. $1 - y \geq 2$ or $-(1 - y) \geq 2$; $-y \geq 1$ or $-1 + y \geq 2$; $y \leq -1$ or $y \geq 3$.
b. $(3m - 6) > 9$ or $-(3m - 6) > 9$; $3m > 15$ or $-3m + 6 > 9$; $m > 5$ or $-3m > 3$; $m > 5$ or $m < -1$.



9. Let l = length in yd; then $(36 - l)$ = width in yd; $2l = 3(36 - l) - 8$; $2l = 108 - 3l - 8$; $5l = 100$; $l = 20$; $36 - l = 16$. 20 yd, 16 yd, *Ans.*

10. Let n = no. B can do in 1 hr; then $2n - 50$ = no. A can do in 1 hr; $n + (2n - 50) \geq 175$; $n + 2n - 50 \geq 175$; $3n \geq 225$; $n \geq 75$; $2n - 50 \geq 100$. 100, *Ans.*

11. Let x = 1st odd integer; then $x + 2$ = 2nd odd integer and $x + 4$ = 3rd odd integer. $x + (x + 4) = (x + 2) + 27$; $2x + 4 = x + 29$; $x = 25$, $x + 2 = 27$, $x + 4 = 29$, *Ans.*

12. Let x = measure in degrees of angle; then $90 - x$ = measure of comp. $(90 - x) = 2x - 18$; $-3x = -108$; $x = 36°$, $90 - x = 54°$, *Ans.*

13. Let x = no. degrees in 1st angle; then $x + 12$ = no. degrees in 2nd angle and $[x + (x + 12) - 12]$ = no. degrees in 3rd angle; $x + (x + 12) + [x + (x + 12) - 12] = 180$; $2x + 12 + [2x] = 180$; $4x + 12 = 180$; $4x = 168$; $x = 42°$, $x + 12 = 54°$, $[x + (x + 12) - 12] = 84°$, *Ans.*

14. Let p = time for plane; then $p - \frac{3}{2}$ = time for jet; $340p = 595(p - \frac{3}{2})$; $2[340p] = 2[595(p - \frac{3}{2})]$; $680p = 1190p - 1785$; $-510p = -1785$; $p = \frac{7}{2}$; $p - \frac{3}{2} = 2$ hr, *Ans.*

	r	$\cdot$ t	$=$ d
Plane	340	p	$340p$
Jet	595	$p - \frac{3}{2}$	$595(p - \frac{3}{2})$

15. Let x = no. adults; then $700 - x$ = no. students; $.50(x) + .20(700 - x) = 305$; $.5x + 140 - .2x = 305$; $.3x = 165$; $10[.3x] = 10[165]$; $3x = 1650$; $x = 550$; $700 - x = 150$. 550 adults, 150 students, *Ans.*

Pages 188–191 · CHAPTER REVIEW

1. directed **2.** equivalent **3.** solution set **4.** Add. **5.** Subt.
6. Div. **7.** Mult.

8. (a) Subt. transf.; (b) div. transf.; (c) subst. principle

9. (a) Add. transf.; (b) Mult. transf.; (c) Div. transf.

10. $13n = -1.3$; $n = -\frac{1}{10}$. $\{-\frac{1}{10}\}$

11. $2 = -\frac{4}{3}w$; $3(2) = 3(-\frac{4}{3}w)$; $6 = -4w$; $w = -\frac{3}{2}$. $\{-\frac{3}{2}\}$

12. $7 + 4t - 12 = t - 5$; $4t - 5 = t - 5$; $3t = 0$; $t = 0$. $\{0\}$

13. $2 - r + 3 = 3r + 3r + 12$; $5 - r = 6r + 12$; $-7 = 7r$; $r = -1$. $\{-1\}$

14. $z = \frac{d}{c}$, $c \neq 0$. $\left\{\frac{d}{c}\right\}$, $c \neq 0$

15. $2y - 5y = 3p$; $-3y = 3p$; $y = -p$. $\{-p\}$

16. $r(t) = r\left(\frac{d}{r}\right)$; $rt = d$; $r = \frac{d}{t} \cdot \left\{\frac{d}{t}\right\}$, $rt \neq 0$

17. $2[m] = 2[\frac{1}{2}(a + b)]$; $2m = a + b$; $2m - b = a$. $\{2m - b\}$

18. $p - a = -i$; $-p + a = i$. $\{a - p\}$

19. $5(F) = 5(\frac{9}{5}C + 32)$; $5F = 9C + 160$; $5F - 160 = 9C$; $\dfrac{5F - 160}{9} = C$. $\left\{\dfrac{5F - 160}{9}\right\}$

20. $n = ac$; $a \neq 0$. $\{ac\}$, $a \neq 0$ **21.** $4a = -2x$; $x = -2a$. $\{-2a\}$

22. $<$ **23.** left **24.** $>$ **25.** $<$ **26.** $<$ **27.** $<$ **28.** $>$ **29.** $=$ **30.** $<$ **31.** $<$ **32.** $>$, $<$

33. $>$ **34.** $-9x$ **35.** reverse **36.** $\leq$

37. $3n < 3$; $n < 1$, *Ans.* **38.** $-8m < 4$; $m > -\frac{1}{2}$, *Ans.*

39. $x > -3$, $x < 2$ **40.** 4, −4

41. $(3x - 2) \geq 4$ or $(3x - 2) \leq -4$; $3x \geq 6$ or $3x \leq -2$; $x \geq 2$ or $x \leq -\frac{2}{3}$, *Ans.*

42. −4, 4

43. $(3x - 1) > 4$ or $3x - 1 < -4$; $3x > 5$ or $3x < -3$; $x > \frac{5}{3}$ or $x < -1$, *Ans.*

44. empty **45.** directed

46. Let n = B's votes; then $\dfrac{n}{2}$ = A's votes and $n + 110$ = C's votes; $n + \dfrac{n}{2} + (n + 110) = 1000$;

$2n + \dfrac{n}{2} + 110 = 1000$; $2n + \dfrac{n}{2} = 890$; $2\left[2n + \dfrac{n}{2}\right] = 2[890]$; $4n + n = 1780$; $5n = 1780$;

$n = 356$; $\dfrac{n}{2} = 178$; $n + 110 = 466$. 178 for A, 356 for B, 466 for C, *Ans.*

47. Let n = length in inches of 2nd side; then $2n + 5$ = length in inches of 1st side and $2n - 5$ = length in inches of 3rd side. $n + (2n + 5) + (2n - 5) \geq 50$; $5n \geq 50$; $n \geq 10$; $2n + 5 \geq 25$. 25, *Ans.*

48. $\{-2, 0, 2, 4, 6\}$

49. Let x = 1st integer; then $x + 1$ = 2nd integer and $x + 2$ = 3rd integer;
$x + (x + 1) + (x + 2) = 2274$; $3x + 3 = 2274$; $3x = 2271$; $x = 757$, $x + 1 = 758$, $x + 2 = 759$, *Ans.*

50. Let x = 1st odd integer; then $x + 2$ = 2nd odd integer, $x + 4$ = 3rd odd integer, $x + 6$ = 4th odd integer, and $x + 8$ = 5th odd integer; $x + (x + 2) + (x + 4) + (x + 6) + (x + 8) = 85$; $5x + 20 = 85$; $5x = 65$; $x = 13$, $x + 2 = 15$, $x + 4 = 17$, $x + 6 = 19$, $x + 8 = 21$, *Ans.*

51. 90

52. Let x = 1st angle; then $x - 24$ = 2nd angle and $x + (x - 24) = 90$; $2x - 24 = 90$; $2x = 114$; $x = 57°$, $x - 24 = 33°$, *Ans.*

53. Let x = one angle; then $5x$ = other angle; $x + 5x = 180$; $6x = 180$; $x = 30°$, $5x = 150°$, *Ans.*

54. Let x = meas. of each of the equal angles; then $180 - 2x$ = meas. of third angle;
$180 - 2x \leq 2(2x) - 12$; $180 - 2x \leq 4x - 12$; $-6x \leq -192$; $x \geq 32°$. 32°, *Ans.*

55. Let y = walking time; then $\frac{3}{2} - y$ = riding time; $2y = 10(\frac{3}{2} - y)$; $2y = 15 - 10y$; $12y = 15$; $y = \frac{5}{4}$; $d = 2y = \frac{5}{2}$ mi, *Ans.*

	r	· t	= d
Walking	2	y	$2y$
Riding	10	$\frac{3}{2} - y$	$10(\frac{3}{2} - y)$

56. Let y = time on 1st leg; then $1 - y$ = time on remainder; $45y = 36(1 - y)$;
$45y = 36 - 36y$; $81y = 36$; $y = \frac{4}{9}$;
$d = 45y = 45(\frac{4}{9}) = 20$ mi, *Ans.*

	r	· t	= d
1st leg	45	y	$45y$
Remain.	36	$1 - y$	$36(1 - y)$

57. Let p = rate of bus; then $2p$ = train's rate. $p(\frac{1}{2}) = \dfrac{p}{2}$ = dist. by bus in half hour and $2p(\frac{1}{2}) =$

p = dist. by train in half hour. $\frac{25}{2} \leq p - \dfrac{p}{2} \leq 15$; $2[\frac{25}{2}] \leq 2\left[p - \dfrac{p}{2}\right] \leq 2[15]$;

$25 \leq 2p - p \leq 30$; $25 \leq p \leq 30$. Max. rate of bus = 30 mph, max. rate of train = 60 mph, min. rate of bus = 25 mph, min. rate of train = 50 mph, *Ans.*

58. Let x = mi by train; then $(80 - x)$ = mi by auto and $(x - 10)$ = mi by train *if* had gone 10 more mi by auto; $2[(80 - x) + 10] = (x - 10)$; $2[90 - x] = x - 10$; $180 - 2x = x - 10$; $190 = 3x$; $x = 63\frac{1}{3}$ mi, *Ans.*

59. Let x = pt of 12¢-oil; then $12x$ = value in cents of 12¢-oil, $5(100)$ = value in cents of 5¢-oil, $7(x + 100)$ = value in cents of final mixture at 7¢ a pt. $12x + 5(100) = 7(x + 100)$; $12x + 500 = 7x + 700$; $5x = 200$; $x = 40$ pt, *Ans.*

60. Let x = lb of creams; then $65x$ = value of creams in cents; $10(125)$ = value of mixture of nut-fruit candy, in cents, $(x + 10)$ = lb in final mixture, and $85(x + 10)$ = value of final mixture, in cents. $65x + 10(125) = 85(x + 10)$; $65x + 1250 = 85x + 850$; $400 = 20x$; $x = 20$ lb, *Ans.*

61. Let n = no. students; then $115 - n$ = no. others; $15n + 35(115 - n) = 2565$; $15n + 4025 - 35n = 2565$; $-20n = -1460$; $n = 73$ students, *Ans.*

Page 195 · EXTRA FOR EXPERTS

1. a. 5 b. 3 c. 1 d. $h - 1$, h is an integer greater than 1.

2. a. $1357_{IX} = 1(9^3) + 3(9^2) + 5(9^1) + 7(9^0) = 1024$
b. $1021_{III} = 1(3^3) + 0(3^2) + 2(3^1) + 1(3^0) = 34$
c. $10101_{II} = 1(2^4) + 0(2^3) + 1(2^2) + 0(2^1) + 1(2^0) = 21$
d. $246_{VII} = 2(7^2) + 4(7^1) + 6(7^0) = 132$

3. a. $\frac{345}{7} = 49$, r = 2; $\frac{49}{7} = 7$, r = 0; $\frac{7}{7} = 1$, r = 0; $\frac{1}{7} = 0$, r = 1; $\therefore 345_X = 1002_{VII}$
b. $\frac{20}{2} = 10$, r = 0; $\frac{10}{2} = 5$, r = 0; $\frac{5}{2} = 2$, r = 1; $\frac{2}{2} = 1$, r = 0; $\frac{1}{2} = 0$, r = 1; $\therefore 20_X = 10100_{II}$
c. $\frac{500}{5} = 100$, r = 0; $\frac{100}{5} = 20$, r = 0; $\frac{20}{5} = 4$, r = 0; $\frac{4}{5} = 0$, r = 4; $\therefore 500_X = 4000_V$
d. $\frac{701}{6} = 116$, r = 5; $\frac{116}{6} = 19$, r = 2; $\frac{19}{6} = 3$, r = 1; $\frac{3}{6} = 0$, r = 3; $\therefore 701_X = 3125_{VI}$

4. 0, 1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111, 10000, 10001, 10010, 10011, 10100, 10101, 10110, 10111, 11000, 11001, 11010, 11011, 11100, 11101, 11110, 11111, 100000, 100001, 100010, 100011, 100100, 100101, 100110, 100111, 101000.

5. a. Odds end with 1; evens with 0. b. $10_{II} = 1(2^1) + 0(2^0) = 2 + 0 = 2_{10}$.

6. $(10)_n = 1(n^1) + 0(n^0) = n^1 + 0 = n$; hence $(10)_n = n(n \neq 0)$.

CHAPTER 6. Working with Polynomials

Pages 199–200 · WRITTEN

A **1.** $-3n - 4h$

2. $-7r - 2.1s$

3.
$$\begin{array}{r} \frac{2}{2}x^3 - \frac{3}{3}y^3 \\ \frac{1}{2}x^3 + \frac{1}{3}y^3 \\ \hline \frac{3}{2}x^3 - \frac{2}{3}y^3 \end{array}$$

4.
$$\begin{array}{r} \frac{4}{4}m^2 + \frac{8}{8}n^2 \\ \frac{1}{4}m^2 - \frac{1}{8}n^2 \\ \hline \frac{5}{4}m^2 + \frac{7}{8}n^2 \end{array}$$

5.
$$\begin{array}{r} -\frac{1}{3}m + \frac{1}{3}n + \frac{1}{6} \\ \frac{3}{3}m - \frac{3}{3}n - \frac{6}{6} \\ \hline \frac{2}{3}m - \frac{2}{3}n - \frac{5}{6} \end{array}$$

6.
$$\begin{array}{r} -\frac{1}{2}x - \frac{1}{2}y - \frac{1}{4} \\ \frac{2}{2}x - \frac{2}{2}y + \frac{4}{4} \\ \hline \frac{1}{2}x - \frac{3}{2}y + \frac{3}{4} \end{array}$$

7. $x - 2y$

8. 0

9. $-3yz + 10$

10. $-13ab + 4$

11. $10x - 10$

12. $13z + 3$

13. $.7xy + 8.8$

14. $.9rs + 13.3$

15. 1.3

16. $9n - .3t + .2$

17. $12h^2 - 4.3dh + .4d^2$

18. $-2xy - .1y^2$

19. $8x^3 - 60x^2 + 150x - 125$

20. $2y^3 - 7y^2 + 5y - 1$

21.
$$\begin{array}{rcl} 5(3) - 6 = 15 - 6 & = & 9 \\ 2(3) + 1 = 6 + 1 & = & \underline{7} \\ 7(3) - 5 = 21 - 5 & = & 16 \end{array}$$

22.
$$\begin{array}{rcl} 3(2) + 7 = 6 + 7 & = & 13 \\ 2(2) - 80 = 4 - 80 & = & \underline{-76} \\ 5(2) - 87 = 10 - 87 & \neq & -63 \end{array}$$
Correct answer is $5y - 73$

23.
$$\begin{array}{rcl} 3(2) - 8 = 6 - 8 & = & -2 \\ -2(2) + 1 = -4 + 1 & = & -3 \\ -(2) + 3 = & & \underline{1} \\ 4(2) - 4 = 8 - 4 & \neq & -4 \end{array}$$
Correct answer is -4

24.
$$\begin{array}{rcl} 5(5) - (2) + 2 = 25 - 2 + 2 & = & 25 \\ -(5) - (2) - 3 = & & -10 \\ 7(5) + 2(2) - 8 = 35 + 4 - 8 & = & \underline{31} \\ 11(5) - 3 = 55 - 3 & \neq & 46 \end{array}$$
Correct answer is $11x - 9$

25. $2^2 + 2 + 6 = 4 + 2 + 6 = 12$
$3(2)^2 - 2 - 8 = 12 - 2 - 8 = \underline{2}$
$4(2)^2 - 2 = 16 - 2 = 14$

26. $3^2 - 3 + 5 = 9 - 3 + 5 = 11$
$2(3)^2 + 2(3) - 5 = 18 + 6 - 5 = \underline{19}$
$3(3)^2 + 3 = 27 + 3 = 30$

27. to **29.** Figures used in computations will vary. Additions are correct as given in text.

B **30.** $12x^3 + 4x^2 - 3$

31. $5y^2 - 10y + 2$

32. $t^5 + 2t^4 - t^3 - t^2 + 3t + 2$

33. $-s^4 - s^3 + 10s + 15$

Pages 202–203 · **WRITTEN EXERCISES**

A **1.** $21 - 14x = 21 - 14(3) = 21 - 42 = -21$
$12 - 6x = 12 - 6(3) = 12 - 18 = \underline{-6}$
$9 - 8x = 9 - 8(3) = 9 - 24 = -15$

2. $3x + 2y = 3(3) + 2(2) = 9 + 4 = 13$
$x - y = (3) - (2) = \underline{1}$
$2x + 3y = 2(3) + 3(2) = 6 + 6 = 12$

3. $2x^2 + 2x + 7 = 2(3^2) + 2(3) + 7 = 18 + 6 + 7 = 31$
$x^2 + 3x - 8 = 3^2 + 3(3) - 8 = 9 + 9 - 8 = \underline{10}$
$x^2 - x + 15 = 3^2 - (3) + 15 = 9 - 3 + 15 = 21$.

4. $3b^2 + b - 1 = 3(4^2) + (4) - 1 = 48 + 4 - 1 = 51$
$2b^2 + 6b + 5 = 2(4^2) + 6(4) + 5 = 32 + 24 + 5 = \underline{61}$
$b^2 - 5b - 6 = 4^2 - 5(4) - 6 = 16 - 20 - 6 = -10$

5. $.4x^2 + 0 - .7 = .4(3^2) + 0 - .7 = 3.6 - .7 = 2.9$
$.5x^2 - 3x - .8 = .5(3^2) - 3(3) - .8 = 4.5 - 9 - .8 = \underline{-5.3}$
$-.1x^2 + 3x + .1 = -.1(3^2) + 3(3) + .1 = -.9 + 9 + .1 = 8.2$

6. $2 - 3a + 2a^2 = 2 - 3(5) + 2(5^2) = 2 - 15 + 50 = 37$
$5 - 6a + a^2 = 5 - 6(5) + 5^2 = 5 - 30 + 25 = \underline{0}$
$-3 + 3a + a^2 = -3 + 3(5) + 5^2 = -3 + 15 + 25 = 37$

7. $x^2 + 0 - 1 = 3^2 + 0 - 1 = 8$
$0 + x + 2 = 0 + 3 + 2 = \underline{5}$
$x^2 - x - 3 = 3^2 - 3 - 3 = 3$

8. $a^2 + a + 0 = 5^2 + 5 = 30$
$0 + a - 1 = 5 - 1 = \underline{4}$
$a^2 + 1 = 5^2 + 1 = 26$

9. $.3x^2 + 7x - 1.8 = .3(3^2) + 7(3) - 1.8 = 2.7 + 21 - 1.8 = 21.9$
$x^2 - 9x - .2 = (3^2) - 9(3) - .2 = 9 - 27 - .2 = \underline{-18.2}$
$-.7x^2 + 16x - 1.6 = -.7(3^2) + 16(3) - 1.6 = -6.3 + 48 - 1.6 = 40.1$

10. $2y + 5 - y = 18; y + 5 = 18; y = 13.$ {13}

11. $5w - 2w - 3 = 12; 3w - 3 = 12; 3w = 15; w = 5.$ {5}

12. $13z - z - 21 = 39; 12z - 21 = 39; 12z = 60; z = 5.$ {5}

13. $3x - 5 + 2x = 35; 5x - 5 = 35; 5x = 40; x = 8.$ {8}

14. $7x + 8x + 54 = 9; 15x + 54 = 9; 15x = -45; x = -3.$ {−3}

15. $9y + 7y + 67 = 3; 16y + 67 = 3; 16y = -64; y = -4.$ {−4}

16. $5x + 2 - 7x = 30; -2x + 2 = 30; -2x = 28; x = -14.$ {−14}

17. $y + 9 - 8y = 65; -7y + 9 = 65; -7y = 56; y = -8.$ {−8}

18. $9n - 3n + 6 = 42; 6n + 6 = 42; 6n = 36; n = 6.$ {6}

19. $6z - 4z + 8 = 24; 2z + 8 = 24; 2z = 16; z = 8.$ {8}

20. $3x + 4 - x - 8 = 18; 2x - 4 = 18; 2x = 22; x = 11.$ {11}

21. $5y + 7 - y - 2 = 17; 4y + 5 = 17; 4y = 12; y = 3.$ {3}

22. $7n - n^2 + n + 9 = 17 - n^2; 8n - n^2 + 9 = 17 - n^2; 8n = 8; n = 1.$ {1}

23. $19x - 1 + 2x + x^2 = 22 + x^2; 21x - 1 + x^2 = 22 + x^2; 21x = 23; x = \frac{23}{21}.$ $\{\frac{23}{21}\}$

24. $2x + 4 - x + 7 = 4 - 3x + 3x + 1; x + 11 = 5; x = -6.$ {−6}

25. $3y + 6 - 2y + 2 = 17 - 2y + 2y - 13; y + 8 = 4; y = -4.$ {−4}

26. $10 - y - 6 + 3y - 2 = -5y + 9 + 5y - 3; 2 + 2y = 6; 2y = 4; y = 2.$ {2}

27. $12 - z - 8 - 5z + 7 = -6z + 16 + 6z - 7; -6z + 11 = 9; -6z = -2; z = \frac{1}{3}.$ $\{\frac{1}{3}\}$

28. $x - .5x - 1 - .3x - 2 = .5x - .5x + 3; .2x - 3 = 3; .2x = 6; 2x = 60; x = 30.$ {30}

29. $.5n + n - 1 - .2n - 10 = .2n - .2n + 28; 1.3n - 11 = 28; 1.3n = 39; 13n = 390;$
$n = 30.$ {30}

B **30.** $2a - [5a - 6a - 2] = 10 - a; 2a - [-a - 2] = 10 - a; 2a + a + 2 = 10 - a;$
$3a + 2 = 10 - a; 4a = 8; a = 2.$ {2}

31. $7x - [x - 2x - 8] = 3x - 2$; $7x - [-x - 8] = 3x - 2$; $7x + x + 8 = 3x - 2$; $8x + 8 = 3x - 2$; $5x = -10$; $x = -2$. $\{-2\}$

32. $20x - [2x - x - 2 + x^2 - 6] = 28 - x - x^2$; $20x - [x - 8 + x^2] = 28 - x - x^2$; $20x - x + 8 - x^2 = 28 - x - x^2$; $19x + 8 = 28 - x$; $20x = 20$; $x = 1$. $\{1\}$

33. $5y - [y - 2y - 8 + 3 - y^2] = y^2 - y - 5$; $5y - [-y - 5 - y^2] = y^2 - y - 5$; $5y + y + 5 + y^2 = y^2 - y - 5$; $6y + 5 = -y - 5$; $7y = -10$; $y = -\frac{10}{7}$. $\{-\frac{10}{7}\}$

34. $-[-5t - 4 + t + 7] - t + 2 = 4t - 1$; $-[-4t + 3] - t + 2 = 4t - 1$; $4t - 3 - t + 2 = 4t - 1$; $3t - 1 = 4t - 1$; $0 = t$. $\{0\}$

35. $-[-7 + 2s - 8 + 3s] - 3s - 1 = -3s - 1$; $-[-15 + 5s] - 3s - 1 = -3s - 1$; $15 - 5s - 3s - 1 = -3s - 1$; $14 - 8s = -3s - 1$; $-5s = -15$; $s = 3$. $\{3\}$

C **36.** $2a + \{5a - [3 - a - 2] + 6\} = -2a - 5$; $2a + \{5a - [1 - a] + 6\} = -2a - 5$; $2a + \{5a - 1 + a + 6\} = -2a - 5$; $2a + \{6a + 5\} = -2a - 5$; $8a + 5 = -2a - 5$; $10a = -10$; $a = -1$. $\{-1\}$

37. $5n + \{n - [2 - 2 - 3n] + n\} = -2n + 6$; $5n + \{n - [-3n] + n\} = -2n + 6$; $5n + \{n + 3n + n\} = -2n + 6$; $5n + 5n = -2n + 6$; $10n = -2n + 6$; $12n = 6$; $n = \frac{1}{2}$. $\{\frac{1}{2}\}$

38. $1.2x - \{10x + [2 - .6x - (3.2 + x)] + 13\} = -1$; $1.2x - \{10x + [2 - .6x - 3.2 - x] + 13\} = -1$; $1.2x - \{10x + [-1.2 - 1.6x] + 13\} = -1$; $1.2x - \{8.4x + 11.8\} = -1$; $1.2x - 8.4x - 11.8 = -1$; $-7.2x - 11.8 = -1$; $-7.2x = 10.8$; $-72x = 108$; $x = -\frac{108}{72} = -\frac{3}{2}$. $\{-\frac{3}{2}\}$

39. $1.4y - \{9y + [3y - .3 - (2.2y - 3)] + 1.7\} = -3$; $1.4y - \{9y + [3y - .3 - 2.2y + 3] + 1.7\} = -3$; $1.4y - \{9y + .8y + 2.7 + 1.7\} = -3$; $1.4y - \{9.8y + 4.4\} = -3$; $1.4y - 9.8y - 4.4 = -3$; $-8.4y - 4.4 = -3$; $-8.4y = 1.4$; $-84y = 14$; $y = -\frac{1}{6}$. $\{-\frac{1}{6}\}$

Page 204 · WRITTEN EXERCISES

A **1.** $-15r^6s^3$ **2.** $-48x^3y^5$ **3.** $27a^3$ **4.** $60a^2b^2$ **5.** $-30xyz$ **6.** $216n^3$
7. $72x^2y^2$ **8.** $-14abc$ **9.** $\frac{1}{4}g^2t^2$ **10.** $-48r^3s^3t$ **11.** $\frac{1}{16}t^2x^2$ **12.** $-r^3s^2$
13. $-a^2b^3$ **14.** r^4t^3 **15.** $12x^6y^6$ **16.** $10a^9b^6$ **17.** $.0064r^4s^{15}$ **18.** $7.29z^4v^9$
19. $c^9d^5f^6$ **20.** $w^5x^{12}y^6$ **21.** $-a^5b^4$ **22.** $-h^4k^7$ **23.** $-.72d^3m^4$ **24.** $-1.28b^3c^4$

B **25.** $70a^2b^2 - 8a^2b^2 = 62a^2b^2$ **26.** $15p^4q^2 - 49p^4q^2 = -34p^4q^2$
27. $-12r^3s^2t^2 + 12r^3s^2t^2 = 0$ **28.** $-18v^3w^3z^4 + 18v^3w^3z^4 = 0$
29. $100c^6d^{10}e - 9c^6d^{10}e = 91c^6d^{10}e$ **30.** $-72m^7n^7p^5 + 64m^7n^7p^5 = -8m^7n^7p^5$

C **31.** $2x^4 + x^3 + 3x^4 + 3x^3 = 5x^4 + 4x^3$ **32.** $3r^4 - 3r^3 - 2r^4 + r^3 = r^4 - 2r^3$
33. $10^{m+n} + 10^{m+r}$ **34.** $a^{m+n} + a^mb^m + a^mb^m + b^{m+n} = a^{m+n} + 2a^mb^m + b^{m+n}$

Page 206 · WRITTEN EXERCISES

A **1.** $-8m^6$ **2.** $-27s^6$ **3.** $-3x(16x^2y^2) = -48x^3y^2$
4. $-6n(25m^2n^2) = -150m^2n^3$ **5.** $(-b)(-27b^3) = 27b^4$ **6.** $(-r)(-216r^3) = 216r^4$
7. $(8m)(8m^3n^3) = 64m^4n^3$ **8.** $(3y)(125x^3y^3) = 375x^3y^4$ **9.** $(5a)(4a^4b^2) = 20a^5b^2$
10. $(-7b)(9b^4c^2) = -63b^5c^2$ **11.** $(25x^2)(.008y^6x^3) = .2x^5y^6$ **12.** $(.09a^2)(-b^6a^3) = -.09a^5b^6$
13. $(.0016b^8d^4)(9d^2b^2) = .0144b^{10}d^6$ **14.** $(-.125k^6p^3)(-8k^3p^3) = k^9p^6$
15. $(9s^2t^4v^2)(9s^4t^2v^6) = 81s^6t^6v^8$

B **16.** $9x^2y^2 - 20x^2y^2 = -11x^2y^2$ **17.** $(-2c)(-27c^3d^3) + (9c^2)(c^2d^2)(-6d) = 54c^4d^3 - 54c^4d^3 = 0$
18. $(a^2)a + (2a^2)(-8a^6) - (16a^8) = a^3 - 16a^8 - 16a^8 = a^3 - 32a^8$
19. $(4b^2)(-25b^4) + (-4b)(b^2) + (100b^6) = -100b^6 - 4b^3 + 100b^6 = -4b^3$
20. $(-z^3t^3)(.2z^2t) + (t^4z^2)(.3z^3) + (t^2z)(z^4t^2) = -.2t^4z^5 + .3t^4z^5 + t^4z^5 = 1.1t^4z^5$
21. $(-m^5k^5)(.3mk^2) + (m^6k^3)(.04k^4) + (m^6k^2)(k^4)k = -.3m^6k^7 + .04m^6k^7 + m^6k^7 = .74m^6k^7$

C **22.** $2x^5y^2 + 6x^3y^4 - x^2y^2(x^3 - xy^2) = 2x^5y^2 + 6x^3y^4 - x^5y^2 + x^3y^4 = x^5y^2 + 7x^3y^4$
23. $-5c^2d + 5c^6d^4 + 6c^2d - 2c^6d^4 = c^2d + 3c^6d^4$
24. $3k^3m^3 - 3k^2m - .2km[k + k^2m^2] = 3k^3m^3 - 3k^2m - .2k^2m - .2k^3m^3 = 2.8k^3m^3 - 3.2k^2m$
25. $a^3b^3(a^2 - ab^2) + (-.3b^2a)[-b^3a^3 + a^4b] = a^5b^3 - a^4b^5 + .3a^4b^5 - .3a^5b^3 = .7a^5b^3 - .7a^4b^5$

Pages 207–208 · **WRITTEN EXERCISES**

A **1.** $-5x^2 + 15x - 35$ **2.** $-6a^2 + 12a - 6$ **3.** $a^3 - 2a^2b + ab^2$ **4.** $x^3 - 2x^2y + xy^2$

5. $-ab^2 + b^3 + b^2c$ **6.** $-r^3 - r^2s + r^2t$ **7.** $-5x^4 + x^3 - 2x^2$ **8.** $-7y^2 + 3y^3 + y^5$

9. $-4v^4 + 6v^5 + 8v^6$ **10.** $-3x^5 + 18x^4 - 27x^3$

11. $15x^2y - 6x^3y^5 + 9x^4y^4 - 3x^2y^6$ **12.** $15rs^2 + 20r^6s^3 - 5r^4s^6 - 35r^2s^9$

13. $-2a^7b^3 + 2a^6b^4 - 4a^5b^6 + 2a^3b^7$ **14.** $-3c^7d^5 - 12c^6d^7 + 3c^4d^8 + 6c^2d^9$

15. $3x + 5x - 15 = 9; 8x - 15 = 9; 8x = 24; x = 3.$ {3}

16. $8n - 6n - 12 = 16; 2n - 12 = 16; 2n = 28; n = 14.$ {14}

17. $-6y - 5 + .5y = 17; -5.5y - 5 = 17; -5.5y = 22; -55y = 220; y = -4.$ {4}

18. $-7x + 5.6 - .8x = 18; -7.8x + 5.6 = 18; -7.8x = 12.4; -78x = 124; x = -\frac{124}{78} = -\frac{62}{39}.$ $\{-\frac{62}{39}\}$

19. $5x + 3 - 3x + 6 = 1; 2x + 9 = 1; 2x = -8; x = -4.$ {−4}

20. $8x + 7 - 5x + 40 = 44; 3x + 47 = 44; 3x = -3; x = -1.$ {−1}

21. $12x + 12 + 12x - 6 = 18; 24x + 6 = 18; 24x = 12; x = \frac{1}{2}.$ $\{\frac{1}{2}\}$

22. $56 - 7x + 10 + 10x = 96; 66 + 3x = 96; 3x = 30; x = 10.$ {10}

B **23.** $2x - 2x - 42 = 3x - 21 + 3x; -42 = 6x - 21; -21 = 6x; x = -\frac{7}{2}.$ $\{-\frac{7}{2}\}$

24. $5n - n + 8 = 14n - 14n - 56; 4n + 8 = -56; 4n = -64; n = -16.$ {−16}

25. $1.5y - 3.5 + 13 = .6y - 2.4 - 6y + 5; 1.5y + 9.5 = -5.4y + 2.6; 6.9y = -6.9; 69y = -69;$ $y = -1.$ {−1}

26. $.8x + x - 5 = -x + 3 - .8 - .8x; 1.8x - 5 = -1.8x + 2.2; 3.6x = 7.2; 36x = 72; x = 2.$ {2}

27. $5 - 3[2n - 10 + 6n] = 8 - 12n; 5 - 6n + 30 - 18n = 8 - 12n; -24n + 35 = 8 - 12n;$ $-12n = -27; n = \frac{9}{4}.$ $\{\frac{9}{4}\}$

Pages 208–209 · **PROBLEMS**

1. $d = 2(45 + x) + 2(200 + x) = 90 + 2x + 400 + 2x; d = (490 + 4x)$ mi, *Ans.*

2. $2x + 4(x + 25) = 1150; 2x + 4x + 100 = 1150; 6x + 100 = 1150; 6x = 1050; x = 175;$ $x + 25 = 200$ mph, *Ans.*

3. $190h + 210(h + 1) = 1210; 190h + 210h + 210 = 1210; 400h + 210 = 1210; 400h = 1000;$ $h = 2.5; h + 1 = 3.5$ hr, *Ans.*

4. $34x + 47(x + 3) = 303; 34x + 47x + 141 = 303; 81x + 141 = 303; 81x = 162; x = 2;$ $x + 3 = 5;$ trip took $2 + 5 = 7$ hr, *Ans.*

5. $A = lw.$ For Harry, $A = 50l;$ for Bob, $A = 60(l + 20); 60(l + 20) = 50l + 1500;$ $60l + 1200 = 50l + 1500; 10l = 300; l = 30$ ft, $l + 20 = 50$ ft, *Ans.*

6. $A = lw.$ For Cayne lot, $A = dw;$ for Bernard lot, $A = d(w + 30); d(w + 30) - dw = 3600;$ $dw + 30d - dw = 3600; 30d = 3600; d = 120$ ft, *Ans.*

7. $2d + 3(d - 1) = 19.50; 2d + 3d - 3 = 19.50; 5d - 3 = 19.50; 5d = 22.50; d = \$4.50,$ $d - 1 = \$3.50,$ *Ans.*

8. Let $p =$ speed in mi/min of piston-driven plane; then $p + 4.6 =$ speed in mi/min of jet. $p(\frac{1}{2} \cdot 60)$ $=$ distance traveled by piston plane and $(p + 4.6)(2 \cdot 60) =$ distance traveled by jet; $30p + 120(p + 4.6) = 1467; 30p + 120p + 552 = 1467; 150p + 552 = 1467; 150p = 915;$ $p = \frac{915}{150} = 6.1$ mi/min, $p + 4.6 = 10.7$ mi/min, *Ans.*

9. $6x + 5(110 + x) = 6600; 6x + 550 + 5x = 6600; 11x + 550 = 6600; 11x = 6050;$ $x = 550, 110 + x = 660,$ *Ans.*

10. Let $h =$ no. of hr one machine operated; then $h + 2 =$ no. of hr other operated; $1200h + 1500(h + 2) = 16500; 1200h + 1500h + 3000 = 16500; 2700h + 3000 = 16500;$ $2700h = 13500; h = \frac{13500}{2700} = 5$ hr, $h + 2 = 7$ hr, *Ans.*

Page 210 · **WRITTEN EXERCISES**

A **1.**
$$\begin{array}{r} x + 2 \\ x + 7 \\ \hline x^2 + 2x \\ 7x + 14 \\ \hline x^2 + 9x + 14 \end{array}$$

2.
$$\begin{array}{r} a + 7 \\ a + 8 \\ \hline a^2 + 7a \\ 8a + 56 \\ \hline a^2 + 15a + 56 \end{array}$$

3.
$$\begin{array}{r} y + 5 \\ y - 9 \\ \hline y^2 + 5y \\ -9y - 45 \\ \hline y^2 - 4y - 45 \end{array}$$

4.
$$\begin{array}{r} n + 4 \\ n - 9 \\ \hline n^2 + 4n \\ -9n - 36 \\ \hline n^2 - 5n - 36 \end{array}$$

5. $x - 6$
$x - 7$
$x^2 - 6x$
$- 7x + 42$
$x^2 - 13x + 42$

6. $y - 8$
$y - 3$
$y^2 - 8y$
$- 3y + 24$
$y^2 - 11y + 24$

7. $5x + 1$
$2x + 3$
$10x^2 + 2x$
$15x + 3$
$10x^2 + 17x + 3$

8. $8n + 9$
$7n + 4$
$56n^2 + 63n$
$32n + 36$
$56n^2 + 95n + 36$

9. $5x + 2$
$7x - 3$
$35x^2 + 14x$
$- 15x - 6$
$35x^2 - x - 6$

10. $5w - 2$
$12w + 5$
$60w^2 - 24w$
$25w - 10$
$60w^2 + w - 10$

11. $4y - 6$
$3y - 4$
$12y^2 - 18y$
$- 16y + 24$
$12y^2 - 34y + 24$

12. $7x - 8$
$6x - 5$
$42x^2 - 48x$
$- 35x + 40$
$42x^2 - 83x + 40$

13. $a + b$
$a + b$
$a^2 + ab$
$ab + b^2$
$a^2 + 2ab + b^2$

14. $x - y$
$x - y$
$x^2 - xy$
$- xy + y^2$
$x^2 - 2xy + y^2$

15. $x + y$
$x - y$
$x^2 + xy$
$- xy - y^2$
$x^2 - y^2$

16. $c - d$
$c + d$
$c^2 - cd$
$cd - d^2$
$c^2 - d^2$

17. $m - 5$
$m - 5$
$m^2 - 5m$
$- 5m + 25$
$m^2 - 10m + 25$

18. $x + 9$
$x + 9$
$x^2 + 9x$
$9x + 81$
$x^2 + 18x + 81$

19. $m + 3n$
$m + 3n$
$m^2 + 3mn$
$3mn + 9n^2$
$m^2 + 6mn + 9n^2$

20. $m - 4n$
$m - 4n$
$m^2 - 4mn$
$- 4mn + 16n^2$
$m^2 - 8mn + 16n^2$

21. $r - 3s$
$r + 3s$
$r^2 - 3rs$
$3rs - 9s^2$
$r^2 - 9s^2$

22. $3a + b$
$3a - b$
$9a^2 + 3ab$
$- 3ab - b^2$
$9a^2 - b^2$

23. $a + 5b$
$2a - .3b$
$2a^2 + 10ab$
$- .3ab - 1.5b^2$
$2a^2 + 9.7ab - 1.5b^2$

24. $a + 3b$
$2a - .5b$
$2a^2 + 6ab$
$- .5ab - 1.5b^2$
$2a^2 + 5.5ab - 1.5b^2$

B **25.** $m^2 - 3m + .4$
$m - .3$
$m^3 - 3m^2 + .4m$
$- .3m^2 + .9m - .12$
$m^3 - 3.3m^2 + 1.3m - .12$

26. $n^2 - .6n - 1$
$2n - .3$
$2n^3 - 1.2n^2 - 2n$
$- .3n^2 + .18n + .3$
$2n^3 - 1.5n^2 - 1.82n + .3$

27. $a^2 + ab + b^2$
$a - b$
$a^3 + a^2b + ab^2$
$- a^2b - ab^2 - b^3$
$a^3 - b^3$

28. $a^2 - ab + b^2$
$a + b$
$a^3 - a^2b + ab^2$
$a^2b - ab^2 + b^3$
$a^3 + b^3$

29. $a^2 + 1$
$a^2 - 8$
$a^4 + a^2$
$- 8a^2 - 8$
$a^4 - 7a^2 - 8$

30. $y^2 - 5$
$y^2 + 40$
$y^4 - 5y^2$
$40y^2 - 200$
$y^4 + 35y^2 - 200$

31. $xy - .8$
$xy - .7$
$x^2y^2 - .8xy$
$- .7xy + .56$
$x^2y^2 - 1.5xy + .56$

32. $ab + .3$
$ab - .1$
$a^2b^2 + .3ab$
$- .1ab - .03$
$a^2b^2 + .2ab - .03$

C **33.** $x^2 + 2x + 1$
$3x^2 - x + 2$
$3x^4 + 6x^3 + 3x^2$
$- x^3 - 2x^2 - x$
$2x^2 + 4x + 2$
$3x^4 + 5x^3 + 3x^2 + 3x + 2$

34. $y^2 - 3y - 2$
$5y^2 - y + 3$
$5y^4 - 15y^3 - 10y^2$
$- y^3 + 3y^2 + 2y$
$3y^2 - 9y - 6$
$5y^4 - 16y^3 - 4y^2 - 7y - 6$

35.
$$\begin{array}{r} a^2 + .3a + 4 \\ 2a^2 + a - .1 \\ \hline 2a^4 + .6a^3 + 8a^2 \\ a^3 + .3a^2 + 4a \\ - .1a^2 - .03a - .4 \\ \hline 2a^4 + 1.6a^3 + 8.2a^2 + 3.97a - .4 \end{array}$$

36.
$$\begin{array}{r} b^2 + 2b + .5 \\ .4b^2 - b + 3 \\ \hline .4b^4 + .8b^3 + .2b^2 \\ - b^3 - 2b^2 - .5b \\ 3b^2 + 6b + 1.5 \\ \hline .4b^4 - .2b^3 + 1.2b^2 + 5.5b + 1.5 \end{array}$$

37.
$$\begin{array}{r} 4a + 4b + 5c \\ 2a - 4b - 3c \\ \hline 8a^2 + 8ab + 10ac \\ - 16ab - 16b^2 - 20bc \\ - 12ac - 12bc - 15c^2 \\ \hline 8a^2 - 8ab - 2ac - 16b^2 - 32bc - 15c^2 \end{array}$$

38.
$$\begin{array}{r} mp + p - p^2 \\ 3m - m^2 \\ \hline 3m^2p + 3mp - 3mp^2 \\ -m^2p - m^3p + m^2p^2 \\ \hline 2m^2p + 3mp - 3mp^2 - m^3p + m^2p^2 \end{array}$$

39.
$$\begin{array}{r} x^4 + x^3 + x^2 + x + 1 \\ x - 1 \\ \hline x^5 + x^4 + x^3 + x^2 + x \\ - x^4 - x^3 - x^2 - x - 1 \\ \hline x^5 - 1 \end{array}$$

40.
$$\begin{array}{r} y^4 - y^3 + y^2 - y + 1 \\ y + 1 \\ \hline y^5 - y^4 + y^3 - y^2 + y \\ y^4 - y^3 + y^2 - y + 1 \\ \hline y^5 + 1 \end{array}$$

41. $(a + b)(c + d)$
$= (a + b)c + (a + b)d$ Distrib. prop.
$= ac + bc + ad + bd$ Distrib. prop.

42. $(a - b)(c + d)$
$= (a - b)c + (a - b)d$ Distrib. prop.
$= ac - bc + ad - bd$ Distrib. prop.

43. $(x + y)(x - y)$
$= (x + y)x - (x + y)y$ Distrib. prop.
$= x^2 + yx - xy - y^2$ Distrib. prop.
$= x^2 + xy - xy - y^2$ Comm. prop. for mult.
$= x^2 + 0 - y^2$ Prop. of opposites
$= x^2 - y^2$ Prop. of 0

44. $(x + y)(x + y)$
$= (x + y)x + (x + y)y$ Distrib. prop.
$= x^2 + yx + xy + y^2$ Distrib. prop.
$= x^2 + xy + xy + y^2$ Comm. prop. for mult.
$= x^2 + 1 \cdot xy + 1 \cdot xy + y^2$ Prop. of 1
$= x^2 + (1 + 1)xy + y^2$ Distrib. prop.
$= x^2 + 2xy + y^2$

Pages 212–213 · **PROBLEMS**

A

1. Let s = length of side of square; $s + 3$ = length of rectangle; $s - 2$ = width of rectangle; $s^2 = (s + 3)(s - 2)$; $s^2 = s^2 + 3s - 2s - 6$; $6 = s$; $s = 6$ ft, *Ans.*

2. Let s = length of side of square; $s + 1$ = length of rectangle; $s - 3$ = width of rectangle; $(s + 1)(s - 3) = s^2 - 45$; $s^2 - 3s + s - 3 = s^2 - 45$; $-2s = -42$; $s = 21$ in, *Ans.*

3. Let s = length of side of baseball diamond; $s - 30$ = length of side of softball diamond; $s^2 - 4500 = (s - 30)^2$; $s^2 - 4500 = s^2 - 30s - 30s + 900$; $-4500 = -60s + 900$; $60s = 5400$; $s = 90$; $s - 30 = 60$ ft, *Ans.*

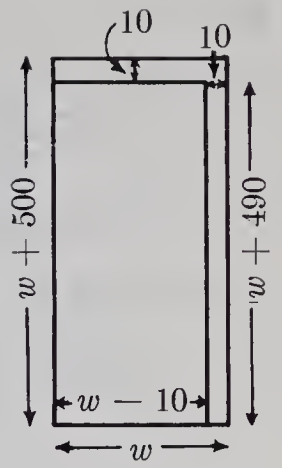
4. Let x = length of side of softball diamond; $x - 33$ = length of side of indoor baseball diamond; $x^2 - 2871 = (x - 33)^2$; $x^2 - 2871 = x^2 - 33x - 33x + 1089$; $-2871 = -66x + 1089$; $66x = 3960$; $x = 60$; $x - 33 = 27$ ft, *Ans.*

5. Let w = width of croquet court; $2w$ = length of croquet court; $2w - 16$ = length of badminton court; $w - 10$ = width of badminton court; $(2w - 16)(w - 10) = 2w \cdot w - 920$; $2w^2 - 20w - 16w + 160 = 2w^2 - 920$; $-36w = -1080$; $w = 30$; $2w - 16 = 44$ ft, $w - 10 = 20$ ft, *Ans.*

6. Let w = width of women's court; $2w$ = length of women's court; $2w + 4$ = length of men's court; $w + 5$ = width of men's court; $(w + 5)(2w + 4) = 2w \cdot w + 650$; $2w^2 + 4w + 10w + 20 = 2w^2 + 650$; $14w = 630$; $w = 45$; $2w + 4 = 94$ ft, $w + 5 = 50$ ft, *Ans.*

7. Let w = width of lacrosse field; $w + 75$ = length of lacrosse field; $(w + 75) - 60$ = length of hockey field; $w - 105$ = width of hockey field; $(w + 15)(w - 105) = w(w + 75) - 38{,}700$; $w^2 - 105w + 15w - 1575 = w^2 + 75w - 38{,}700$; $-165w = -37{,}125$; $w = 225$ ft, $w + 75 = 300$ ft, *Ans.*

8. Let w = width of field; $w + 500$ = length of field; $w(w + 500)$ = area of field; $(w - 10)(w + 490)$ = area planted; $(w - 10)(w + 490) + 44{,}900 = w(w + 500)$; $w^2 + 490w - 10w - 4900 + 44{,}900 = w^2 + 500w$; $480w + 40{,}000 = 500w$; $40{,}000 = 20w$; $w = 2000$ ft, $w + 500 = 2500$ ft, *Ans.*



B **9.** Let s = length of side of square; $s + 3$ = length of base of triangle; $s - 2$ = height of triangle; $\frac{1}{2}(s + 3)(s - 2) = \frac{1}{2}s^2$; $(s + 3)(s - 2) = s^2$; $s^2 - 2s + 3s - 6 = s^2$; $s = 6$ in, *Ans.*

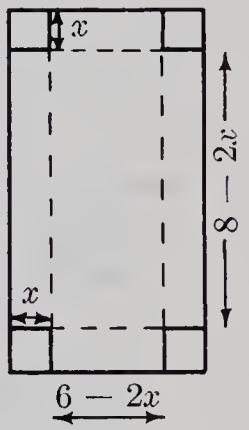
10. Let e = length of edge of smaller crate; e^2 = area of one face; $6e^2$ = total surface area; $e + 1$ = length of edge of larger crate; $6(e + 1)^2$ = total surface area; $6e^2 + 54 = 6(e + 1)^2$; $6e^2 + 54 = 6(e^2 + 2e + 1)$; $6e^2 + 54 = 6e^2 + 12e + 6$; $48 = 12e$; $e = 4$ ft, *Ans.*

11. Let r = inner radius; $r + \frac{3}{2}$ = outer radius; $\pi r^2 + 478\frac{1}{2} = \pi(r + \frac{3}{2})^2$; $\frac{22}{7}r^2 + \frac{957}{2} = \frac{22}{7}(r^2 + 3r + \frac{9}{4})$; $\frac{22}{7}r^2 + \frac{957}{2} = \frac{22}{7}r^2 + \frac{66}{7}r + \frac{198}{28}$; $\frac{957}{2} = \frac{66}{7}r + \frac{198}{28}$; $\frac{13398}{28} = \frac{66}{7}r + \frac{198}{28}$; $\frac{13200}{28} = \frac{66}{7}r$; $\frac{7}{66}(\frac{13200}{28}) = \frac{7}{66}(\frac{66}{7}r)$; $50 = r$. Diam. $= 2r = 2(50) = 100$ in, *Ans.*

12. Let r = inner radius; $r + 2$ = outer radius; $\pi r^2 + 88 = \pi(r + 2)^2$; $\frac{22}{7}r^2 + 88 = \frac{22}{7}(r^2 + 4r + 4)$; $\frac{22}{7}r^2 + 88 = \frac{22}{7}r^2 + \frac{88}{7}r + \frac{88}{7}$; $88 = \frac{88}{7}r + \frac{88}{7}$; $\frac{616}{7} = \frac{88}{7}r + \frac{88}{7}$; $\frac{528}{7} = \frac{88}{7}r$; $\frac{7}{88}(\frac{528}{7}) = \frac{7}{88}(\frac{88}{7}r)$; $6 = r$. Diam. $= 2r = 2(6) = 12$ in, *Ans.*

13. Let l = length of box; $l + 2$ = height of box; $l - 2$ = width of box; $l(l + 2)(l - 2) = l^3 - 80$; $l(l^2 - 2l + 2l - 4) = l^3 - 80$; $l^3 - 4l = l^3 - 80$; $-4l = -80$; $l = 20$ ft, $l + 2 = 22$ ft, $l - 2 = 18$ ft, *Ans.*

14. Let x = length of side of square (also height of box); x^2 = area of one square; $4x^2$ = total area to be cut out; $8 - 2x$ = length of box; $6 - 2x$ = width of box; $(8 - 2x)(6 - 2x) = 4x^2 - 8$; $48 - 16x - 12x + 4x^2 = 4x^2 - 8$; $48 - 28x = -8$; $-28x = -56$; $x = 2$ in, $8 - 2x = 4$ in, $6 - 2x = 2$ in, *Ans.*



Page 214 · WRITTEN EXERCISES

A **1.** $(a + b)(a + b) = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$

2. $(a - b)(a - b) = a^2 - ab - ab + b^2 = a^2 - 2ab + b^2$

3.
$$\begin{array}{r} a - b \\ a - b \\ \hline a^2 - ab \\ \quad - ab + b^2 \\ \hline a^2 - 2ab + b^2 \\ a - b \\ \hline a^3 - 2a^2b + ab^2 \\ \quad - a^2b + 2ab^2 - b^3 \\ \hline a^3 - 3a^2b + 3ab^2 - b^3 \end{array}$$

4.
$$\begin{array}{r} a + b \\ a + b \\ \hline a^2 + ab \\ \quad ab + b^2 \\ \hline a^2 + 2ab + b^2 \\ a + b \\ \hline a^3 + 2a^2b + ab^2 \\ \quad a^2b + 2ab^2 + b^3 \\ \hline a^3 + 3a^2b + 3ab^2 + b^3 \end{array}$$

5. $(3x + 2)(3x + 2) = 9x^2 + 6x + 6x + 4 = 9x^2 + 12x + 4$

6. $(5x + 2)(5x + 2) = 25x^2 + 10x + 10x + 4 = 25x^2 + 20x + 4$

7. $(7y + 3z)(7y + 3z) = 49y^2 + 21yz + 21yz + 9z^2 = 49y^2 + 42yz + 9z^2$

8. $(2m + 6n)(2m + 6n) = 4m^2 + 12mn + 12mn + 36n^2 = 4m^2 + 24mn + 36n^2$

9. $(r - 8s)(r - 8s) = r^2 - 8rs - 8rs + 64s^2 = r^2 - 16rs + 64s^2$

10. $(c - 4d)(c - 4d) = c^2 - 4cd - 4cd + 16d^2 = c^2 - 8cd + 16d^2$

11.
$$\begin{array}{r} 3x - 1 \\ 3x - 1 \\ \hline 9x^2 - 3x \\ \quad - 3x + 1 \\ \hline 9x^2 - 6x + 1 \\ 3x - 1 \\ \hline 27x^3 - 18x^2 + 3x \\ \quad - 9x^2 + 6x - 1 \\ \hline 27x^3 - 27x^2 + 9x - 1 \end{array}$$

12.
$$\begin{array}{r} 2y - 1 \\ 2y - 1 \\ \hline 4y^2 - 2y \\ \quad - 2y + 1 \\ \hline 4y^2 - 4y + 1 \\ 2y - 1 \\ \hline 8y^3 - 8y^2 + 2y \\ \quad - 4y^2 + 4y - 1 \\ \hline 8y^3 - 12y^2 + 6y - 1 \end{array}$$

B **13.**
$$\begin{array}{rrrrrr} a & + & b & + & c & \\ a & + & b & + & c & \\ \hline a^2 & + & ab & + & ac & \\ & & ab & & + b^2 + & bc \\ & & & & ac & + bc + c^2 \\ \hline a^2 & + & 2ab & + & 2ac + b^2 & + 2bc + c^2 \end{array}$$

14.
$$\begin{array}{rrrrrr} a & - & b & + & c & \\ a & - & b & + & c & \\ \hline a^2 & - & ab & + & ac & \\ & - & ab & & + b^2 - & bc \\ & & & & ac & - bc + c^2 \\ \hline a^2 & - & 2ab & + & 2ac + b^2 & - 2bc + c^2 \end{array}$$

15.
$$\begin{array}{rrrrrr} .2x^2 & + & x & - & .3 & \\ .2x^2 & + & x & - & .3 & \\ \hline .04x^4 & + & .2x^3 & - & .06x^2 & \\ & & .2x^3 & + & x^2 - & .3x \\ & & & & .06x^2 - & .3x + .09 \\ \hline .04x^4 & + & .4x^3 & + & .88x^2 - & .6x + .09 \end{array}$$

16.
$$\begin{array}{rrrrrr} .3y^2 & - & y & + & .2 & \\ .3y^2 & - & y & + & .2 & \\ \hline .09y^4 & - & .3y^3 & + & .06y^2 & \\ & - & .3y^3 & + & y^2 - & .2y \\ & & & & .06y^2 - & .2y + .04 \\ \hline .09y^4 & - & .6y^3 & + & 1.12y^2 - & .4y + .04 \end{array}$$

17.
$$\begin{array}{l} (x-y)(x-y) = x^2 - xy - xy + y^2 = x^2 - 2xy + y^2 \\ (x-y)(x-y) = x^2 - xy - xy + y^2 = x^2 - 2xy + y^2 \\ \hline x^4 - 2x^3y + x^2y^2 \\ \quad - 2x^3y + 4x^2y^2 - 2xy^3 \\ \quad\quad x^2y^2 - 2xy^3 + y^4 \\ \hline x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4 \end{array}$$

18.
$$\begin{array}{l} (x+y)(x+y) = x^2 + xy + xy + y^2 = x^2 + 2xy + y^2 \\ (x+y)(x+y) = x^2 + xy + xy + y^2 = x^2 + 2xy + y^2 \\ \hline x^4 + 2x^3y + x^2y^2 \\ \quad 2x^3y + 4x^2y^2 + 2xy^3 \\ \quad\quad x^2y^2 + 2xy^3 + y^4 \\ \hline x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4 \end{array}$$

19.
$$\begin{array}{rrrrrrr} 2x^2 & + & x & - & 2 & & \\ 2x^2 & + & x & - & 2 & & \\ \hline 4x^4 & + & 2x^3 & - & 4x^2 & & \\ & & 2x^3 & + & x^2 - & 2x & \\ & & & - & 4x^2 - & 2x & + 4 \\ \hline 4x^4 & + & 4x^3 & - & 7x^2 - & 4x & + 4 \\ 2x^2 & + & x & - & 2 & & \\ \hline 8x^6 & + & 8x^5 & - & 14x^4 - & 8x^3 & + 8x^2 \\ & & 4x^5 & + & 4x^4 - & 7x^3 - & 4x^2 + 4x \\ & & & - & 8x^4 - & 8x^3 + & 14x^2 + 8x - 8 \\ \hline 8x^6 & + & 12x^5 & - & 18x^4 - & 23x^3 + & 18x^2 + 12x - 8 \end{array}$$

20.
$$\begin{array}{rrrrrrr} 3y^2 & - & y & + & 2 & & \\ 3y^2 & - & y & + & 2 & & \\ \hline 9y^4 & - & 3y^3 & + & 6y^2 & & \\ & - & 3y^3 & + & y^2 - & 2y & \\ & & & & 6y^2 - & 2y & + 4 \\ \hline 9y^4 & - & 6y^3 & + & 13y^2 - & 4y & + 4 \\ 3y^2 & - & y & + & 2 & & \\ \hline 27y^6 & - & 18y^5 & + & 39y^4 - & 12y^3 & + 12y^2 \\ & - & 9y^5 & + & 6y^4 - & 13y^3 + & 4y^2 - 4y \\ & & & & 18y^4 - & 12y^3 + & 26y^2 - 8y + 8 \\ \hline 27y^6 & - & 27y^5 & + & 63y^4 - & 37y^3 + & 42y^2 - 12y + 8 \end{array}$$

21.
$$\begin{array}{rrrrrrr} .8x^2 & - & .9x & & & & \\ .8x^2 & - & .9x & & & & \\ \hline .64x^4 & - & .72x^3 & & & & \\ & - & .72x^3 & + & .81x^2 & & \\ \hline .64x^4 & - & 1.44x^3 & + & .81x^2 & & \\ .8x^2 & - & .9x & & & & \\ \hline .512x^6 & - & 1.152x^5 & + & .648x^4 & & \\ & - & .576x^5 & + & 1.296x^4 & - & .729x^3 \\ \hline .512x^6 & - & 1.728x^5 & + & 1.944x^4 & - & .729x^3 \end{array}$$

Pages 214–215 · **PROBLEMS**

A

1. Let x = length of side of smaller opening; $x + 3$ = length of side of larger opening; $(x + 3)^2 - x^2 = 81$; $x^2 + 3x + 3x + 9 - x^2 = 81$; $6x + 9 = 81$; $6x = 72$; $x = 12$ in, $x + 3 = 15$ in, *Ans.*

2. Let n = length of side of larger square; $n - 10$ = length of side of smaller square; $(n - 10)^2 = n^2 - 1100$; $n^2 - 10n - 10n + 100 = n^2 - 1100$; $-20n + 100 = -1100$; $-20n = -1200$; $n = 60$; $n - 10 = 50$ in. 50 in $\times$ 50 in, *Ans.*

3. Let n = first integer; $n + 1$ = second integer; $(n + 1)^2 - n^2 = 103$; $n^2 + 2n + 1 - n^2 = 103$; $2n = 102$; $n = 51$, $n + 1 = 52$, *Ans.*

4. Let n = first integer; $n + 1$ = second integer; $(n + 1)^2 - n^2 = 95$; $n^2 + 2n + 1 - n^2 = 95$; $2n = 94$; $n = 47$, $n + 1 = 48$, *Ans.*

5. Let n = width of picture; $n + 2$ = width of picture and frame; $n^2 + 24 = (n + 2)^2$; $n^2 + 24 = n^2 + 4n + 4$; $20 = 4n$; $n = 5$ in, *Ans.*

6. Let n = length of checkerboard; $n + 4$ = length of checkerboard and border; $n^2 + 176 = (n + 4)^2$; $n^2 + 176 = n^2 + 8n + 16$; $160 = 8n$; $n = 20$ in, *Ans.*

7. Let n = smaller integer; $n + 1$ = larger integer; $n(n + 1) - n^2 = 13$; $n^2 + n - n^2 = 13$; $n = 13$, $n + 1 = 14$, *Ans.*

8. Let n = smaller integer; $n + 1$ = larger integer; $n(n + 1) = (n + 1)^2 - 17$; $n^2 + n = n^2 + 2n + 1 - 17$; $n = 2n - 16$; $-n = -16$; $n = 16$, $n + 1 = 17$, *Ans.*

B

9. Let w = width of living room (and also of bedroom); $w + 5$ = length of living room; $w^2 = w(w + 5) - 75$; $w^2 = w^2 + 5w - 75$; $75 = 5w$; $w = 15$ ft, $w + 5 = 20$ ft, *Ans.*

10. Let w = width of rectangle; $w + 5$ = length of rectangle (and also of square); $w(w + 5) + 105 = (w + 5)^2$; $w^2 + 5w + 105 = w^2 + 10w + 25$; $-5w = -80$; $w = 16$ ft, $w + 5 = 21$ ft, *Ans.*

11. Let r = radius of restaurant; $r + 14$ = radius of skating rink; πr^2 = area of restaurant; $\pi(r + 14)^2$ = area of skating rink; $\pi r^2 + 3080 = \pi(r + 14)^2$; $\frac{22}{7}r^2 + 3080 = \frac{22}{7}(r^2 + 28r + 196)$; $\frac{22}{7}r^2 + 3080 = \frac{22}{7}r^2 + 88r + 616$; $3080 = 88r + 616$; $2464 = 88r$; $r = 28$; $r + 14 = 42$; (a) $2(42) = 84$ ft; (b) $.35(\pi r^2) = .35(\frac{22}{7})(28)^2 = \$862.40$

12. Let r = radius of larger rod; $r - \frac{1}{2}$ = radius of smaller rod; $60\pi r^2$ = volume of larger rod; $60\pi(r - \frac{1}{2})^2$ = volume of smaller rod; $60\pi(r - \frac{1}{2})^2 = 60\pi r^2 - 45\pi$; $\frac{1}{15}\pi[60\pi(r - \frac{1}{2})^2] = \frac{1}{15}\pi[60\pi r^2 - 45\pi]$; $4(r - \frac{1}{2})^2 = 4r^2 - 3$; $4(r^2 - r + \frac{1}{4}) = 4r^2 - 3$; $4r^2 - 4r + 1 = 4r^2 - 3$; $-4r = -4$; $r = 1$; $r - \frac{1}{2} = \frac{1}{2}$; (a) $2(1) = 2$ in, $2(\frac{1}{2}) = 1$ in; (b) $60\pi r^2 = 60\pi(1)^2 = 60\pi$ cu in, $60\pi(r - \frac{1}{2})^2 = 60\pi(\frac{1}{2})^2 = 15\pi$ cu in; (c) $\frac{5}{18}(60\pi) = 16\frac{2}{3}\pi$ lb, $\frac{5}{18}(15\pi) = 4\frac{1}{6}\pi$ lb

Page 218 · **WRITTEN EXERCISES**

A

1. $8ab$ **2.** $\frac{8b^2}{c^2}$ **3.** $\frac{8d}{e}$ **4.** $\frac{12x}{y}$ **5.** $-\frac{9z}{y^2}$ **6.** $-6r^2s$

7. $-\frac{12s}{t}$ **8.** $-\frac{12m}{n}$ **9.** $\frac{b^4}{7a^{10}}$ **10.** $\frac{h^2}{3g}$ **11.** $-\frac{3y^6}{50x^6}$ **12.** $-\frac{3s^3}{40r}$

13. $\frac{-5a^2b}{21a^2b^2c^2} = -\frac{5}{21bc^2}$ **14.** $\frac{-4m^2n}{64m^3n^3p^3} = -\frac{1}{16mn^2p^3}$ **15.** $\frac{3ab}{2c}$ **16.** $\frac{2yz}{3x}$

17. $\frac{2.7p^5r^2}{-27pr^4} = -\frac{p^4}{10r^2}$ **18.** $\frac{16vw^9}{1.6v^2w^7} = \frac{10w^2}{v}$ **19.** $\frac{-64x^3z^5}{8x^{12}z^{10}} = -\frac{8}{x^9z^5}$ **20.** $\frac{81d^{11}h^4}{81d^8h^4} = d^3$

B

21. $-\frac{3a^{x-1}b^{y-1}}{2}$, if x and y are integers > 1 **22.** $-\frac{x^{a-b}}{3y^{a-b}}$, if $a > b$; $-\frac{y^{b-a}}{3x^{b-a}}$, if $b > a$

23. $\frac{32a^mb^m}{8a^mb^n} = 4b^{m-n}$, if $m > n$; $\frac{4}{b^{n-m}}$, if $n > m$

24. $\frac{51a^{mn}b^n}{-13a^mb^{mn}} = -\frac{51a^{mn-m}}{13b^{mn-n}}$, if $mn > m$ and $mn > n$, i.e., if $n > 1$ and $m > 1$

Page 219 · **WRITTEN EXERCISES**

1. $3x^3 - 4 + 7 - 12x^3 = 3 - 9x^3$

2. $-4y^2 + 6y^2 - 4 + 4 = 2y^2$

3. $10e^6 + 25e^6 - 1 = 35e^6 - 1$, if $e \neq 0$

4. $-7s^4 + 49s^4 + 1 = 42s^4 + 1$, if $s \neq 0$

5. $\frac{-81x^2y^6z^4}{9x^2y^6z^4} + 1 = -9 + 1 = -8$

6. $1 + \frac{4a^6b^4z^8}{-16a^6b^4z^8} = 1 - \frac{1}{4} = \frac{3}{4}$

7. $x^2 + 1 - 1 = x^2$

8. $1 - y^2 + 1 = 2 - y^2$, if $y \neq 0$

9. $3x - 5 - 1 = 3x - 6$, if $(3x - 5) \neq 0$

10. $4 - y + 1 = 5 - y$, if $(y - 4) \neq 0$

Pages 220–221 · **WRITTEN EXERCISES**

A **1.** $\frac{12x}{3} + \frac{15y}{3} = 4x + 5y$ **2.** $\frac{5x}{5} + \frac{10y}{5} = x + 2y$ **3.** $\frac{2x}{x} + \frac{3x}{x} = 2 + 3 = 5$

4. $\frac{6y}{y} + \frac{16y}{y} = 6 + 16 = 22$ **5.** $\frac{5a}{a} + \frac{b}{a} = 5 + \frac{b}{a}$ **6.** $\frac{8m}{n} - \frac{n}{n} = \frac{8m}{n} - 1$

7. $\frac{r^2}{r} - \frac{7r}{r} = r - 7$ **8.** $\frac{2s^2}{s} + \frac{4s}{s} = 2s + 4$ **9.** $\frac{4c^2}{3} + \frac{3c}{3} = \frac{4c^2}{3} + c$

10. $\frac{3b^2}{2} + \frac{2b}{2} = \frac{3b^2}{2} + b$ **11.** $\frac{8a^3}{2a} - \frac{4a}{2a} = 4a^2 - 2$ **12.** $\frac{7c^3}{7c} + \frac{14c}{7c} = c^2 + 2$

13. $\frac{12x^3}{6x} + \frac{6x}{6x} = 2x^2 + 1$ **14.** $\frac{15y^2}{5y} - \frac{5y}{5y} = 3y - 1$

15. $\frac{24n^3}{3n} - \frac{12n^2}{3n} + \frac{15n}{3n} = 8n^2 - 4n + 5$ **16.** $\frac{50r^3}{5r} + \frac{10r^2}{5r} - \frac{35r}{5r} = 10r^2 + 2r - 7$

17. $\frac{8x^3}{-x} - \frac{4x^2}{-x} - \frac{2x}{-x} = -8x^2 - (-4x) - (-2) = -8x^2 + 4x + 2$

18. $\frac{2n^4}{-n} - \frac{3n^3}{-n} - \frac{4n^2}{-n} = -2n^3 - (-3n^2) - (-4n) = -2n^3 + 3n^2 + 4n$

19. $\frac{-60m^3n^3}{-6mn} + \frac{36m^2n^2}{-6mn} - \frac{6mn}{-6mn} = 10m^2n^2 + (-6mn) - (-1) = 10m^2n^2 - 6mn + 1$

20. $\frac{12x^3y^3}{-6xy} - \frac{6x^2y^2}{-6xy} + \frac{18xy}{-6xy} = -2x^2y^2 - (-xy) + (-3) = -2x^2y^2 + xy - 3$

21. $\frac{5x^4}{5x} - \frac{15x^3}{5x} + \frac{45x^2}{5x} - \frac{10x}{5x} = x^3 - 3x^2 + 9x - 2$

22. $\frac{35a^4}{7a} - \frac{28a^3}{7a} - \frac{56a^2}{7a} - \frac{14a}{7a} = 5a^3 - 4a^2 - 8a - 2$

23. $\frac{2.4a^2b^2}{.3ab} + \frac{.6ab^2}{.3ab} + \frac{30a^2b}{.3ab} = 8ab + 2b + 100a$ **24.** $\frac{3.2st^2}{.4st} + \frac{.8s^2t}{.4st} + \frac{40s^2t^2}{.4st} = 8t + 2s + 100st$

B **25.** $\frac{32a^5}{-8a^2} - \frac{6a^4b}{-8a^2} + \frac{24a^3b^2}{-8a^2} = -4a^3 - (-\frac{3}{4}a^2b) + (-3ab^2) = -4a^3 + \frac{3}{4}a^2b - 3ab^2$

26. $\frac{12h^4}{-36h^2} - \frac{24h^3k}{-36h^2} - \frac{6h^2k^2}{-36h^2} = -\frac{h^2}{3} - \left(-\frac{2hk}{3}\right) - \left(-\frac{k^2}{6}\right) = -\frac{h^2}{3} + \frac{2hk}{3} + \frac{k^2}{6}$

27. $\frac{30x^3y^3}{75xy^2} - \frac{45x^2y^2}{75xy^2} + \frac{15xy}{75xy^2} = \frac{2x^2y}{5} - \frac{3x}{5} + \frac{1}{5y}$

28. $\frac{12r^2t^2}{21r^2t^4} + \frac{18r^2t^3}{21r^2t^4} - \frac{6r^3t^3}{21r^2t^4} = \frac{4}{7t^2} + \frac{6}{7t} - \frac{2r}{7t}$

29. $\frac{21m^3n}{-35mn^3} - \frac{28m^2n^2}{-35mn^3} + \frac{35mn^3}{-35mn^3} = -\frac{3m^2}{5n^2} - \left(-\frac{4m}{5n}\right) + (-1) = -\frac{3m^2}{5n^2} + \frac{4m}{5n} - 1$

30. $\frac{9r^2s}{-27rs^2} + \frac{18rs^2}{-27rs^2} - \frac{27s^3}{-27rs^2} = -\frac{r}{3s} + \left(-\frac{2}{3}\right) - \left(-\frac{s}{r}\right) = -\frac{r}{3s} - \frac{2}{3} + \frac{s}{r}$

C **31.** $\frac{a}{b} + \frac{b}{b} = 3.5; \frac{a}{b} + 1 = 3.5; \frac{a}{b} = 2.5$

32. $\frac{n^2}{x^2} - \frac{x^2}{x^2} = -.91; \frac{n^2}{x^2} - 1 = -.91; \frac{n^2}{x^2} = .09; \frac{n}{x} = .3$

33. $\frac{5a^2}{a} + \frac{2a}{a} = 3a - 8; 5a + 2 = 3a - 8; 2a = -10; a = -5.$ $\{-5\}$

34. $\frac{3x^2}{x} - \frac{7x}{x} = x - 13; 3x - 7 = x - 13; 2x = -6; x = -3.$ $\{-3\}$

Page 223 · **WRITTEN EXERCISES**

A **1.**

$$\begin{array}{r} x + 3 \\ x + 2 \overline{)\, x^2 + 5x + 6} \\ \underline{x^2 + 2x} \\ 3x + 6 \\ \underline{3x + 6} \\ 0 \end{array}$$

Q: $x + 3$

2.

$$\begin{array}{r} x + 2 \\ x + 1 \overline{)\, x^2 + 3x + 2} \\ \underline{x^2 + \ x} \\ 2x + 2 \\ \underline{2x + 2} \\ 0 \end{array}$$

Q: $x + 2$

3.
$$\begin{array}{r} x+4 \\ x+3\overline{)\,x^2+7x+12} \\ \underline{x^2+3x} \\ 4x+12 \\ \underline{4x+12} \\ 0 \end{array}$$
Q: $x+4$

4.
$$\begin{array}{r} x+4 \\ x+7\overline{)\,x^2+11x+28} \\ \underline{x^2+7x} \\ 4x+28 \\ \underline{4x+28} \\ 0 \end{array}$$
Q: $x+4$

5.
$$\begin{array}{r} y-7 \\ y-6\overline{)\,y^2-13y+42} \\ \underline{y^2-6y} \\ -7y+42 \\ \underline{-7y+42} \\ 0 \end{array}$$
Q: $y-7$

6.
$$\begin{array}{r} x-8 \\ x-7\overline{)\,x^2-15x+56} \\ \underline{x^2-7x} \\ -8x+56 \\ \underline{-8x+56} \\ 0 \end{array}$$
Q: $x-8$

7.
$$\begin{array}{r} r-6 \\ r+1\overline{)\,r^2-5r-7} \\ \underline{r^2+r} \\ -6r-7 \\ \underline{-6r-6} \\ -1 \end{array}$$
Q: $r-6+\dfrac{-1}{r+1}$

8.
$$\begin{array}{r} n-8 \\ n+1\overline{)\,n^2-7n-9} \\ \underline{n^2+n} \\ -8n-9 \\ \underline{-8n-8} \\ -1 \end{array}$$
Q: $n-8+\dfrac{-1}{n+1}$

9.
$$\begin{array}{r} n-2 \\ 3n-2\overline{)\,3n^2-8n+4} \\ \underline{3n^2-2n} \\ -6n+4 \\ \underline{-6n+4} \\ 0 \end{array}$$
Q: $n-2$

10.
$$\begin{array}{r} z-8 \\ z-8\overline{)\,z^2-16z+64} \\ \underline{z^2-8z} \\ -8z+64 \\ \underline{-8z+64} \\ 0 \end{array}$$
Q: $z-8$

11.
$$\begin{array}{r} 4x+7 \\ 4x-7\overline{)\,16x^2+0-49} \\ \underline{16x^2-28x} \\ 28x-49 \\ \underline{28x-49} \\ 0 \end{array}$$
Q: $4x+7$

12.
$$\begin{array}{r} 5x-9 \\ 5x+9\overline{)\,25x^2+0-81} \\ \underline{25x^2+45x} \\ -45x-81 \\ \underline{-45x-81} \\ 0 \end{array}$$
Q: $5x-9$

13.
$$\begin{array}{r} a-9 \\ a+9\overline{)\,a^2+0+81} \\ \underline{a^2+9a} \\ -9a+81 \\ \underline{-9a-81} \\ 162 \end{array}$$
Q: $a-9+\dfrac{162}{a+9}$

14.
$$\begin{array}{r} x-3 \\ x+3\overline{)\,x^2+0+9} \\ \underline{x^2+3x} \\ -3x+9 \\ \underline{-3x-9} \\ 18 \end{array}$$
Q: $x-3+\dfrac{18}{x+3}$

15.
$$\begin{array}{r} y+3z \\ y-9z\overline{)\,y^2-6yz-27z^2} \\ \underline{y^2-9yz} \\ 3yz-27z^2 \\ \underline{3yz-27z^2} \\ 0 \end{array}$$
Q: $y+3z$

16.
$$\begin{array}{r} w+6x \\ w-17x\overline{)\,w^2-11wx-102x^2} \\ \underline{w^2-17wx} \\ 6wx-102x^2 \\ \underline{6wx-102x^2} \\ 0 \end{array}$$
Q: $w+6x$

17.
$$\begin{array}{r} 3x-5y \\ x-3y\overline{)\,3x^2-14xy+25y^2} \\ \underline{3x^2-9xy} \\ -5xy+25y^2 \\ \underline{-5xy+15y^2} \\ 10y^2 \end{array}$$
Q: $3x-5y+\dfrac{10y^2}{x-3y}$

18.
$$\begin{array}{r} 5x-3y \\ x-2y\overline{)\,5x^2-13xy+16y^2} \\ \underline{5x^2-10xy} \\ -3xy+16y^2 \\ \underline{-3xy+6y^2} \\ 10y^2 \end{array}$$
Q: $5x-3y+\dfrac{10y^2}{x-2y}$

19.
$$\begin{array}{r} 3y - 2 \\ 2y + 5\overline{)\,6y^2 + 11y - 10} \\ \underline{6y^2 + 15y} \\ -\,4y - 10 \\ \underline{-\,4y - 10} \\ 0 \end{array}$$
Q: $3y - 2$

20.
$$\begin{array}{r} 5n - 4 \\ 2n + 3\overline{)\,10n^2 + 7n - 12} \\ \underline{10n^2 + 15n} \\ -\,8n - 12 \\ \underline{-\,8n - 12} \\ 0 \end{array}$$
Q: $5n - 4$

21.
$$\begin{array}{r} 4x^2 + 2xy + y^2 \\ 2x - y\overline{)\,8x^3 + 0 + 0 - y^3} \\ \underline{8x^3 - 4x^2y} \\ 4x^2y + 0 \\ \underline{4x^2y - 2xy^2} \\ 2xy^2 - y^3 \\ \underline{2xy^2 - y^3} \\ 0 \end{array}$$
Q: $4x^2 + 2xy + y^2$

22.
$$\begin{array}{r} a^2 - 6ab + 36b^2 \\ a + 6b\overline{)\,a^3 + 0 + 0 + 216b^3} \\ \underline{a^3 + 6a^2b} \\ -\,6a^2b + 0 \\ \underline{-\,6a^2b - 36ab^2} \\ 36ab^2 + 216b^3 \\ \underline{36ab^2 + 216b^3} \\ 0 \end{array}$$
Q: $a^2 - 6ab + 36b^2$

23.
$$\begin{array}{r} x^2 - 7xy + 49y^2 \\ x + 7y\overline{)\,x^3 + 0 + 0 + 343y^3} \\ \underline{x^3 + 7x^2y} \\ -\,7x^2y + 0 \\ \underline{-\,7x^2y - 49xy^2} \\ 49xy^2 + 343y^3 \\ \underline{49xy^2 + 343y^3} \\ 0 \end{array}$$
Q: $x^2 - 7xy + 49y^2$

24.
$$\begin{array}{r} x + 7 \\ 2x - 3\overline{)\,2x^2 + 11x - 18} \\ \underline{2x^2 - 3x} \\ 14x - 18 \\ \underline{14x - 21} \\ 3 \end{array}$$
Q: $x + 7 + \dfrac{3}{2x - 3}$

25.
$$\begin{array}{r} 6x - 7 \\ 2x + 3\overline{)\,12x^2 + 4x - 18} \\ \underline{12x^2 + 18x} \\ -\,14x - 18 \\ \underline{-\,14x - 21} \\ 3 \end{array}$$
Q: $6x - 7 + \dfrac{3}{2x + 3}$

B

26.
$$\begin{array}{r} 3x^2 + 2x + 5 \\ x + 3\overline{)\,3x^3 + 11x^2 + 11x + 15} \\ \underline{3x^3 + 9x^2} \\ 2x^2 + 11x \\ \underline{2x^2 + 6x} \\ 5x + 15 \\ \underline{5x + 15} \\ 0 \end{array}$$
Q: $3x^2 + 2x + 5$

27.
$$\begin{array}{r} 2x^2 + 3x - 2 \\ x - 5\overline{)\,2x^3 - 7x^2 - 17x + 10} \\ \underline{2x^3 - 10x^2} \\ 3x^2 - 17x \\ \underline{3x^2 - 15x} \\ -\,2x + 10 \\ \underline{-\,2x + 10} \\ 0 \end{array}$$
Q: $2x^2 + 3x - 2$

28.
$$\begin{array}{r} 3x^2 + 8x + 11 \\ 2x - 5\overline{)\,6x^3 + x^2 - 18x - 33} \\ \underline{6x^3 - 15x^2} \\ 16x^2 - 18x \\ \underline{16x^2 - 40x} \\ 22x - 33 \\ \underline{22x - 55} \\ 22 \end{array}$$

Q: $3x^2 + 8x + 11 + \dfrac{22}{2x - 5}$

29.
$$\begin{array}{r} n^2 - n + 9 \\ 2n - 3\overline{)\,2n^3 - 5n^2 + 21n - 14} \\ \underline{2n^3 - 3n^2} \\ -2n^2 + 21n \\ \underline{-2n^2 + 3n} \\ 18n - 14 \\ \underline{18n - 27} \\ 13 \end{array}$$

Q: $n^2 - n + 9 + \dfrac{13}{2n - 3}$

30.
$$\begin{array}{r} x^2 - 2x + 3 \\ x^2 - 2x + 3\overline{)\,x^4 - 4x^3 + 10x^2 - 12x + 9} \\ \underline{x^4 - 2x^3 + 3x^2} \\ -2x^3 + 7x^2 - 12x \\ \underline{-2x^3 + 4x^2 - 6x} \\ 3x^2 - 6x + 9 \\ \underline{3x^2 - 6x + 9} \\ 0 \end{array}$$

Q: $x^2 - 2x + 3$

31.
$$\begin{array}{r} y^2 + 2y - 1 \\ y^2 + 2y - 1\overline{)\,y^4 + 4y^3 + 2y^2 - 4y + 1} \\ \underline{y^4 + 2y^3 - y^2} \\ 2y^3 + 3y^2 - 4y \\ \underline{2y^3 + 4y^2 - 2y} \\ -y^2 - 2y + 1 \\ \underline{-y^2 - 2y + 1} \\ 0 \end{array}$$

Q: $y^2 + 2y - 1$

32.
$$\begin{array}{r} x^2 - x + 1 \\ x + 1\overline{)\,x^3 + 0 + 0 + 1} \\ \underline{x^3 + x^2} \\ -x^2 + 0 \\ \underline{-x^2 - x} \\ x + 1 \\ \underline{x + 1} \\ 0 \end{array}$$

Other factor: $x^2 - x + 1$

33.
$$\begin{array}{r} y^2 + y + 1 \\ y - 1\overline{)\,y^3 + 0 + 0 - 1} \\ \underline{y^3 - y^2} \\ y^2 + 0 \\ \underline{y^2 - y} \\ y - 1 \\ \underline{y - 1} \\ 0 \end{array}$$

Other factor: $y^2 + y + 1$

34.
$$\begin{array}{r} 3y^2 \\ 2y + 3\overline{)\,6y^3 + 2y^2 + y - 9} \\ \underline{6y^3 + 9y^2} \\ -7y^2 + y \end{array}$$

No; there does not exist a polynomial with integral coeff. such that $(2y + 3) \times$ this polynomial $= 6y^3 + 2y^2 + y - 9$

35.
$$\begin{array}{r} 3z^2 \\ 3z - 2\overline{)\,9z^3 - 4z^2 - 5z + 8} \\ \underline{9z^3 - 6z^2} \\ 2z^2 - 5z \end{array}$$

No; there does not exist a polynomial with integral coeff. such that $(3z - 2) \times$ this polynomial $= 9z^3 - 4z^2 - 5z + 8$

36. Let $x =$ the no.; $59(x) + 10 = 718$; $59x = 708$; $x = 12$, *Ans.*

37. Let $x =$ the no.; $28(x) + 3 = 395$; $28x = 392$; $x = 14$, *Ans.*

Page 226 · CHAPTER TEST

1. $69x + 45$

2. $-a^2 + a - 13$

3. $(5 \cdot 2 - 6) + (2 \cdot 2 - 80) + (3 \cdot 2 + 17) \stackrel{?}{=} 10 \cdot 2 - 69$;
$(10 - 6) + (4 - 80) + (6 + 17) \stackrel{?}{=} 20 - 69$;
$4 + (-76) + (23) \stackrel{?}{=} -49$;
$27 + (-76) = -49$

4. $(3^2 + 3 + 6) + (2 \cdot 3^2 - 2 \cdot 3 - 5) \stackrel{?}{=} 3 \cdot 3^2 - 3 + 1$;
$18 + (18 - 6 - 5) \stackrel{?}{=} 27 - 3 + 1$;
$18 + 7 = 25$

5. $\begin{array}{rrrrrrrrr} 2x & - & 3y & - & 6z & = & 2(2) - 3(3) - 6(4) & = & 4 - 9 - 24 = -29 \\ -5x & + & 7y & - & 2z & = & -5(2) + 7(3) - 2(4) & = & -10 + 21 - 8 = 3 \\ \hline 7x & - & 10y & - & 4z & = & 7(2) - 10(3) - 4(4) & = & 14 - 30 - 16 = -32 \end{array}$

6. $\begin{array}{rrrrrrrrr} x^2 & + & y^2 & - & z^2 & = & 2^2 + 3^2 - 4^2 & = & 4 + 9 - 16 = -3 \\ -x^2 & - & y^2 & - & z^2 & = & -(2^2) - (3^2) - (4^2) & = & -4 - 9 - 16 = -29 \\ \hline 2x^2 & + & 2y^2 & & & = & 2(2^2) + 2(3^2) & = & 8 + 18 = 26 \end{array}$

7. $28a + 6 = 34;\ 28a = 28;\ a = 1.\ \{1\}$

8. $2n - [n - 2n - 8 - 1] = 0;\ 2n - [-n - 9] = 0;\ 2n + n + 9 = 0;\ 3n + 9 = 0;\ 3n = -9;$ $n = -3.\ \{-3\}$

9. $-3m^3n^3t^5$ **10.** $-6r^6s^6$ **11.** $-64m^6n^9$

12. $(9x^2)(8x^3y^6z^{15}) = 72x^5y^6z^{15}$ **13.** $-60x^2 + 88xy$ **14.** $-6t^2 - 2t^3 + 3t^5$

15.
$$\begin{array}{r} c - 2 \\ c + 8 \\ \hline c^2 - 2c \\ 8c - 16 \\ \hline c^2 + 6c - 16 \end{array}$$

16.
$$\begin{array}{r} 2a - 4y \\ 8a - 5y \\ \hline 16a^2 - 32ay \\ -10ay + 20y^2 \\ \hline 16a^2 - 42ay + 20y^2 \end{array}$$

17. Let x = length of garden; $x + 4$ = length of garden and path; $(x + 4)^2 = x^2 + 184$; $x^2 + 8x + 16 = x^2 + 184$; $8x = 168$; $x = 21$ ft, *Ans.*

18. Let x = smaller number; $x + 5$ = larger number; $(x + 5)^2 - x^2 = 135$; $x^2 + 10x + 25 - x^2 = 135$; $10x = 110$; $x = 11$; $x + 5 = 16$, *Ans.*

19. $\dfrac{16h^2}{k^2}$ **20.** (Optional) a. 1; b. $3 \cdot 1 = 3$ **21.** $\dfrac{12a^3}{3a} - \dfrac{3a}{3a} = 4a^2 - 1$

22. $\dfrac{16bc}{-4bc^3d} - \dfrac{8b^2c^2}{-4bc^3d} - \dfrac{c^3d}{-4bc^3d} = -\dfrac{4}{c^2d} - \left(-\dfrac{2b}{cd}\right) - \left(-\dfrac{1}{4b}\right) = -\dfrac{4}{c^2d} + \dfrac{2b}{cd} + \dfrac{1}{4b}$

B **23.**
$$\begin{array}{r|l} & 4n - 9 \\ \hline n - 5 & 4n^2 - 29n + 45 \\ & 4n^2 - 20n \\ \hline & -9n + 45 \\ & -9n + 45 \\ \hline & 0 \end{array}$$

Q: $4n - 9$

24.
$$\begin{array}{r|l} & 2x^2 - 3x + 1 \\ \hline 5x - 3 & 10x^3 - 21x^2 + 14x + 12 \\ & 10x^3 - 6x^2 \\ \hline & -15x^2 + 14x \\ & -15x^2 + 9x \\ \hline & 5x + 12 \\ & 5x - 3 \\ \hline & 15 \end{array}$$

Q: $2x^2 - 3x + 1 + \dfrac{15}{5x - 3}$

Pages 227–229 · **CHAPTER REVIEW**

1. monomials **2.** trinomial **3.** 5 **4.** third

5. $3n^3 + n^2 - 8n - 3$ **6.** $-2mn^3 + m^2n^2 - 5m^3n$ **7.** 0, 1

8. $\begin{array}{rrrrrrrrr} 3(3) & + & 2 & + & 4 & = & 9 + 2 + 4 & = & 15 \\ 2(3) & - & 3(2) & - & 2 & = & 6 - 6 - 2 & = & -2 \\ -6(3) & - & 2(2) & - & 5 & = & -18 - 4 - 5 & = & -27 \\ \hline 3 & & & - & 3 & & & \neq & -14 \end{array}$

Correct Ans. is $-a - 4b - 3$

9. $\begin{array}{rrrrrrrrr} 2(3^2)(2) & - & 3(3)(2^2) & + & (2^3) & = & 36 - 36 + 8 & = & 8 \\ -(3^2)(2) & + & (3)(2^2) & - & 2(2^3) & = & -18 + 12 - 16 & = & -22 \\ -(3^2)(2) & + & 2(3)(2^2) & + & (2^3) & = & -18 + 24 + 8 & = & 14 \\ \hline 0 & & & & & & & = & 0 \end{array}$

10. opposite

In Ex. 11–14 values used in checking will vary.

11. $3t^2 - 2t + 1 + (-t^2 + 2t - 2) = 2t^2 - 1$ **12.** $2 - 4c + 2c^2$ **13.** $-h^2 + 2h - 1$

14. $(a - 2b + 3) - (2a + b + 7) = (a - 2b + 3) + (-2a - b - 7) = -a - 3b - 4$

15. $7k - 2k - 3 = 3k - 9; 5k - 3 = 3k - 9; 2k = -6; k = -3.$ $\{-3\}$

16. $3 - [2n - 5 - n + 3] = 5 + 6n; 3 - [n - 2] = 5 + 6n; 3 - n + 2 = 5 + 6n;$
$5 - n = 5 + 6n; -7n = 0; n = 0.$ $\{0\}$

17. 2, 18 **18.** $c^3 \cdot c^1 = c^4$ **19.** $3a^{11}b^9c^7$ **20.** 2, 3, b^3, $8a^3b^3$ **21.** b^n

22. $(9p^6r^8)(-p^{10}r^{15}s^5) = -9p^{16}r^{23}s^5$ **23.** $2a$, $3b$

24. $-9y^2z^3 + 6y^3z^2$ **25.** $-2x + 4x - 8 = -7; 2x - 8 = -7; 2x = 1; x = \frac{1}{2}.$ $\{\frac{1}{2}\}$

26. $8a - 5a + 15 = 18a - 18a - 6; 3a + 15 = -6; 3a = -21; a = -7.$ $\{-7\}$

27. $A = n(2n - 5) = 2n^2 - 5n$

28. $2a$, -3, $8a^2 - 2a - 15$. $2a(4a + 5) - 3(4a + 5) = 8a^2 + 10a - 12a - 15 = 8a^2 - 2a - 15$

29.
$$\begin{array}{llll} (5x + 2)(3x + 7) & = (5x)(3x) + 5x(7) + 2(3x) + 2(7) & = & 15x^2 + 41x + 14 \\ [5(3) + 2][3(3) + 7] & \stackrel{?}{=} & & 15(9) + 41(3) + 14 \\ 17 \cdot 16 & \stackrel{?}{=} & & 135 + 123 + 14 \\ 272 & = & & 272 \end{array}$$

30.
$$\begin{array}{rcrcrl} 7a - 3x & = & 7(2) - 3(3) & = & 14 - 9 & = 5 \\ 3a - x & = & 3(2) - 3 & = & 6 - 3 & = 3 \\ \hline 21a^2 - 9ax & & & & & \\ \quad - 7ax + 3x^2 & & & & & \\ \hline 21a^2 - 16ax + 3x^2 & = 21(2^2) - 16(2)(3) + 3(3^2) & = & 84 - 96 + 27 & = & 15 \end{array}$$

31.
$$\begin{array}{rcrcl} a + 9 & = & 2 + 9 & = & 11 \\ 2a - 4 & = & 2(2) - 4 & = & 0 \\ \hline 2a^2 + 18a & & & & \\ \quad - 4a - 36 & & & & \\ \hline 2a^2 + 14a - 36 & = 2(2^2) + 14(2) - 36 & = & 8 + 28 - 36 & = 0 \end{array}$$

32. $3s$, $3s^2$ **33.** $A = (s + 2)(3s - 4) = 3s^2 - 4s + 6s - 8 = 3s^2 + 2s - 8$

34. $3s^2 = 3s^2 + 2s - 8$ **35.** $8 = 2s; s = 4$ in, $3s = 12$ in

36. Let w = width of pool; $w + 30$ = length of pool; $w + 10$ = width of pool and walk; $w + 40$ = length of pool and walk; $w(w + 30) + 1200 = (w + 10)(w + 40)$; $w^2 + 30w + 1200 = w^2 + 40w + 10w + 400; 800 = 20w; w = 40$ ft, $w + 30 = 70$ ft, *Ans.*

37.
$$\begin{array}{r} a + 1 \\ a + 1 \\ \hline a^2 + a \\ \quad a + 1 \\ \hline a^2 + 2a + 1 \end{array}$$

38.
$$\begin{array}{r} x^2 - 5x - 1 \\ x^2 - 5x - 1 \\ \hline x^4 - 5x^3 - x^2 \\ \quad - 5x^3 + 25x^2 + 5x \\ \quad\quad - x^2 + 5x + 1 \\ \hline x^4 - 10x^3 + 23x^2 + 10x + 1 \\ x^2 - 5x - 1 \\ \hline x^6 - 10x^5 + 23x^4 + 10x^3 + x^2 \\ \quad - 5x^5 + 50x^4 - 115x^3 - 50x^2 - 5x \\ \quad\quad - x^4 + 10x^3 - 23x^2 - 10x - 1 \\ \hline x^6 - 15x^5 + 72x^4 - 95x^3 - 72x^2 - 15x - 1 \end{array}$$

39. Let r = length of inner radius of wheel; $r + .35$ = length of outer radius of wheel; πr^2 = area of inner ring; $\pi(r + .35)^2$ = area of outer ring; $\pi(r + .35)^2 - \pi r^2 = 5.115; \frac{22}{7}(r^2 + .7r + .1225) \cdot \frac{22}{7}r^2 = 5.115; \frac{22}{7}r^2 + 2.2r + .385 - \frac{22}{7}r^2 = 5.115; 2.2r = 4.73; 22r = 47.3; r = 2.15; r + .35 = 2.5; d = 2(2.5) = 5$ cm, *Ans.*

40. $-\dfrac{2n^3}{5m^3}$ **41.** $\dfrac{1}{2st^2}$ **42.** 1 **43.** 1

44. $\dfrac{5a}{-3} - \dfrac{3}{-3} = -\dfrac{5a}{3} - (-1) = -\dfrac{5a}{3} + 1$

45. $\dfrac{5m^2n}{-50m^3n^3} + \dfrac{10mn}{-50m^3n^3} - \dfrac{25mn^2}{-50m^3n^3} = -\dfrac{1}{10mn^2} + \left(-\dfrac{1}{5m^2n^2}\right) - \left(-\dfrac{1}{2m^2n}\right)$

$$= -\frac{1}{10mn^2} - \frac{1}{5m^2n^2} + \frac{1}{2m^2n}$$

46. increasing, decreasing

47.
$$\begin{array}{r} 2a - 7 \\ 3a - 2\overline{)\,6a^2 - 25a + 14} \\ \underline{6a^2 - 4a} \\ -21a + 14 \\ \underline{-21a + 14} \\ 0 \end{array}$$
Q: $2a - 7$

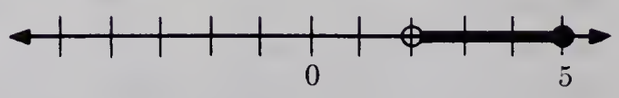
48.
$$\begin{array}{r} 4c^2 - 4c + 1 \\ 2c - 1\overline{)\,8c^3 - 12c^2 + 6c - 1} \\ \underline{8c^3 - 4c^2} \\ -8c^2 + 6c \\ \underline{-8c^2 + 4c} \\ 2c - 1 \\ \underline{2c - 1} \\ 0 \end{array}$$
Q: $4c^2 - 4c + 1$

49.
$$\begin{array}{r} 2a^2 - 5ab - 3b^2 \\ a + 2b\overline{)\,2a^3 - a^2b - 13ab^2 - 6b^3} \\ \underline{2a^3 + 4a^2b} \\ -5a^2b - 13ab^2 \\ \underline{-5a^2b - 10ab^2} \\ -3ab^2 - 6b^3 \\ \underline{-3ab^2 - 6b^3} \\ 0 \end{array}$$
Q: $2a^2 - 5ab - 3b^2$

50.
$$\begin{array}{r} 3r^2 + 4rs - s^2 \\ 3r - 2s\overline{)\,9r^3 + 6r^2s - 11rs^2 + s^3} \\ \underline{9r^3 - 6r^2s} \\ 12r^2s - 11rs^2 \\ \underline{12r^2s - 8rs^2} \\ -3rs^2 + s^3 \\ \underline{-3rs^2 + 2s^3} \\ -s^3 \end{array}$$
Q: $3r^2 + 4rs - s^2 + \dfrac{-s^3}{3r - 2s}$

Pages 230–232 · CUMULATIVE REVIEW

1. proper subset, *or*, subset

2.


3. $(2 \cdot 3 + 4)(2 \cdot 3 - 4) = (6 + 4)(6 - 4) = (10)(2) = 20$

4. closed, multiplication

5. distributive

6. equivalent, same. $2x - 2 = 3$; $2x = 5$; $x = \frac{5}{2}$. $\{\frac{5}{2}\}$. $y = \frac{5}{2}$. $\{\frac{5}{2}\}$

7. associative (for add.)

8. $\{-2, 0, 1, 2\}$, since $x \neq -1$

9. $2n^3$

10. $n + n^2$

11. $n - \dfrac{1}{n}$

12. $a^2 + b^2$

13. $5m^3 - 5n^2 - 9m^3 + 5n^2 = -4m^3$

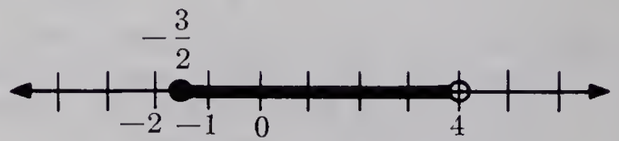
14. $.6y = -48$; $6y = -480$; $y = -80$. $\{-80\}$

15. $3n - \frac{10}{3}n + 4 = 2$; $3[3n - \frac{10}{3}n + 4] = 3(2)$; $9n - 10n + 12 = 6$; $-n = -6$; $n = 6$. $\{6\}$

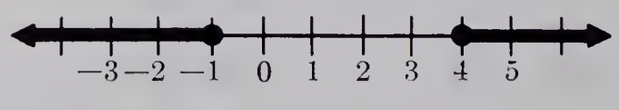
16. $12x + 20 - 2x = 12x - 15 + 25$; $10x + 20 = 12x + 10$; $10 = 2x$; $5 = x$. $\{5\}$

17. $3t - [2t - 3t - 5 - 7] = 0$; $3t - [-t - 12] = 0$; $3t + t + 12 = 0$; $4t = -12$; $t = -3$. $\{-3\}$

18. $-6 \leq 2x - 3$ and $2x - 3 < 5$; $-3 \leq 2x$ and $2x < 8$; $-\frac{3}{2} \leq x$ and $x < 4$; $-\frac{3}{2} \leq x < 4$



19. $(2r - 3) \geq 5$ or $-(2r - 3) \geq 5$; $2r - 3 \geq 5$ or $2r - 3 \leq -5$; $2r \geq 8$ or $2r \leq -2$; $r \geq 4$ or $r \leq -1$



20. $4(\frac{5}{4}m + 6) \geq 4\left(\dfrac{m}{2} + 3\right)$; $5m + 24 \geq 2m + 12$; $3m \geq -12$; $m \geq -4$

21. $(-3)^3 = -27$ **22.** factor **23.** -1

24. $0 - (2a - b) = 0 + (-2a + b) = -2a + b = b - 2a$ **25.** $(2a - b) - 0 = 2a - b$

26. $-\frac{1}{p}$ **27.** $\frac{12a^2}{-3a} - \frac{15ab}{-3a} = -4a - (-5b) = -4a + 5b = 5b - 4a$

28. $-8r^6s^9$ **29.** $(a^3 + b^3) + (a^3 - b^3) = 2a^3$

30. $(a^3 + b^3) - (a^3 - b^3) = (a^3 + b^3) + (-a^3 + b^3) = 2b^3$

31.
$$\begin{array}{r} a^3 + b^3 \\ a^3 - b^3 \\ \hline a^6 + a^3b^3 \\ \quad - a^3b^3 - b^6 \\ \hline a^6 \qquad - b^6 \end{array}$$

32. $n^2 - 8n + 2n^2 - 3 = 3n^2 - 8n - 3$

33. $-6a^2x^3 + 30a^3x^2 - 4a^4x$

34. $4m^5 - 12m^2n^4 - 5m^4n^3 + 10m^3n^4$

C **35.** $\frac{-6x^2y^2}{-6x^2y^2} + \frac{3xy^2}{-6x^2y^2} - \frac{2x^2y}{-6x^2y^2} = 1 + \left(-\frac{1}{2x}\right) - \left(-\frac{1}{3y}\right) = 1 - \frac{1}{2x} + \frac{1}{3y}$

36.
$$\begin{array}{r} x^2 - 5x + 2 \\ 3x - 2 \\ \hline 3x^3 - 15x^2 + 6x \\ \quad - 2x^2 + 10x - 4 \\ \hline 3x^3 - 17x^2 + 16x - 4 \end{array}$$

37.
$$\begin{array}{r|l} & 2x - 3 \qquad Q: 2x - 3 \\ \cline{2-2} 4x - 5 & 8x^2 - 22x + 15 \\ & \underline{8x^2 - 10x} \\ & \quad - 12x + 15 \\ & \underline{- 12x + 15} \\ & \qquad 0 \end{array}$$

38.
$$\begin{array}{r} 2r - 3s \\ 2r - 3s \\ \hline 4r^2 - 6rs \\ \quad - 6rs + 9s^2 \\ \hline 4r^2 - 12rs + 9s^2 \\ 2r - 3s \\ \hline 8r^3 - 24r^2s + 18rs^2 \\ \quad - 12r^2s + 36rs^2 - 27s^3 \\ \hline 8r^3 - 36r^2s + 54rs^2 - 27s^3 \end{array}$$

39.
$$\begin{array}{r|l} & 2x^2 + 3x - 4 \\ \cline{2-2} 3x - 5 & 6x^3 - x^2 - 27x + 10 \\ & \underline{6x^3 - 10x^2} \\ & \quad 9x^2 - 27x \\ & \underline{9x^2 - 15x} \\ & \qquad - 12x + 10 \\ & \qquad - 12x + 20 \\ & \qquad\qquad -10 \end{array}$$

$$Q: 2x^2 + 3x - 4 + \frac{-10}{3x - 5}$$

40. $P = 8a + 2b + 2c$

41. $A = (b + 2a)(c + 2a) - 4a^2 = bc + 2ab + 2ac + 4a^2 - 4a^2 = bc + 2ab + 2ac$
or $A = b(c + 2a) + 2(ca) = bc + 2ab + 2ac$

42. $C = 2\pi r$; $P = 4[\frac{1}{4}(2\pi n)] = 2\pi n$

43. $2n$ = length of side of square; $(2n)^2$ = area of square; $\frac{1}{4}\pi n^2$ = area of each quarter-circle;
$A = (2n)^2 - 4(\frac{1}{4}\pi n^2) = 4n^2 - \pi n^2$

44. $T = \frac{3(lwh)}{94}$; $T = \frac{3lwh}{94}$

45. $t(L) = t\left(\frac{mt - k}{t}\right)$; $tL = mt - k$; $k = mt - tL$; $k = 100(4) - 4(99) = 4(100 - 99) = 4(1) = 4$

46. $F - 32 = \frac{9C}{5}$; $\frac{5}{9}(F - 32) = \frac{5}{9}\left(\frac{9C}{5}\right)$; $C = \frac{5}{9}(F - 32)$ *or* $C = \frac{5F - 160}{9}$;
$C = \frac{5}{9}(40 - 32) = \frac{5}{9}(8) = \frac{40}{9} = 4\frac{4}{9}°$

47. $m\left(\frac{x}{m}\right) = m\left(\frac{1}{m}\right)$; $x = 1$. {1}

48. $b = \frac{lwd}{2.5}$

49. Let p = rate of plane headed east; $p + 25$ = rate of plane headed west; $4p + 4(p + 25) = 4900$; $4p + 4p + 100 = 4900$; $8p = 4800$; $p = 600$ mph, $p + 25 = 625$ mph, *Ans.*

50. Let n = larger part; $75 - n$ = smaller part; $3(75 - n) = 2n - 5$; $225 - 3n = 2n - 5$; $-5n = -230$; $n = 46$, $75 - n = 29$, *Ans.*

51. Let n = first no.; $3n$ = second no.; $n + 20$ = third no.; $n + 3n + (n + 20) = 360$; $5n + 20 = 360$; $5n = 340$; $n = 68$, $3n = 204$, $n + 20 = 88$, *Ans.*

52. Let n = no. of nickels; $23 - n$ = no. of dimes; $10(23 - n) - 5n = 95$; $230 - 10n - 5n = 95$; $230 - 15n = 95$; $-15n = -135$; $n = 9$; $23 - n = 14$. 9 nickels, 14 dimes, *Ans.*

53. Let w = width of flower bed; $2w$ = length of flower bed; $w + 6$ = width of flower bed and walk; $2w + 6$ = length of flower bed and walk; $w \cdot 2w + 216 = (w + 6)(2w + 6)$; $2w^2 + 216 = 2w^2 + 6w + 12w + 36$; $216 = 18w + 36$; $180 = 18w$; $w = 10$ ft; $2w = 20$ ft. 10 ft × 20 ft, *Ans.*

54. Let x = cost per lb of black tea; $x + 75$ = cost per lb of orange tea; $20(x + 75) + 40x = 150(60)$; $20x + 1500 + 40x = 9000$; $60x + 1500 = 9000$; $60x = 7500$; $x = 125$ cents = \$1.25; $x + 75 = 200$ cents = \$2.00. Black, \$1.25; orange, \$2.00, *Ans.*

Page 233 · **EXTRA FOR EXPERTS**

1. $\frac{1}{a} \cdot b^2 = \frac{b^2}{a}$ **2.** $2 \cdot \frac{1}{a^3} = \frac{2}{a^3}$ **3.** $4h^3 \cdot \frac{1}{k^{-2}} = 4h^3k^2$

4. $3m^{-1} \cdot \frac{1}{2n^{-4}} = 3 \cdot \frac{1}{m} \cdot \frac{1}{2} \cdot n^4 = \frac{3n^4}{2m}$ **5.** $a \cdot \frac{1}{b^2} = ab^{-2}$ **6.** $2r^{-2} \cdot \frac{1}{s^{-5}} = 2r^{-2}s^5$

7. $x^2 \cdot \frac{1}{y^3z^{-4}} = x^2 \cdot \frac{1}{y^3} \cdot \frac{1}{z^{-4}} = x^2y^{-3}z^4$ **8.** $p^{-2} \cdot \frac{1}{q^{-1}t^{-3}} = p^{-2} \cdot \frac{1}{q^{-1}} \cdot \frac{1}{t^{-3}} = p^{-2}qt^3$

9. $\frac{1}{(\frac{1}{4})^3} = \frac{1}{\frac{1}{64}} = 64$ **10.** $\frac{1}{(-4)^3} = \frac{1}{-64} = -\frac{1}{64}$ **11.** $\frac{1}{10^5} = \frac{1}{100{,}000}$ *or* .00001

12. $\frac{1}{(-\frac{3}{4})^2} = \frac{1}{\frac{9}{16}} = \frac{16}{9}$ **13.** $9c^{-8}$

14. $-3a^{1-(-3)}b^{-2-(-6)} = -3a^4b^4$ **15.** $2.4 \times 10^{-8-(-10)} = 2.4 \times 10^2 = 240$

CHAPTER 7. Special Products and Factoring

Page 240 · **WRITTEN EXERCISES**

A **1.** $2 \cdot 3 \cdot 5 \cdot 7$ **2.** $2 \cdot 7 \cdot 13$ **3.** $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11$ **4.** $3 \cdot 5 \cdot 7 \cdot 11$ **5.** $2^2 \cdot 3 \cdot 5^3$

6. $2^4 \cdot 5^3$ **7.** prime **8.** prime **9.** 1, 5, 7, 35 **10.** 1, 7, 13, 91

11. 1, 3, 7, 9, 21, 63 **12.** 1, 2, 4, 13, 26, 52 **13.** 1, 2, 4, 5, 10, 20, 25, 50, 100

14. 1, 3, 7, 9, 21, 49, 63, 147, 441 **15.** 1, 2, 3, 6, 11, 22, 33, 66, 121, 242, 363, 726

16. 1, 2, 5, 10, 13, 26, 65, 130, 169, 338, 845, 1690

17. $144 = 2^4 \cdot 3^2$; $630 = 2 \cdot 3^2 \cdot 5 \cdot 7$; greatest common factor is $2 \cdot 3^2 = 18$

18. $231 = 3 \cdot 7 \cdot 11$; $294 = 2 \cdot 3 \cdot 7^2$; greatest common factor is $3 \cdot 7 = 21$

19. $180 = 2^2 \cdot 3^2 \cdot 5$; $1368 = 2^3 \cdot 3^2 \cdot 19$; greatest common factor is $2^2 \cdot 3^2 = 36$

20. $4200 = 2^3 \cdot 3 \cdot 5^2 \cdot 7$; $3850 = 2 \cdot 5^2 \cdot 7 \cdot 11$; greatest common factor is $2 \cdot 5^2 \cdot 7 = 350$

21. xy **22.** $3ab$ **23.** $5abc^2$ **24.** $7d^2e$ **25.** $3mn^2p$

26. $-3r^3s$ **27.** $-19p^3d^2q^4$ **28.** $-13u^2v$ **29.** $-4mj$ **30.** $-5m^3$

B **31.** $(x)^5 = x^5$ and $7x^5y^2 = (x^5)(7y^2)$; $(x)^5$ **32.** $(y)^7 = y^7$ and $-3x^2y^7 = (-3x^2)(y^7)$; $(y)^7$

33. $(b^2)^2 = b^4$ and $15b^5c^4 = (b^4)(15bc^4)$; $(b^2)^2$ **34.** $(h^2)^3 = h^6$ and $34a^5h^7 = (34a^5h)(h^6)$; $(h^2)^3$

35. $(5r)^3 = 125r^3$ and $250r^4t^3 = (125r^3)(2rt^3)$; $(5r)^3$

36. $(3v)^3 = 27v^3$ and $27v^6w^2 = (27v^3)(v^3w^2)$; $(3v)^3$

37. $(3rp)^1 = 3rp$ and $192r^4p^6 = (3rp)(64r^3p^5)$; $(3rp)^1$

38. $(7rq^3)^2 = 49r^2q^6$ and $98r^3q^7 = (49r^2q^6)(2rq)$; $(7rq^3)^2$

39. $(2x^2y^2)^1 = 2x^2y^2$ and $6x^6y^6 = (2x^2y^2)(3x^4y^4)$; $(2x^2y^2)^1$

40. $(3u^3v^3)^2 = 9u^6v^6$ and $9u^9v^9 = (9u^6v^6)(u^3v^3)$; $(3u^3v^3)^2$

41. $(5a^2b^3)^2 = 25a^4b^6$ and $625a^8b^7 = (25a^4b^6)(25a^4b)$; $(5a^2b^3)^2$

42. $(4r^3s^2)^3 = 64r^9s^6$ and $256r^{12}s^6 = (64r^9s^6)(4r^3)$; $(4r^3s^2)^3$

43. $(2pq^4)^4 = 16p^4q^{16}$; $(2pq^4)^4$ **44.** $(7m^3n)^3 = 343m^9n^3$; $(7m^3n)^3$

45. $(2c^5d^3e)^3 = 8c^{15}d^9e^3$ and $8c^{20}d^{12}e^4 = (8c^{15}d^9e^3)(c^5d^3e)$; $(2c^5d^3e)^3$

Pages 242–243 · **WRITTEN EXERCISES**

A **1.** $3(x^2 + 4y^2)$ **2.** $7(a^2 + 2b^2)$ **3.** $6x(3x - 2)$ **4.** $9r(3r - 10)$

5. $x(x + 7)$ **6.** $n(n + 13)$ **7.** $3x^2(1 - 7x)$ **8.** $5b^2(1 - 14b)$

9. $2x(3x - 2)$ **10.** $4a(2a + 3)$ **11.** $b(b^2 + b + 1)$ **12.** $a(a^2 - a - 1)$

13. $ab(a + b)$ **14.** $xy(y - x)$ **15.** $3c(5a^2 - 1)$ **16.** $2y(6x^2 + 1)$

17. $5rs(r - 2s)$ **18.** $3ab(a - 3b)$ **19.** $t(1 + 2m)$ **20.** $x(3n - 1)$

21. $-6x(2x + 1)$ **22.** $(y + 2)(y - 1)$ **23.** $(a + 9)(a - 8)$ **24.** $(4c + 5d)(x - y)$

25. $(x + 1)(2x + 3 - 1) = (x + 1)(2x + 2) = 2(x + 1)(x + 1)$

26. $(x - y + x + y)(x - y) = (2x)(x - y)$

27. $[2m - (m + n)](m - n) = (2m - m - n)(m - n) = (m - n)(m - n)$

28. $(b - 3c)(x^2 + y^2)$ **29.** $(k + 2)k + (k + 2)t = (k + 2)(k + t)$

30. $(x - 1)x + (x - 1)y = (x - 1)(x + y)$ **31.** $d(g + m) + (-f)(g + m) = (d - f)(g + m)$

32. $r(s - t) + (-k)(s - t) = (r - k)(s - t)$ **33.** $(3b + 2)b + (3b + 2)4 = (3b + 2)(b + 4)$

B **34.** $4x(3x^3 - 2x + 5)$ **35.** $6y(3y^3 + 5y^2 - 7)$ **36.** $5rt(r - 2 + t)$ **37.** $3ac(6a^2c - 3a + 2c)$

38. $x(3x - 2) + 2(3x - 2) = (x + 2)(3x - 2)$

39. $10y^2 - 2y + 15y - 3 = 2y(5y - 1) + 3(5y - 1) = (2y + 3)(5y - 1)$

40. $x(x + a + b)$ **41.** $3a[a + 2(x - y)] = 3a(a + 2x - 2y)$

42. $m^2 + 2m + 7am + 14a = m(m + 2) + 7a(m + 2) = (m + 7a)(m + 2)$

43. $2p^2 + 4ap + p + 2a = 2p(p + 2a) + (p + 2a) = (2p + 1)(p + 2a)$

44. $x^3 - 3x^2 + 7x - 21 = x^2(x - 3) + 7(x - 3) = (x^2 + 7)(x - 3)$

45. $r^3 - sr^2 + rs^2 - s^3 = r^2(r - s) + s^2(r - s) = (r^2 + s^2)(r - s)$

Page 244 · PROBLEMS

1. A = area of circle − area of square; $A = \pi r^2 - r^2 = r^2(\pi - 1)$

2. A = area of square − area of circle; $A = (4r)^2 - \pi r^2 = 16r^2 - \pi r^2 = r^2(16 - \pi)$

3. A = area of circle − area of square; $A = \pi(2S)^2 - S^2 = 4\pi S^2 - S^2 = S^2(4\pi - 1)$

4. A = area of outer circle − area of inner circle; $A = \pi R^2 - \pi r^2 = \pi(R^2 - r^2)$

5. A = area of large circle − 4(area of a small circle); $A = \pi R^2 - 4\pi r^2 = \pi(R^2 - 4r^2)$

6. A = area of square − 2(area of a circle); $A = (4r)^2 - 2(\pi r^2) = 16r^2 - 2\pi r^2 = 2r^2(8 - \pi)$

7. A = area of square + area of semi-circle; $A = (2r)^2 + \frac{1}{2}(\pi r^2) = 4r^2 + \frac{1}{2}\pi r^2 = \frac{1}{2}r^2(8 + \pi)$

8. A = area of rectangle − area of semi-circle; $A = (4r)(2r) - \frac{1}{2}(\pi r^2) = 8r^2 - \frac{1}{2}\pi r^2 = \frac{1}{2}r^2(16 - \pi)$

Page 246 · WRITTEN EXERCISES

A

1. a. $(10 - 1)(10 + 1) = 100 - 1 = 99$;
b. $9(10) + 9(1) = 90 + 9 = 99$

2. a. $(50 + 1)(50 - 1) = 2500 - 1 = 2499$;
b. $50(49) + 1(49) = 2450 + 49 = 2499$

3. a. $(20 + 2)(20 - 2) = 400 - 4 = 396$;
b. $20(18) + 2(18) = 360 + 36 = 396$

4. a. $(60 - 2)(60 + 2) = 3600 - 4 = 3596$;
b. $58(60) + 58(2) = 3480 + 116 = 3596$

5. a. $(30 + 3)(30 - 3) = 900 - 9 = 891$;
b. $30(27) + 3(27) = 810 + 81 = 891$

6. a. $(70 - 3)(70 + 3) = 4900 - 9 = 4891$;
b. $67(70) + 67(3) = 4690 + 201 = 4891$

7. a. $(40 - 4)(40 + 4) = 1600 - 16 = 1584$;
b. $36(40) + 36(4) = 1440 + 144 = 1584$

8. a. $(80 + 4)(80 - 4) = 6400 - 16 = 6384$;
b. $80(76) + 4(76) = 6080 + 304 = 6384$

9. a. $(50 + 5)(50 - 5) = 2500 - 25 = 2475$;
b. $50(45) + 5(45) = 2250 + 225 = 2475$

10. a. $(90 + 5)(90 - 5) = 8100 - 25 = 8075$;
b. $90(85) + 5(85) = 7650 + 425 = 8075$

11. a. $(40 + 6)(40 - 6) = 1600 - 36 = 1564$;
b. $40(34) + 6(34) = 1360 + 204 = 1564$

12. a. $(100 - 6)(100 + 6) = 10{,}000 - 36 = 9964$;
b. $94(100) + 94(6) = 9400 + 564 = 9964$

B

13. a. $(1000 + 10)(1000 - 10) = 1{,}000{,}000 - 100 = 999{,}900$;
b. $1000(990) + 10(990) = 990{,}000 + 9900 = 999{,}900$

14. a. $(500 + 20)(500 - 20) = 250{,}000 - 400 = 249{,}600$;
b. $500(480) + 20(480) = 240{,}000 + 9600 = 249{,}600$

15. a. $(5 + \frac{1}{2})(5 - \frac{1}{2}) = 25 - \frac{1}{4} = 24\frac{3}{4}$;
b. $5(4\frac{1}{2}) + \frac{1}{2}(4\frac{1}{2}) = 22\frac{1}{2} + 2\frac{1}{4} = 24\frac{3}{4}$

16. a. $(1100 + 30)(1100 - 30) = 1{,}210{,}000 - 900 = 1{,}209{,}100$;
b. $1100(1070) + 30(1070) = 1000(1070) + 100(1070) + 30(1070) =$
$1{,}070{,}000 + 107{,}000 + 32{,}100 = 1{,}209{,}100$

17. a. $(600 + 40)(600 - 40) = 360{,}000 - 1600 = 358{,}400$;
b. $600(560) + 40(560) = 336{,}000 + 22{,}400 = 358{,}400$

18. a. $(9 - \frac{2}{3})(9 + \frac{2}{3}) = 81 - \frac{4}{9} = 80\frac{5}{9}$;
b. $8(9\frac{2}{3}) + \frac{1}{3}(9\frac{2}{3}) = 8(9) + 8(\frac{2}{3}) + \frac{1}{3}(9) + \frac{1}{3}(\frac{2}{3}) = 72 + 5\frac{1}{3} + 3 + \frac{2}{9} = 80\frac{5}{9}$

19. a. $(1200 - 50)(1200 + 50) = 1{,}440{,}000 - 2500 = 1{,}437{,}500$;
b. $1150(1200) + 1150(50) = 1000(1200) + 100(1200) + 50(1200) + 1150(50) =$
$1{,}200{,}000 + 120{,}000 + 60{,}000 + 57{,}500 = 1{,}437{,}500$

20. a. $(300 + 60)(300 - 60) = 90{,}000 - 3600 = 86{,}400$
b. $300(240) + 60(240) = 72{,}000 + 14{,}400 = 86{,}400$

21. a. $(4 + \frac{1}{4})(4 - \frac{1}{4}) = 16 - \frac{1}{16} = 15\frac{15}{16}$;
b. $4(3\frac{3}{4}) + \frac{1}{4}(3\frac{3}{4}) = 4(3) + 4(\frac{3}{4}) + \frac{1}{4}(3) + \frac{1}{4}(\frac{3}{4}) = 12 + 3 + \frac{3}{4} + \frac{3}{16} = 15\frac{15}{16}$

Pages 247–248 · WRITTEN EXERCISES

A **1.** $(x + 4)(x - 4)$ **2.** $(r + 3)(r - 3)$ **3.** $(x + 2y)(x - 2y)$
4. $(a + 2b)(a - 2b)$ **5.** $(R + r)(R - r)$ **6.** $(a + by^3)(a - by^3)$
7. $(4a + b^2)(4a - b^2)$ **8.** $(16x^2 + y)(16x^2 - y)$ **9.** $(2x^2 + z)(2x^2 - z)$
10. $(m + 4n^2)(m - 4n^2)$ **11.** $(14b + 11x)(14b - 11x)$ **12.** $(17x + 26y)(17x - 26y)$
13. $4r^2t^2 - 9 = (2rt + 3)(2rt - 3)$ **14.** $m^2n^2 - 144 = (mn + 12)(mn - 12)$
15. $(1 + 3n)(1 - 3n)$ **16.** $(5m + 1)(5m - 1)$ **17.** $(.3a + 2)(.3a - 2)$ **18.** $(.2b + 7)(.2b - 7)$
19. $(c + .8)(c - .8)$ **20.** $(.9 + d)(.9 - d)$ **21.** $(x + \frac{1}{3})(x - \frac{1}{3})$ **22.** $(\frac{1}{4} + y)(\frac{1}{4} - y)$
23. $\left(a + \frac{b}{2}\right)\left(a - \frac{b}{2}\right)$ **24.** $\frac{d^2}{36} - \frac{c^2}{25} = \left(\frac{d}{6} + \frac{c}{5}\right)\left(\frac{d}{6} - \frac{c}{5}\right)$

B **25.** $25x^6 - x^4 = x^4(25x^2 - 1) = x^4(5x + 1)(5x - 1)$
26. $s^2(16 - s^2) = s^2(4 + s)(4 - s)$ **27.** $a^2(1 - a^2) = a^2(1 + a)(1 - a)$
28. $c^3(1 - c^2) = c^3(1 + c)(1 - c)$ **29.** $xy(x^2 - y^4) = xy(x + y^2)(x - y^2)$
30. $a^2b(a^4 - b^2) = a^2b(a^2 + b)(a^2 - b)$ **31.** $2pq^2(p^2q^2 - 36) = 2pq^2(pq + 6)(pq - 6)$
32. $3x^2y(49 - x^2y^2) = 3x^2y(7 + xy)(7 - xy)$
33. $2(a^4 - 16) = 2(a^2 + 4)(a^2 - 4) = 2(a^2 + 4)(a + 2)(a - 2)$
34. $n(1 - n^4) = n(1 + n^2)(1 - n^2) = n(1 + n^2)(1 + n)(1 - n)$
35. $1 - a^{2n} = (1 + a^n)(1 - a^n)$ **36.** $t^2 - n^{2x} = (t + n^x)(t - n^x)$
37. $m^a - m^{5a} = m^a(1 - m^{4a}) = m^a(1 + m^{2a})(1 - m^{2a}) = m^a(1 + m^{2a})(1 + m^a)(1 - m^a)$
38. $b^{6x} - b^{2x} = b^{2x}(b^{4x} - 1) = b^{2x}(b^{2x} + 1)(b^{2x} - 1) = b^{2x}(b^{2x} + 1)(b^x + 1)(b^x - 1)$
39. $.02(9r^4 - 4s^6t^2) = .02(3r^2 + 2s^3t)(3r^2 - 2s^3t)$

C **40.** $(a + 2b + x^2)(a + 2b - x^2)$ **41.** $(3a - 1 + y^3)(3a - 1 - y^3)$
42. $(x - 1)^2 - 1 = (x - 1 + 1)(x - 1 - 1) = x(x - 2)$
43. $(x + 3)^2 - 9 = (x + 3 + 3)(x + 3 - 3) = x(x + 6)$
44. a. Let $n =$ an integer; then $n + 1 =$ the next consecutive integer;
$(n + 1)^2 - n^2 = n^2 + 2n + 1 - n^2 = 2n + 1 = n + (n + 1)$
b. Let $2n + 1 =$ an odd integer; then $2n + 3 =$ the next consecutive odd integer; $2(4n + 4) = 8n + 8$
$=$ twice sum of integers; $(2n + 3)^2 - (2n + 1)^2 = 4n^2 + 12n + 9 - 4n^2 - 4n - 1 = 8n + 8$

Pages 250–251 · PROBLEMS

A **1.** Area of entire square = sum of areas of nine regions;
$(a + b + c)^2 = a^2 + ab + ac + ba + b^2 + bc + ca + cb + c^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$
2. Area of white region = area of entire square − areas of remaining three regions;
$(a - b)^2 = a^2 - (a - b)b - b^2 - b(a - b) = a^2 - ab + b^2 - b^2 - ba + b^2 = a^2 - 2ab + b^2$

B **3.** Area of entire region − area of white region = area of remaining four regions;
$(a + b)^2 - (a - b)^2 = ba + ba + ab + ab = 4ab$
4. Area of entire region + area of white region = sum of areas of upper left square, lower right square; upper right square and lower left square;
$(a + b)^2 + (a - b)^2 = a^2 + a^2 + b^2 + b^2 = 2a^2 + 2b^2 = 2(a^2 + b^2)$
5. Area of entire square − area of dark red region = sum of areas of remaining three regions;
$(a + b)^2 - b^2 = ba + a^2 + ab = a^2 + 2ab$
6. Area of entire square − area of white region = sum of areas of remaining three regions;
$a^2 - (a - b)^2 = b(a - b) + b^2 + (a - b)b = ba - b^2 + b^2 + ab - b^2 = 2ab - b^2$

C **7.** *Part 1.* The shaded areas of the first column on the left and the first column on the right are equal to one complete shaded column. The areas of the second from the left plus the second from the right

also equal one complete shaded column. Continuing in this way gives four complete shaded columns with 9 squares in each column. Thus, $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8$ or $(1 + 8) + (2 + 7) + (3 + 6) + (4 + 5) = 4(9) = \frac{8(9)}{2}$.

Part 2. Consider a region with n columns and $(n + 1)$ rows. There is 1 shaded square in the first column on the left, 2 in the second, . . . , n in the last column on the right. Following the process described in part 1 gives $\frac{n}{2}$ complete shaded columns with $(n + 1)$ squares in each column. Thus, $1 + 2 + 3 + \cdots n = \frac{n}{2}(n + 1) = \frac{n(n + 1)}{2}$.

8. *Part 1.* The area of the first region in the upper left plus the area of the first region in the lower right is equal in area to $2 \cdot 8$. The area of the second from the upper left plus the second from the lower right also equals $2 \cdot 8$. Continuing in this way gives four regions, each equal in area to $2 \cdot 8$. Thus, $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15$ or $(1 + 15) + (3 + 13) + (5 + 11) + (7 + 9) = 4(2 \cdot 8) = 8 \cdot 8 = 8^2$.

Part 2. Consider n such regions placed together as shown in the figure. The dimensions of the entire region will be n by n. Following the process described in part 1 gives $\frac{n}{2}$ regions, each equal in area to $2 \cdot n$. Thus, $1 + 3 + 5 + \cdots (2n - 1) = \frac{n}{2}(2 \cdot n) = n^2$.

Pages 252–253 · WRITTEN EXERCISES

A

1. $(x + b)^2$ **2.** $(g - h)^2$ **3.** $(a - 6)^2$ **4.** $(y + 8)^2$ **5.** $(b + 7)^2$
6. $(n + 9)^2$ **7.** $(2a - b)^2$ **8.** $(5a - b)^2$ **9.** $(3x + 1)^2$ **10.** $(4x + 1)^2$
11. $(1 + n)^2$ **12.** $(1 + 2b)^2$ **13.** $(7x - 2)^2$ **14.** $(5x - 3)^2$ **15.** $(5a + 6b)^2$
16. $(12n + 5x)^2$ **17.** $(12x - 1)^2$ **18.** $(4x - 3y)^2$ **19.** $(6a + 5b)^2$ **20.** $(4r + 5t)^2$
21. $r^2 - 10r + 25 = (r - 5)^2$ **22.** $4m^2 + 4mn + n^2 = (2m + n)^2$
23. $(11ab - 1)^2$ **24.** $3(81m^2n^2 + 18mn + 1) = 3(9mn + 1)^2$
25. $3k(1 + 14k + 49k^2) = 3k(1 + 7k)^2$ **26.** $2n(4 + 4n + n^2) = 2n(2 + n)^2$
27. $(z - 2a^2)^2$ **28.** $(x^2 + y)^2$ **29.** $(y^3 - 4)^2$
30. $(n^2 - 1)^2 = [(n + 1)(n - 1)]^2 = (n + 1)^2(n - 1)^2$

B

31. $(x^2 - 9)^2 = [(x + 3)(x - 3)]^2 = (x + 3)^2(x - 3)^2$ **32.** $(a^4 - 3)^2$
33. $(x - 2y)^2 - 9 = (x - 2y + 3)(x - 2y - 3)$ **34.** $(a + 3b)^2 - 1 = (a + 3b + 1)(a + 3b - 1)$
35. $m^2 - (x^2 - 2x + 1) = m^2 - (x - 1)^2 = [m + (x - 1)][m - (x - 1)] = (m + x - 1)(m - x + 1)$
36. $n^2 - (y^2 + 6y + 9) = n^2 - (y + 3)^2 = [n + (y + 3)][n - (y + 3)] = (n + y + 3)(n - y - 3)$

C

37. $(x + 3)^2 = x^2 + 6x + 9$; $k = 6$ or $(x - 3)^2 = x^2 - 6x + 9$; $k = -6$
38. $(x - 5)^2 = x^2 - 10x + 25$; $-10 = -2k$; $k = 5$ or $(x + 5)^2 = x^2 + 10x + 25$; $10 = -2k$; $k = -5$
39. $(n - 8)^2 = n^2 - 16n + 64$; $k = 64$

Pages 254–255 · WRITTEN EXERCISES

A

1. $12a^2 + 16a + 9a + 12 = 12a^2 + 25a + 12$ **2.** $27z^2 + 9z + 6z + 2 = 27z^2 + 15z + 2$
3. $10b^2 - 20b - 18b + 36 = 10b^2 - 38b + 36$ **4.** $14y^2 - 10y - 21y + 15 = 14y^2 - 31y + 15$
5. $12c^2 + 9c - 8c - 6 = 12c^2 + c - 6$ **6.** $42x^2 + 30x - 63x - 45 = 42x^2 - 33x - 45$
7. $6w^2 - 10vw + 33vw - 55v^2 = 6w^2 + 23vw - 55v^2$
8. $28h^2 - 48hk + 49hk - 84k^2 = 28h^2 + hk - 84k^2$
9. $16x^2 - 6xy + 40xy - 15y^2 = 16x^2 + 34xy - 15y^2$
10. $48x^2 + 30xy - 56xy - 35y^2 = 48x^2 - 26xy - 35y^2$
11. $-x^2 + 3x - 7x + 21 = -x^2 - 4x + 21$ **12.** $-x^2 - 6x + 5x + 30 = -x^2 - x + 30$
13. $s^2 + 5s + 3s + 15 = s^2 + 8s + 15$ **14.** $t^2 + 2t + 8t + 16 = t^2 + 10t + 16$

B

15. $-2y^2 + 16y + 3y - 24 = -2y^2 + 19y - 24$
16. $-3y^2 - 4y - 15y - 20 = -3y^2 - 19y - 20$
17. $6n^2 + 10n + 12n + 20 = 6n^2 + 22n + 20$ **18.** $10m^2 + 15m + 4m + 6 = 10m^2 + 19m + 6$

19. $-.48m^2 + .84m + .92m - 1.61 = -.48m^2 + 1.76m - 1.61$

20. $-6.51h^2 + 13.64h - 6.72h + 14.08 = -6.51h^2 + 6.92h + 14.08$

C **21.** $x^2 - 25 = x^2 + x - 42$; $-25 = x - 42$; $17 = x$. $\{17\}$

22. $x^2 - 16 = x^2 + 2x - 48$; $-16 = 2x - 48$; $32 = 2x$; $x = 16$. $\{16\}$

23. $16r^2 + 2r - 3 = 16r^2 - 8r + 1$; $2r - 3 = -8r + 1$; $10r = 4$; $r = \frac{2}{5}$. $\{\frac{2}{5}\}$

24. $36t^2 + 14t - 2 = 36t^2 + 12t + 1$; $14t - 2 = 12t + 1$; $2t = 3$; $t = \frac{3}{2}$. $\{\frac{3}{2}\}$

25. $x^2 - 1 = x^2 - 2x + 1 + 2x$; $-1 = 1$. $\emptyset$

26. $x^2 - 3x + 2 = x^2 + 2x + 1 - 5x + 1$; $-3x + 2 = -3x + 2$; $0 = 0$. {directed numbers}

Pages 256–257 · WRITTEN EXERCISES

A

1. $(n + 3)(n + 11)$	**2.** $(z + 3)(z + 9)$	**3.** $(x + 2)(x + 9)$	**4.** $(x + 8)(x + 2)$
5. $(h - 9)(h - 4)$	**6.** $(y - 7)(y - 8)$	**7.** $(x - 9)(x - 10)$	**8.** $(a - 45)(a - 2)$
9. $(m + 15)(m + 6)$	**10.** $(r + 30)(r + 3)$	**11.** $(33 - b)(1 - b)$	**12.** $(1 - k)(14 - k)$
13. $(3 + c)(14 + c)$	**14.** $(4 + s)(13 + s)$	**15.** $(x + 3)(x + 17)$	**16.** $(y + 51)(y + 1)$
17. $(x + 2y)(x + 12y)$	**18.** $(y + 24z)(y + 2z)$	**19.** $(m - 18n)(m - 4n)$	**20.** $(s - t)(s - 20t)$
21. $(b - 19c)(b - 4c)$	**22.** $(a - 3b)(a - 19b)$	**23.** $(z - 5b)(z - 24b)$	**24.** $(z - 15d)(z - 8d)$

B **25.** $20 = 1 \cdot 20, 2 \cdot 10, 4 \cdot 5, (-1)(-20), (-2)(-10), (-4)(-5)$. Values for b: 21, 12, 9, −21, −12, −9

26. $63 = 1 \cdot 63, 3 \cdot 21, 7 \cdot 9, (-1)(-63), (-3)(-21), (-7)(-9)$. Values for b: 64, 24, 16, −64, −24, −16

27. $1 = 1 \cdot 1, (-1)(-1)$. Values for b: 2, −2

28. $4 = 1 \cdot 4, 2 \cdot 2, (-1)(-4), (-2)(-2)$. Values for b: 5, 4, −5, −4

29. $36 = 1 \cdot 36, 2 \cdot 18, 3 \cdot 12, 4 \cdot 9, 6 \cdot 6, (-1)(-36), (-2)(-18), (-3)(-12), (-4)(-9), (-6)(-6)$; values for $2b$: 37, 20, 15, 13, 12, −37, −20, −15, −13, −12. Integral values for b: 10, 6, −10, −6

30. $44 = 1 \cdot 44, 2 \cdot 22, 4 \cdot 11, (-1)(-44), (-2)(-22), (-4)(-11)$; values for $2b$: 45, 24, 15, −45, −24, −15. Integral values for b: 12, −12

31. $5 = 1 + 4, 2 + 3$. Values for c: 4, 6

32. $7 = 1 + 6, 2 + 5, 3 + 4$. Values for c: 6, 10, 12

33. $-6 = -1 + (-5), -2 + (-4), -3 + (-3)$. Values for c: 5, 8, 9

34. $7 = 1 \cdot 7, (-1)(-7)$; $1 + 7 \neq 3, -1 + (-7) \neq 3$

35. $9 = 1 \cdot 9, 3 \cdot 3, (-1)(-9), (-3)(-3)$; $1 + 9 \neq 15; 3 + 3 \neq 15, -1 + (-9) \neq 15, -3 + (-3) \neq 15$

36. $1 = 1 \cdot 1, (-1)(-1)$; $1 + 1 \neq -4, -1 + (-1) \neq -4$

37. $3 = 1 \cdot 3, (-1)(-3)$; $1 + 3 \neq -3, -1 + (-3) \neq -3$

38. $x^2 + 0x + 4$; $4 = 1 \cdot 4, 2 \cdot 2, (-1)(-4), (-2)(-2)$; $1 + 4 \neq 0, 2 + 2 \neq 0, -1 + (-4) \neq 0, -2 + (-2) \neq 0$

39. $x^2 + 0x + 9$; $9 = 1 \cdot 9, 3 \cdot 3, (-1)(-9), (-3)(-3)$; $1 + 9 \neq 0, 3 + 3 \neq 0, -1 + (-9) \neq 0, -3 + (-3) \neq 0$

Pages 258–259 · WRITTEN EXERCISES

A

1. $(x + 5)(x - 2)$	**2.** $(y + 7)(y - 2)$	**3.** $(y - 7)(y + 3)$	**4.** $(x - 5)(x + 3)$
5. $(h - 9)(h + 7)$	**6.** $(n + 8)(n - 7)$	**7.** $(k + 11)(k - 10)$	**8.** $(w - 12)(w + 8)$
9. $(r - 11)(r + 9)$	**10.** $(x - 15)(x + 2)$	**11.** $(u - 11)(u + 5)$	**12.** $(b + 12)(b - 11)$
13. $(x - 11)(x + 3)$	**14.** $(n - 12)(n + 4)$	**15.** $(x - 10)(x + 9)$	**16.** $(b - 8)(b + 3)$
17. $(z - 4t)(z + t)$	**18.** $(t - 5u)(t + 2u)$	**19.** $(y + 2z)(-y + z)$	**20.** $(x + 3y)(-x + 2y)$
21. $(a + 8)(a - 5)$	**22.** $(c + 12)(c - 3)$	**23.** $(d - 8)(d + 2)$	**24.** $(m - 6)(m + 5)$

B **25.** $-12 = (1)(-12), (2)(-6), (3)(-4), (-1)(12), (-2)(6), (-3)(4)$.
Values for b: −11, −4, −1, 11, 4, 1

26. $-14 = (1)(-14), (2)(-7), (-1)(14), (-2)(7)$. Values for b: −13, −5, 13, 5

27. $-4 = (1)(-4), (2)(-2), (-1)(4)$. Values for b: −3, 0, 3

28. $-16 = (1)(-16), (2)(-8), (4)(-4), (-1)(16), (-2)(8)$. Values for b: −15, −6, 0, 15, 6

29. $-28 = (1)(-28), (2)(-14), (4)(-7), (-1)(28), (-2)(14), (-4)(7)$; values for $2b$: −27, −12, −3, 27, 12, 3. Integral values for b: −6, 6

30. $-20 = (1)(-20), (2)(-10), (4)(-5), (-1)(20), (-2)(10), (-4)(5)$; values for $2b$: −19, −8, −1, 19, 8, 1. Integral values for b: −4, 4

31. $5 = 6 + (-1), 7 + (-2)$. Values for c: −6, −14

32. $7 = 8 + (-1), 9 + (-2)$. Values for c: $-8, -18$

33. $-6 = 1 + (-7), 2 + (-8)$. Values for c: $-7, -16$

C **34.** $-7 = (1)(-7), (-1)(7); 1 + (-7) \neq 8, (-1) + 7 \neq 8$

35. $-4 = (1)(-4), (2)(-2), (-1)(4); 1 + (-4) \neq 5, 2 + (-2) \neq 5, (-1) + 4 \neq 5$

36. $-3 = (1)(-3), (-1)(3); 1 + (-3) \neq -4, (-1) + 3 \neq -4$

37. $-5 = (1)(-5), (-1)(5); 1 + (-5) \neq -6, (-1) + 5 \neq -6$

38. $-3 = (1)(-3), (-1)(3); 1 + (-3) \neq 0, (-1) + 3 \neq 0$

39. $-6 = (1)(-6), (2)(-3), (-1)(6), (-2)(3); 1 + (-6) \neq 0, 2 + (-3) \neq 0, (-1) + 6 \neq 0, (-2) + 3 \neq 0$

Page 260 · WRITTEN EXERCISES

A

1. $(2y + 1)(y + 3)$	**2.** $(3x + 1)(x + 2)$	**3.** $(3n - 1)(n - 1)$	**4.** $(3x - 2)(x - 1)$
5. $(5x - 7)(x + 1)$	**6.** $(2x + 1)(x - 5)$	**7.** $(3a - 1)(a + 1)$	**8.** $(5a - 1)(a + 1)$
9. $(4x - 1)(2x - 3)$	**10.** $(3x - 1)(x + 7)$	**11.** $(7y - 3)(5y - 1)$	**12.** $(13y + 6)(y - 1)$

B

13. $(6x + 7)(x + 3)$	**14.** $(14x + 5)(x + 2)$	**15.** $(7x - 11)(2x + 1)$	**16.** $(2x + 5y)(6x - y)$
17. $(4x - 3y)(6x + y)$	**18.** $(6n + 7s)(n - 9s)$	**19.** $(4m - 3)(2m + 5)$	**20.** $(5y + 2)(2y - 3)$

C **21.** $(3x + 4)$ and $(2x - 3)$ **22.** $(4n - 5)$ and $(n + 2)$

23. $A = \frac{1}{2}bh = 15a^2 + 38a - 21; \frac{1}{2}bh = (15a - 7)(a + 3); 2[\frac{1}{2}bh] = 2[(15a - 7)(a + 3)]$; $bh = (30a - 14)(a + 3)$ or $(15a - 7)(2a + 6)$; $[(30a - 14)$ and $(a + 3)]$ or $[(15a - 7)$ and $(2a + 6)]$, *Ans.*

24. $A = \frac{1}{2}bh = 14b^2 - 25b - 25; \frac{1}{2}bh = (7b + 5)(2b - 5)$; $2[\frac{1}{2}bh] = 2[(7b + 5)(2b - 5)]; bh = (14b + 10)(2b - 5)$ or $(7b + 5)(4b - 10)$ $[(14b + 10)$ and $(2b - 5)]$ or $[(7b + 5)$ and $(4b - 10)]$, *Ans.*

Pages 262–263 · WRITTEN EXERCISES

A

1. $3(x^2 - 9) = 3(x + 3)(x - 3)$		**2.** $14(y^2 - 4) = 14(y + 2)(y - 2)$	
3. $2(w^2 + 18w + 81) = 2(w + 9)^2$		**4.** $8(z^2 + 14z + 49) = 8(z + 7)^2$	
5. $(6x - 5)(x - 1)$	**6.** $(5r - 3)(r - 2)$	**7.** $(7n - 1)^2$	**8.** $(5x - 2)^2$
9. $-5a(x^2 + y^2)$	**10.** $-rt(t^2 + r^2)$	**11.** $(3x - 4)^2$	**12.** $(2x - 7)^2$
13. $(13a + 7b^2)(13a - 7b^2)$		**14.** $(7y^2 + 12z)(7y^2 - 12z)$	
15. $(n + 2)(n - 20)$	**16.** $(n - 12)(n + 2)$	**17.** $(x - 7)(x + 12)$	
18. $(x - 6)(x + 8)$	**19.** $(7 + m)(3 - m)$	**20.** $(161 + u)(1 - u)$	
21. $104 + 5s - 2^2 = (8 + s)(13 - s)$		**22.** $180 + 3r - r^2 = (12 + r)(15 - r)$	

23. $5(1 - 5y - 6y^2) = 5(1 - 6y)(1 + y)$ *or* $-5(6y^2 + 5y - 1) = -5(6y - 1)(y + 1)$

24. $3(1 - 7y - 8y^2) = 3(1 - 8y)(1 + y)$ *or* $-3(8y^2 + 7y - 1) = -3(8y - 1)(y + 1)$

25. $(7n + 6)(4n + 9)$ **26.** $6(w^2 - 16w - 57) = 6(w - 19)(w + 3)$

27. $4(t^2 - 7t - 120) = 4(t - 15)(t + 8)$

B

28. $a(z^2 - 12zw + 36w^2) = a(z - 6w)^2$	**29.** $b(x^2 - 14xy + 49y^2) = b(x - 7y)^2$	
30. $3y(2y^2 + y - 1) = 3y(2y - 1)(y + 1)$	**31.** $2v(3v^2 + 13v + 4) = 2v(3v + 1)(v + 4)$	
32. $-1(x^2 - 2bx + b^2) = -1(x - b)^2$	**33.** $-1(k^2 - 2kh + h^2) = -1(k - h)^2$	
34. $-1(n^2 - 2n + 1) = -1(n - 1)^2$	**35.** $-1(4b^2 - 4b + 1) = -1(2b - 1)^2$	
36. $-1(14 + 9z + z^2) = -1(7 + z)(2 + z)$	**37.** $-1(16 + 10k + k^2) = -1(8 + k)(2 + k)$	

38. $(7a^2 - 5a - 2)(a + 1) = (7a + 2)(a - 1)(a + 1)$

39. $(4b^2 - 3b - 1)(b + 2) = (4b + 1)(b - 1)(b + 2)$

40. $(18x^2 + 24x + 8)(x + 1) = 2(9x^2 + 12x + 4)(x + 1) = 2(3x + 2)^2(x + 1)$

41. $(147y^2 - 42y + 3)(y - 1) = 3(49y^2 - 14y + 1)(y - 1) = 3(7y - 1)^2(y - 1)$

42. $(x^2 - 1)(1 + 5x + 4x^2) = (x + 1)(x - 1)(1 + 4x)(1 + x) = (x + 1)^2(x - 1)(1 + 4x)$

43. $(y^2 - 4)(1 + y - 2y^2) = (y + 2)(y - 2)(1 + 2y)(1 - y)$

44. $2(w^4 - 81) = 2(w^2 + 9)(w^2 - 9) = 2(w^2 + 9)(w + 3)(w - 3)$

45. $3(m^4 - 625) = 3(m^2 + 25)(m^2 - 25) = 3(m^2 + 25)(m + 5)(m - 5)$

46. $-1(20y^2 - 43xy + 14x^2) = -1(4y - 7x)(5y - 2x)$

47. $-1(8m^2 - 50mn + 33n^2) = -1(4m - 3n)(2m - 11n)$

C **48.** $(x^2 + 5)(x^2 - 12)$ **49.** $(n^2 + 15)(n^2 - 2)$

50. $(y^2 - 4)(y^2 - 1) = (y + 2)(y - 2)(y + 1)(y - 1)$

51. $(m^2 - 9)(m^2 - 4) = (m + 3)(m - 3)(m + 2)(m - 2)$

52. $5ax^2 - 5ay^2 + bx^2 - by^2 = 5a(x^2 - y^2) + b(x^2 - y^2) = (5a + b)(x^2 - y^2) = (5a + b)(x + y)(x - y)$

53. $m^2r^2 + m^2s^2 - 9r^2 - 9s^2 = m^2(r^2 + s^2) + (-9)(r^2 + s^2) = (m^2 - 9)(r^2 + s^2) = (m + 3)(m - 3)(r^2 + s^2)$

54. $y^2(y - 1) + (y - 1) = (y^2 + 1)(y - 1)$

55. $x^2(x + 1) + (-1)(x + 1) = (x^2 - 1)(x + 1) = (x + 1)(x - 1)(x + 1) = (x + 1)^2(x - 1)$

56. $10x^2 - 3x + c = (2x - 3)(5x + ?)$; if $(2x)(?) + (-15x) = -3x$, then $? = 6$; if $10x^2 - 3x + c = (2x - 3)(5x + 6)$, then $c = (-3)(6) = -18$

57. $21y^2 - y + c = (3y + 2)(7y + ?)$; if $(3y)(?) + 14y = -y$, then $? = -5$; if $21y^2 - y + c = (3y + 2)(7y - 5)$, then $c = (2)(-5) = -10$

58. $cw^2 + 3w - 1 = (5w + 1)((?)w - 1)$; if $(-5w) + (1(?)w) = 3w$, then $? = 8$; if $cw^2 + 3w - 1 = (5w + 1)(8w - 1)$, then $c = (5)(8) = 40$

59. $cn^2 - 5n - 21 = (n - 3)((?)n + 7)$; if $7n + (-3(?)n) = -5n$, then $? = 4$; if $cn^2 - 5n - 21 = (n - 3)(4n + 7)$, then $c = (1)(4) = 4$

Pages 263–264 · WRITTEN EXERCISES

A **1.** $(a - 2) = 0$; $a = 2$. $\{2\}$ **2.** $(a - 8) = 0$; $a = 8$. $\{8\}$ **3.** $(5 + c) = 0$; $c = -5$. $\{-5\}$

4. $\left(16 - \frac{2}{y}\right) = 0$; $16 = \frac{2}{y}$; $16y = 2$; $y = \frac{1}{8}$. $\{\frac{1}{8}\}$

5. $\left(\frac{1}{b} + \frac{2}{7}\right) = 0$; $\frac{1}{b} = -\frac{2}{7}$; $7b\left(\frac{1}{b}\right) = 7b\left(-\frac{2}{7}\right)$; $7 = -2b$; $b = -\frac{7}{2}$. $\{-\frac{7}{2}\}$

6. $\left(\frac{1}{b} + \frac{3}{7}\right) = 0$; $\frac{1}{b} = -\frac{3}{7}$; $7b\left(\frac{1}{b}\right) = 7b\left(-\frac{3}{7}\right)$; $7 = -3b$; $b = -\frac{7}{3}$. $\{-\frac{7}{3}\}$

7. $12 + c = 0$; $c = -12$. $\{-12\}$ **8.** $0 = 0$. {directed numbers} **9.** $0 = 0$. {directed numbers}

10. $\left(18 - \frac{3}{y}\right) = 0$; $18 = \frac{3}{y}$; $18y = 3$; $y = \frac{1}{6}$. $\{\frac{1}{6}\}$

11. $0 = 0$. {any number other than 0} **12.** $0 = 0$. {any number other than 0}

B **13.** $(x - 3) = 0$ or $(x - 5) = 0$; $x = 3$ or $x = 5$. $\{3, 5\}$

14. $(x + 2) = 0$ or $(x + 7) = 0$; $x = -2$ or $x = -7$. $\{-2, -7\}$

15. $(y + 10) = 0$ or $(y - 7) = 0$; $y = -10$ or $y = 7$. $\{-10, 7\}$

16. $(y - 8) = 0$ or $(y + 3) = 0$; $y = 8$ or $y = -3$. $\{8, -3\}$

17. $v = 0$ or $(2v + 1) = 0$; $v = 0$ or $2v = -1$; $v = 0$ or $v = -\frac{1}{2}$. $\{0, -\frac{1}{2}\}$

18. $v = 0$ or $(3v + 1) = 0$; $v = 0$ or $3v = -1$; $v = 0$ or $v = -\frac{1}{3}$. $\{0, -\frac{1}{3}\}$

19. $(5r + 1) = 0$ or $(2r - 6) = 0$; $5r = -1$ or $2r = 6$; $r = -\frac{1}{5}$ or $r = 3$. $\{-\frac{1}{5}, 3\}$

20. $(6r + 3) = 0$ or $(7r + 14) = 0$; $6r = -3$ or $7r = -14$; $r = -\frac{1}{2}$ or $r = -2$. $\{-\frac{1}{2}, -2\}$

Pages 266–267 · WRITTEN EXERCISES

A **1.** $(w + 10)(w - 9) = 0$; $(w + 10) = 0$ or $(w - 9) = 0$; $w = -10$ or $w = 9$. $\{-10, 9\}$

2. $(w + 7)(w - 2) = 0$; $(w + 7) = 0$ or $(w - 2) = 0$; $w = -7$ or $w = 2$. $\{-7, 2\}$

3. $t^2 - 2t - 15 = 0$; $(t - 5)(t + 3) = 0$; $(t - 5) = 0$ or $(t + 3) = 0$; $t = 5$ or $t = -3$. $\{5, -3\}$

4. $s^2 - 3s - 18 = 0$; $(s - 6)(s + 3) = 0$; $(s - 6) = 0$ or $(s + 3) = 0$; $s = 6$ or $s = -3$. $\{6, -3\}$

5. $x^2 + 8x + 15 = 0$; $(x + 3)(x + 5) = 0$; $(x + 3) = 0$ or $(x + 5) = 0$; $x = -3$ or $x = -5$. $\{-3, -5\}$

6. $y^2 + 11y + 18 = 0$; $(y + 2)(y + 9) = 0$; $(y + 2) = 0$ or $(y + 9) = 0$; $y = -2$ or $y = -9$. $\{-2, -9\}$

7. $y^2 - 4y + 3 = 0$; $(y - 3)(y - 1) = 0$; $(y - 3) = 0$ or $(y - 1) = 0$; $y = 3$ or $y = 1$. $\{3, 1\}$

8. $x^2 - 6x + 5 = 0$; $(x - 1)(x - 5) = 0$; $(x - 1) = 0$ or $(x - 5) = 0$; $x = 1$ or $x = 5$. $\{1, 5\}$

9. $x^2 - 3x - 108 = 0$; $(x + 9)(x - 12) = 0$; $(x + 9) = 0$ or $(x - 12) = 0$; $x = -9$ or $x = 12$. $\{-9, 12\}$

10. $m^2 - 5m - 66 = 0$; $(m - 11)(m + 6) = 0$; $(m - 11) = 0$ or $(m + 6) = 0$; $m = 11$ or $m = -6$. $\{11, -6\}$

11. $y^2 - 4 = 0$; $(y + 2)(y - 2) = 0$; $(y + 2) = 0$ or $(y - 2) = 0$; $y = -2$ or $y = 2$. $\{-2, 2\}$

12. $(h + 16)(h - 16) = 0$; $(h + 16) = 0$ or $(h - 16) = 0$; $h = -16$ or $h = 16$. $\{-16, 16\}$

13. $4z^2 - 25 = 0$; $(2z + 5)(2z - 5) = 0$; $(2z + 5) = 0$ or $(2z - 5) = 0$; $z = -\frac{5}{2}$ or $z = \frac{5}{2}$. $\{-\frac{5}{2}, \frac{5}{2}\}$

14. $9z^2 - 16 = 0$; $(3z + 4)(3z - 4) = 0$; $(3z + 4) = 0$ or $(3z - 4) = 0$; $z = -\frac{4}{3}$ or $z = \frac{4}{3}$. $\{-\frac{4}{3}, \frac{4}{3}\}$

15. $x(x - 24) = 0$; $x = 0$ or $(x - 24) = 0$; $x = 0$ or $x = 24$. $\{0, 24\}$

16. $y(y - 16) = 0$; $y = 0$ or $(y - 16) = 0$; $y = 0$ or $y = 16$. $\{0, 16\}$

17. $(r + 2)(2r + 5) = 0$; $(r + 2) = 0$ or $(2r + 5) = 0$; $r = -2$ or $r = -\frac{5}{2}$. $\{-2, -\frac{5}{2}\}$

18. $(r + 2)(3r + 7) = 0$; $(r + 2) = 0$ or $(3r + 7) = 0$; $r = -2$ or $r = -\frac{7}{3}$. $\{-2, -\frac{7}{3}\}$

19. $2n^2 - 11n + 5 = 0$; $(2n - 1)(n - 5) = 0$; $(2n - 1) = 0$ or $(n - 5) = 0$; $n = \frac{1}{2}$ or $n = 5$. $\{\frac{1}{2}, 5\}$

20. $x^2 + 3x = 0$; $x(x + 3) = 0$; $x = 0$ or $(x + 3) = 0$; $x = 0$ or $x = -3$. $\{0, -3\}$

21. $2n^2 - 5n = 0$; $n(2n - 5) = 0$; $n = 0$ or $(2n - 5) = 0$; $n = 0$ or $n = \frac{5}{2}$. $\{0, \frac{5}{2}\}$

22. $2r^2 - 17r + 21 = 0$; $(r - 7)(2r - 3) = 0$; $(r - 7) = 0$ or $(2r - 3) = 0$; $r = 7$ or $r = \frac{3}{2}$. $\{7, \frac{3}{2}\}$

23. $3k^2 - 17k + 10 = 0$; $(k - 5)(3k - 2) = 0$; $(k - 5) = 0$ or $(3k - 2) = 0$; $k = 5$ or $k = \frac{2}{3}$. $\{5, \frac{2}{3}\}$

24. $10t^2 - 11t + 3 = 0$; $(5t - 3)(2t - 1) = 0$; $(5t - 3) = 0$ or $(2t - 1) = 0$; $t = \frac{3}{5}$ or $t = \frac{1}{2}$. $\{\frac{3}{5}, \frac{1}{2}\}$

25. $3x^2 + 6x - 144 = 0$; $x^2 + 2x - 48 = 0$; $(x + 8)(x - 6) = 0$; $(x + 8) = 0$ or $(x - 6) = 0$; $x = -8$ or $x = 6$. $\{-8, 6\}$

26. $4x^2 + 8x - 140 = 0$; $x^2 + 2x - 35 = 0$; $(x + 7)(x - 5) = 0$; $(x + 7) = 0$ or $(x - 5) = 0$; $x = -7$ or $x = 5$. $\{-7, 5\}$

27. $7t^2 - 35t - 168 = 0$; $t^2 - 5t - 24 = 0$; $(t - 8)(t + 3) = 0$; $(t - 8) = 0$ or $(t + 3) = 0$; $t = 8$ or $t = -3$. $\{8, -3\}$

28. $8y^2 - 48y - 216 = 0$; $y^2 - 6y - 27 = 0$; $(y - 9)(y + 3) = 0$; $(y - 9) = 0$ or $(y + 3) = 0$; $y = 9$ or $y = -3$. $\{9, -3\}$

29. $x^2 - 6x + 9 = 0$; $(x - 3)(x - 3) = 0$; $(x - 3) = 0$ or $(x - 3) = 0$; $x = 3$ or $x = 3$. $\{3\}$

30. $(4 + m)(4 + m) = 0$; $(4 + m) = 0$ or $(4 + m) = 0$; $m = -4$ or $m = -4$. $\{-4\}$

31. $0 = m^2 - 4m$; $0 = m(m - 4)$; $m = 0$ or $(m - 4) = 0$; $m = 0$ or $m = 4$. $\{0, 4\}$

32. $s(6s + 1) = 0$; $s = 0$ or $(6s + 1) = 0$; $s = 0$ or $s = -\frac{1}{6}$. $\{0, -\frac{1}{6}\}$

B

33. $n(n^2 - 6n - 40) = 0$; $n(n - 10)(n + 4) = 0$; $n = 0$ or $(n - 10) = 0$ or $(n + 4) = 0$; $n = 0$ or $n = 10$ or $n = -4$. $\{0, 10, -4\}$

34. $t(t^2 + 8t - 84) = 0$; $t(t + 14)(t - 6) = 0$; $t = 0$ or $(t + 14) = 0$ or $(t - 6) = 0$; $t = 0$ or $t = -14$ or $t = 6$. $\{0, -14, 6\}$

35. $r(4r^2 + 4r + 1) = 0$; $r(2r + 1)(2r + 1) = 0$; $r = 0$ or $(2r + 1) = 0$ or $(2r + 1) = 0$; $r = 0$ or $r = -\frac{1}{2}$ or $r = -\frac{1}{2}$. $\{0, -\frac{1}{2}\}$

36. $s(9s^2 - 12s + 4) = 0$; $s(3s - 2)(3s - 2) = 0$; $s = 0$ or $(3s - 2) = 0$ or $(3s - 2) = 0$; $s = 0$ or $s = \frac{2}{3}$ or $s = \frac{2}{3}$. $\{0, \frac{2}{3}\}$

37. $(x^2 - 16)(x^2 - 1) = 0$; $(x + 4)(x - 4)(x + 1)(x - 1) = 0$; $(x + 4) = 0$ or $(x - 4) = 0$ or $(x + 1) = 0$ or $(x - 1) = 0$; $x = -4$ or $x = 4$ or $x = -1$ or $x = 1$. $\{-4, 4, -1, 1\}$

38. $(y^2 - 9)(y^2 - 1) = 0$; $(y + 3)(y - 3)(y + 1)(y - 1) = 0$; $(y + 3) = 0$ or $(y - 3) = 0$ or $(y + 1) = 0$ or $(y - 1) = 0$; $y = -3$ or $y = 3$ or $y = -1$ or $y = 1$. $\{-3, 3, -1, 1\}$

C

39. $4x^2 - 4x + 1 + 3x^2 - 9x = 3(x^2 - 3x + 2) - 5$; $4x^2 - 4x + 1 + 3x^2 - 9x = 3x^2 - 9x + 6 - 5$; $7x^2 - 13x + 1 = 3x^2 - 9x + 1$; $4x^2 - 4x = 0$; $x^2 - x = 0$; $x(x - 1) = 0$; $x = 0$ or $(x - 1) = 0$; $x = 0$ or $x = 1$. $\{0, 1\}$

40. $9y^4 - 12y^2 + 4 - 2 - 5y^2 = 5y^4 - 9y^2 - 2$; $9y^4 - 17y^2 + 2 = 5y^4 - 9y^2 - 2$; $4y^4 - 8y^2 + 4 = 0$; $y^4 - 2y^2 + 1 = 0$; $(y^2 - 1)(y^2 - 1) = 0$; $(y + 1)(y - 1)(y + 1)(y - 1) = 0$; $(y + 1) = 0$ or $(y - 1) = 0$ or $(y + 1) = 0$ or $(y - 1) = 0$; $y = -1$ or $y = 1$ or $y = -1$ or $y = 1$. $\{-1, 1\}$

41. $x(x^4 - 5x^2 + 4) = 0$; $x(x^2 - 1)(x^2 - 4) = 0$; $x(x + 1)(x - 1)(x + 2)(x - 2) = 0$; $x = 0$ or $(x + 1) = 0$ or $(x - 1) = 0$ or $(x + 2) = 0$ or $(x - 2) = 0$; $x = 0$ or $x = -1$ or $x = 1$ or $x = -2$ or $x = 2$. $\{0, -1, 1, -2, 2\}$

42. $y(y^4 - 8y^2 + 16) = 0$; $y(y^2 - 4)(y^2 - 4) = 0$; $y(y + 2)(y - 2)(y + 2)(y - 2) = 0$; $y = 0$ or $(y + 2) = 0$ or $(y - 2) = 0$ or $(y + 2) = 0$ or $(y - 2) = 0$; $y = 0$ or $y = -2$ or $y = 2$ or $y = -2$ or $y = 2$. $\{0, -2, 2\}$

43. Since $y = 1$ or $y = 3$, then $y - 1 = 0$ or $y - 3 = 0$; $(y - 1)(y - 3) = 0$; $y^2 - 4y + 3 = 0$

44. Since $z = -1$ or $z = -2$, then $z + 1 = 0$ or $z + 2 = 0$; $(z + 1)(z + 2) = 0$; $z^2 + 3z + 2 = 0$

45. Since $w = 0$, $w = 1$, or $w = -2$, then $w = 0$, $w - 1 = 0$, or $w + 2 = 0$; $w(w - 1)(w + 2) = 0$; $w(w^2 + w - 2) = 0$; $w^3 + w^2 - 2w = 0$

46. Since $r = 0$, $r = 2$, or $r = 4$, then $r = 0$, $r - 2 = 0$, or $r - 4 = 0$; $r(r - 2)(r - 4) = 0$; $r(r^2 - 6r + 8) = 0$; $r^3 - 6r^2 + 8r = 0$

47. Since $p = -1$, $p = 1$, or $p = 3$, then $p + 1 = 0$, $p - 1 = 0$, or $p - 3 = 0$; $(p + 1)(p - 1)(p - 3) = 0$; $(p^2 - 1)(p - 3) = 0$; $p^3 - 3p^2 - p + 3 = 0$

48. Since $y = -3$, $y = -2$, $y = 2$, or $y = 3$, then $y + 3 = 0$, $y + 2 = 0$, $y - 2 = 0$, or $y - 3 = 0$; $(y + 3)(y + 2)(y - 2)(y - 3) = 0$; $(y^2 - 9)(y^2 - 4) = 0$; $y^4 - 13y^2 + 36 = 0$

Pages 269–270 · **PROBLEMS**

A

1. Let w = width of rectangle; $w + 5$ = length of rectangle; $w(w + 5) = 66$; $w^2 + 5w - 66 = 0$; $(w + 11)(w - 6) = 0$; $(w + 11) = 0$ or $(w - 6) = 0$; $w = -11$ (reject) or $w = 6$ ft, $w + 5 = 11$ ft, *Ans.*

2. Let w = width of rectangle; $w + 2$ = length of rectangle; $w(w + 2) = 99$; $w^2 + 2w - 99 = 0$; $(w + 11)(w - 9) = 0$; $(w + 11) = 0$ or $(w - 9) = 0$; $w = -11$ (reject) or $w = 9$ yd, $w + 2 = 11$ yd, *Ans.*

3. Let t = time it takes to hit ground; $7680 = 64t + 16t^2$; $480 = 4t + t^2$; $t^2 + 4t - 480 = 0$; $(t + 24)(t - 20) = 0$; $(t + 24) = 0$ or $(t - 20) = 0$; $t = -24$ (reject) or $t = 20$ sec, *Ans.*

4. Let t = time it takes to reach ground; $11{,}200 = 48t + 16t^2$; $700 = 3t + t^2$; $t^2 + 3t - 700 = 0$; $(t + 28)(t - 25) = 0$; $(t + 28) = 0$ or $(t - 25) = 0$; $t = -28$ (reject) or $t = 25$ sec, *Ans.*

5. $325 = \frac{1}{2}n(n - 1)$; $650 = n^2 - n$; $n^2 - n - 650 = 0$; $(n + 25)(n - 26) = 0$; $(n + 25) = 0$ or $(n - 26) = 0$; $n = -25$ (reject) or $n = 26$, *Ans.*

6. $15 = \dfrac{p(p - 1)}{2}$; $30 = p^2 - p$; $p^2 - p - 30 = 0$; $(p + 5)(p - 6) = 0$; $(p + 5) = 0$ or $(p - 6) = 0$; $p = -5$ (reject) or $p = 6$, *Ans.*

7. Let x = first integer; $x + 2$ = second integer; $x^2 + (x + 2)^2 = 202$; $x^2 + x^2 + 4x + 4 = 202$; $2x^2 + 4x - 198 = 0$; $x^2 + 2x - 99 = 0$; $(x + 11)(x - 9) = 0$; $(x + 11) = 0$ or $(x - 9) = 0$; $x = -11$, $x + 2 = -9$ or $x = 9$, $x + 2 = 11$, *Ans.*

8. Let x = first integer; $x + 2$ = second integer; $x^2 + (x + 2)^2 = 100$; $x^2 + x^2 + 4x + 4 = 100$; $2x^2 + 4x - 96 = 0$; $x^2 + 2x - 48 = 0$; $(x + 8)(x - 6) = 0$; $(x + 8) = 0$ or $(x - 6) = 0$; $x = -8$, $x + 2 = -6$ or $x = 6$, $x + 2 = 8$, *Ans.*

9. Let n = height of triangle; $n + 1$ = length of base of triangle; $\frac{1}{2}n(n + 1) = 36$; $n^2 + n = 72$; $n^2 + n - 72 = 0$; $(n + 9)(n - 8) = 0$; $(n + 9) = 0$ or $(n - 8) = 0$; $n = -9$ (reject) or $n = 8$m, $n + 1 = 9$m. Base, 9m; height, 8m, *Ans.*

10. Let h = height of triangle; $19 - h$ = length of base of triangle; $42 = \frac{1}{2}(19 - h)h$; $84 = 19h - h^2$; $h^2 - 19h + 84 = 0$; $(h - 12)(h - 7) = 0$; $(h - 12) = 0$ or $(h - 7) = 0$; $h = 12$ cm, $19 - h = 7$ cm or $h = 7$ cm, $19 - h = 12$ cm, *Ans.*

11. Let t = number of seconds; $64 = 64t - 16t^2$; $4 = 4t - t^2$; $t^2 - 4t + 4 = 0$; $(t - 2)(t - 2) = 0$; $(t - 2) = 0$ or $(t - 2) = 0$; $t = 2$ sec, *Ans.*

12. Let t = number of seconds; $144 = 96t - 16t^2$; $9 = 6t - t^2$; $t^2 - 6t + 9 = 0$; $(t - 3)(t - 3) = 0$; $(t - 3) = 0$ or $(t - 3) = 0$; $t = 3$ sec, *Ans.*

13. Let t = number of seconds; $4656 = 1600t - 16t^2$; $291 = 100t - t^2$; $t^2 - 100t + 291 = 0$; $(t - 97)(t - 3) = 0$; $(t - 97) = 0$ or $(t - 3) = 0$; $t = 97$ (reject) or $t = 3$ sec, *Ans.*

14. Let t = number of seconds; $2080 = 2096t - 16t^2$; $130 = 131t - t^2$; $t^2 - 131t + 130 = 0$; $(t - 130)(t - 1) = 0$; $(t - 130) = 0$ or $(t - 1) = 0$; $t = 130$ (reject) or $t = 1$ sec, *Ans.*

B

15. Let x = width of strip; $2x$ = total no. of feet added to each dimension; $(80 + 2x)(120 + 2x)$ = new plowed area; $(80 + 2x)(120 + 2x) = 2(80)(120)$; $9600 + 400x + 4x^2 = 19200$; $4x^2 + 400x - 9600 = 0$; $x^2 + 100x - 2400 = 0$; $(x + 120)(x - 20) = 0$; $(x + 120) = 0$ or $(x - 20) = 0$; $x = -120$ (reject) or $x = 20$ ft; $2x = 40$ ft, *Ans.*

16. Let x = total no. of feet to reduce each dimension; $(30 - x)(40 - x)$ = new area; $(30 - x)(40 - x) = \frac{1}{2}(30)(40)$; $1200 - 70x + x^2 = 600$; $x^2 - 70x + 600 = 0$; $(x - 60)(x - 10) = 0$; $(x - 60) = 0$ or $(x-10) = 0$; $x = 60$ (reject) or $x = 10$ ft, *Ans.*

17. a. Let t = time at which it is 32 ft above ground; $32 = 48t - 16t^2$; $16t^2 - 48t + 32 = 0$; $t^2 - 3t + 2 = 0$; $(t - 2)(t - 1) = 0$; $(t - 2) = 0$ or $(t - 1) = 0$; $t = 2$ sec or $t = 1$ sec; there are two roots because the ball is at given height on the way up and on the way down.

b. Ball returns when $h = 0$; let t = time at which it returns; $0 = 48t - 16t^2$; $16t^2 - 48t = 0$; $t^2 - 3t = 0$; $t(t - 3) = 0$; $t = 0$ or $(t - 3) = 0$; $t = 0$ (reject) or $t = 3$ sec, *Ans.*

18. a. Let t = time when bullet is 3200 ft above ground; $3200 = 3216t - 16t^2$; $16t^2 - 3216t + 3200 = 0$; $t^2 - 201t + 200 = 0$; $(t - 200)(t - 1) = 0$; $(t - 200) = 0$ or $(t - 1) = 0$; $t = 200$ sec or $t = 1$ sec; explanation: see #17

b. Let t = time at which it returns to ground; $0 = 3216t - 16t^2$; $16t^2 - 3216t = 0$; $t^2 - 201t = 0$; $t(t - 201) = 0$; $t = 0$ or $(t - 201) = 0$; $t = 0$ (reject) or $t = 201$ sec $= 3\frac{7}{20}$ min, *Ans.*

19. If the product of two numbers equals a nonzero number, it does not follow that each factor equals a factor of the number. It is only true for 0 that if $a \cdot b = 0$, then $a = 0$ or $b = 0$, for one of the factors *must* be 0 if the product is zero. Further, $\{3, 5\}$ does not satisfy the original equation.

20. $n^2 + 8 = 2(n + 8)$; $n^2 + 8 = 2n + 16$; $n^2 - 2n - 8 = 0$; $(n + 2)(n - 4) = 0$; $(n + 2) = 0$ or $(n - 4) = 0$; $n = -2$ (reject) or $n = 4$, $n^2 = 16$ yr, *Ans.*

21. Let x = length of one leg of triangle; $x + 2$ = length of hypotenuse of triangle; $60 - (x + x + 2) = 58 - 2x$ = length of third leg of triangle; $\frac{1}{2}(x)(58 - 2x) = 120$; $\frac{1}{2}(58x - 2x^2) = 120$; $29x - x^2 = 120$; $x^2 - 29x + 120 = 0$; $(x - 24)(x - 5) = 0$; $(x - 24) = 0$ or $(x - 5) = 0$; $x = 5$ (reject, since hypotenuse must be longest side) or $x = 24$, $x + 2 = 26$ ft, *Ans.*

22. Let n = any whole number; $n + 1$ = next whole number; $n^2 + (n + 1)^2 = n^2 + n^2 + 2n + 1 = 2n^2 + 2n + 1 = 2(n^2 + n) + 1$. Since $2(n^2 + n)$ is even, then $2(n^2 + n) + 1$ is odd.

Page 273 · CHAPTER TEST

1. $792 = 2^3 \cdot 3^2 \cdot 11$; $2520 = 2^3 \cdot 3^2 \cdot 5 \cdot 7$. Greatest common factor of 792 and 2520: $2^3 \cdot 3^2 = 72$, *Ans.*

2. $7yz^2$ **3.** $16a(5a - b)$ **4.** $(x - 2)(x + 1)$

5. $y^3 - 15 - 5y^2 + 3y = (y^3 - 5y^2) + (3y - 15) = y^2(y - 5) + 3(y - 5) = (y^2 + 3)(y - 5)$
or $y^3 - 15 - 5y^2 + 3y = (y^3 + 3y) - (5y^2 + 15) = y(y^2 + 3) - 5(y^2 + 3) = (y - 5)(y^2 + 3)$

6. $81x^6$ **7.** $(rs)^2 - (t^2)^2 = r^2s^2 - t^4$ **8.** $2(4m^2 - 25) = 2(2m + 5)(2m - 5)$

9. $x^2 - 144 = (x + 12)(x - 12)$ **10.** $9a^2 + 6ab + b^2$

11. $25k^2 - 20km + 4m^2$ **12.** $(5x + 9y)^2$

13. $12x^2 - 17x - 5 = 12x^2 - 20x + 7$; $3x = 12$; $x = 4$. $\{4\}$
Check: $(3 \cdot 4 - 5)(4 \cdot 4 + 1) \stackrel{?}{=} (6 \cdot 4 - 7)(2 \cdot 4 - 1)$; $7 \cdot 17 \stackrel{?}{=} 17 \cdot 7$; $119 = 119$

14. $(n + 14)(n + 3)$ **15.** $(r - 18s)(r - 5s)$ **16.** $(m + 9)(m - 4)$ **17.** $(k - 9)(k + 2)$

18. $(9n + 8)(4n + 7)$ **19.** $(3a + 4b)(a - 9b)$ **20.** $10(3t^2 + t - 10) = 10(3t - 5)(t + 2)$

21. $x - 2 = 0$ or $x + 5 = 0$; $x = 2$ or $x = -5$. $\{2, -5\}$
Check: $x = 2$, $(2 - 2)(2 + 5) \stackrel{?}{=} 0$, $0 \cdot 7 = 0$; $x = -5$, $(-5 - 2)(-5 + 5) \stackrel{?}{=} 0$, $(-7) \cdot 0 = 0$

22. $y(2y - 1) = 0$; $y = 0$ or $2y - 1 = 0$; $y = 0$ or $y = \frac{1}{2}$. $\{0, \frac{1}{2}\}$
Check: $y = 0$, $0(2 \cdot 0 - 1) \stackrel{?}{=} 0$, $0 \cdot (-1) = 0$; $y = \frac{1}{2}$, $\frac{1}{2}(2 \cdot \frac{1}{2} - 1) \stackrel{?}{=} 0$, $\frac{1}{2}(1 - 1) \stackrel{?}{=} 0$, $\frac{1}{2} \cdot 0 = 0$

23. $z^2 - z - 90 = 0$; $(z - 10)(z + 9) = 0$; $(z - 10) = 0$ or $(z + 9) = 0$; $z = 10$ or $z = -9$. $\{10, -9\}$
Check: $z = 10$, $10^2 - 10 \stackrel{?}{=} 90$, $100 - 10 = 90$; $z = -9$, $(-9)^2 - (-9) \stackrel{?}{=} 90$, $81 + 9 = 90$

24. $(7n + 13)(7n - 13) = 0$; $7n + 13 = 0$ or $(7n - 13) = 0$; $n = -\frac{13}{7}$ or $n = \frac{13}{7}$. $\{-\frac{13}{7}, \frac{13}{7}\}$
Check: $n = -\frac{13}{7}$ $49(-\frac{13}{7})^2 - 169 \stackrel{?}{=} 0$; $49 \cdot \frac{169}{49} - 169 \stackrel{?}{=} 0$; $169 - 169 = 0$;
$n = \frac{13}{7}$ $49(\frac{13}{7})^2 - 169 \stackrel{?}{=} 0$; $49 \cdot \frac{169}{49} - 169 \stackrel{?}{=} 0$; $169 - 169 = 0$

25. Let x = width in feet; $x + 8$ = length in feet; $x(x + 8) = 105$; $x^2 + 8x - 105 = 0$; $x^2 + 8x - 105 = 0$; $(x + 15)(x - 7) = 0$; $(x + 15) = 0$ or $(x - 7) = 0$; $x = -15$ (rejected); $x = 7$ ft, $x + 8 = 15$ ft, *Ans.*

Pages 274–276 · CHAPTER REVIEW

1. $2^3 \cdot 3^2 \cdot 5^1$ **2.** $(-5bc^2)$ **3.** $360 = 9 \cdot 40$; $400 = 10 \cdot 40$; 40, *Ans.*

4. $(2x^3y^3)^1 = 2x^3y^3$; $(2x^3y^3)^2 = 4x^6y^6$, not a factor of $6x^9y^9$; $\therefore$ $(2x^3y^3)$, *Ans.*

5. $5x^2(x^2 - 2x + 3)$ **6.** $x(x - 2x^2 + 1) = -x(2x^2 - x - 1) = -x(2x + 1)(x - 1)$

7. n^2, 2 **8.** $(y + 5)(y - 3)$ **9.** $r(r + 3) + s(r + 3) = (r + s)(r + 3)$

10. $(2x - 1)(3x + 1 - 1) = (2x - 1)(3x) = 3x(2x - 1)$

11. $t(2r + 5s) - (3u)(2r + 5s) = (t - 3u)(2r + 5s)$ **12.** $(-16)^2 = 256$ **13.** $(8x)^2 = 64x^2$

14. $(-7a^2b^3)^2 = 49a^4b^6$ **15.** $25n^2 - x^2$ **16.** $9y^2 - \frac{1}{9}$

17. $(2x + 5)(2x - 5)$ **18.** $8(x^2 - 4y^2) = 8(x + 2y)(x - 2y)$ **19.** $d(2d^2 - 1)$

20. $(x^2 + 9)(x^2 - 9) = (x^2 + 9)(x + 3)(x - 3)$ **21.** $a^2 + 2ab + b^2$

22. $a^2 - 2ab + b^2$ **23.** $4x^2 - 4x + 1$ **24.** $25h^2 + 30hk + 9k^2$

25. (a) $x^2 - 20x - 100$ is not a trinomial square because the third term is not a square.

(b) $4m^2 - 24m + 9$ is not a trinomial square because the middle term is not double the product of $2m$ and 3.

(c) $y^2 + 2y + 1 = (y + 1)^2$

26. $(4x - 1)(4x - 1) = (4x - 1)^2$ **27.** $(3z + 5a)(3z + 5a) = (3z + 5a)^2$

28. quadratic **29.** linear **30.** constant **31.** $8x^2 + 59x + 21$ **32.** $5n^2 - 64n + 48$

33. $(5x - 8)(2x - 3) = (x + 2)(10x - 9)$; $10x^2 - 31x + 24 = 10x^2 + 11x - 18$; $42 = 42x$; $x = 1$. $\{1\}$

34. 12, 8 **35.** positive, negative **36.** $r = 6$, $s = 2$ **37.** $(n + 11)(n + 4)$

38. $(x - 21)(x - 2)$ **39.** -42, 1 **40.** opposite **41.** $r = 7$, $s = -6$

42. $(x + 6)(x - 7)$ **43.** $(x + 5)(x - 7)$ **44.** $(x + 7)(x - 5)$

45. $(3x + 1)(x + 2)$ if $b = 7$; $(3x - 1)(x - 2)$ if $b = -7$; $(3x + 2)(x + 1)$ if $b = 5$; $(3x - 2)(x - 1)$ if $b = -5$. $b \in \{7, -7, 5, -5\}$

46. $(14y - 1)(y - 3)$ if $b = 43$; $(14y + 1)(y + 3)$ if $b = -43$; $(14y - 3)(y - 1)$ if $b = 17$; $(14y + 3)(y + 1)$ if $b = -17$; $(7y - 1)(2y - 3)$ if $b = 23$; $(7y + 1)(2y + 3)$ if $b = -23$; $(7y - 3)(2y - 1)$ if $b = 13$; $(7y + 3)(2y + 1)$ if $b = -13$.
$b \in \{43, -43, 17, -17, 23, -23, 13, -13\}$

47. $(10n + 6)(n - 1) = 2(5n + 3)(n - 1)$ if $b = 4$; $(10n - 6)(n + 1) = 2(5n - 3)(n + 1)$ if $b = -4$; $(10n + 1)(n - 6)$ if $b = 59$; $(10n - 1)(n + 6)$ if $b = -59$; $(10n + 3)(n - 2)$ if $b = 17$; $(10n - 3)(n + 2)$ if $b = -17$; $(10n + 2)(n - 3) = 2(5n + 1)(n - 3)$ if $b = 28$; $(10n - 2)(n + 3) = 2(5n - 1)(n + 3)$ if $b = -28$; $(5n - 6)(2n + 1)$ if $b = 7$; $(5n + 6)(2n - 1)$ if $b = -7$; $(5n + 2)(2n - 3)$ if $b = 11$; $(5n - 2)(2n + 3)$ if $b = -11$.
$b \in \{4, -4, 59, -59, 17, -17, 28, -28, 7, -7, 11, -11\}$

48. $(4y - 3)(5y - 1)$ **49.** $(2z + 1)(13z - 6)$ **50.** difference, squares

51. perfect square, not a perfect square **52.** $x + 2 = 0$; $x = -2$. $\{-2\}$

53. $y + 2 = 0$ or $2y - 5 = 0$; $y = -2$ or $y = \frac{5}{2}$. $\{-2, \frac{5}{2}\}$ **54.** $x^2 - 5x - 50 = 0$

55. $x^2 + 9x - 70 = 0$; $(x + 14)(x - 5) = 0$; $x + 14 = 0$ or $x - 5 = 0$; $x = -14$ or $x = 5$. $\{-14, 5\}$

56. $x^2 - 19x + 84 = 0$; $(x - 12)(x - 7) = 0$; $x - 12 = 0$ or $x - 7 = 0$; $x = 12$ or $x = 7$. $\{12, 7\}$

57. $x(x^2 - 7x - 30) = 0$; $x(x - 10)(x + 3) = 0$; $x = 0$, or $x - 10 = 0$, or $x + 3 = 0$; $x = 0$ or $x = 10$ or $x = -3$. $\{0, 10, -3\}$

58. Let x = the integer; $2x^2 = 7x - 3$; $2x^2 - 7x + 3 = 0$; $(2x - 1)(x - 3) = 0$; $2x - 1 = 0$ or $x - 3 = 0$; $x = \frac{1}{2}$ (rejected); $x = 3$, *Ans.*

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1. 1.86×10^8 **2.** 1.7×10^{-7} **3.** 2.5×10^{-5} **4.** 6.02×10^{23}

5. 1.3×10^{11} **6.** 1.6×10^{-19} **7.** 93,000,000 **8.** 186,000

9. 96,000,000,000,000,000,000,000 **10.** .0000095

11. .0000000667 **12.** .00000000000000000016

13. $\frac{6.9}{2.3} \times \frac{10^{-6}}{10^{-2}} = 3 \times 10^{-6-(-2)} = 3 \times 10^{-4}$ **14.** $(5.83 \times 1.39) \times (10^{-9} \times 10^{-8}) = 8.1037 \times 10^{-17}$

15. $\frac{1.82}{9.1} \times \frac{10^5}{10^{17}} = .2 \times 10^{5-17} = .2 \times 10^{-12}$

16. $(9.72 \times 4.8) \times (10^8 \times 10^{-20}) = 46.656 \times 10^{-12} = 4.6656 \times 10^{-11}$

17. $(8.3 - 6.7)10^3 + (2.7 + 3.1 - 8.1)10^4 = (1.6)10^3 + (-2.3)10^4 = (.16 - 2.3)10^4 = -2.14 \times 10^4$

CHAPTER 8. Working with Fractions

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A **1.** $\frac{5}{y}$; $y \neq 0$ **2.** $\frac{-7}{z}$; $z \neq 0$ **3.** $\frac{17}{100}$; no exclusions

4. $\frac{9}{10}$; no exclusions **5.** $\frac{2x}{10} = \frac{x}{5}$; no exclusions **6.** $\frac{3y}{1}$; no exclusions

7. $\frac{b-1}{b}$; $b \neq 0$ **8.** $\frac{a}{a-1}$; $a - 1 \neq 0$; $a \neq 1$ **9.** $\frac{k}{6k-12}$; $6k - 12 \neq 0$; $k \neq 2$

10. $\frac{f}{3f+21}$; $3f + 21 \neq 0$; $f \neq -7$ **11.** $\frac{12x^2+4}{3}$; no exclusions

12. $\frac{15y^2-10}{5} = \frac{3y^2-2}{1}$; no exclusions **13.** $\frac{g-2}{14g+7}$; $14g + 7 \neq 0$; $g \neq -\frac{1}{2}$

14. $\frac{h-5}{33h-11}$; $33h - 11 \neq 0$; $h \neq \frac{1}{3}$

15. $\frac{1}{x(x-1)}$; $x(x - 1) \neq 0$; $x \neq 0$ and $x - 1 \neq 0$; $x \neq 0$ and $x \neq 1$

16. $\frac{1}{y(y+4)}$; $y(y + 4) \neq 0$; $y \neq 0$ and $y + 4 \neq 0$; $y \neq 0$ and $y \neq -4$

B **17.** $c^2 - 8c + 12 = 0$; $(c - 6)(c - 2) = 0$; $c - 6 = 0$ or $c - 2 = 0$; $c = 6$ or $c = 2$. $\{6, 2\}$

18. $d^2 + 9d + 14 = 0$; $(d + 7)(d + 2) = 0$; $d + 7 = 0$ or $d + 2 = 0$; $d = -7$ or $d = -2$. $\{-7, -2\}$

19. $5p^2 + 14p - 3 = 0$; $(5p - 1)(p + 3) = 0$; $5p - 1 = 0$ or $p + 3 = 0$; $5p = 1$ or $p = -3$; $p = \frac{1}{5}$ or $p = -3$. $\{\frac{1}{5}, -3\}$

20. $7g^2 - 5g - 2 = 0$; $(7g + 2)(g - 1) = 0$; $7g + 2 = 0$ or $g - 1 = 0$; $7g = -2$ or $g = 1$; $g = -\frac{2}{7}$ or $g = 1$. $\{-\frac{2}{7}, 1\}$

21. $a^2 - 4 = 0$; $(a + 2)(a - 2) = 0$; $a + 2 = 0$ or $a - 2 = 0$; $a = -2$ or $a = 2$. $\{-2, 2\}$

22. $b^2 - 9 = 0$; $(b + 3)(b - 3) = 0$; $b + 3 = 0$ or $b - 3 = 0$; $b = -3$ or $b = 3$. $\{-3, 3\}$

23. $y^2 + 9 = 0$; $\emptyset$ **24.** $x^2 + 4 = 0$; $\emptyset$

25. $x^2 - 16 = 0$; $(x + 4)(x - 4) = 0$; $x + 4 = 0$ or $x - 4 = 0$; $x = -4$ or $x = 4$. $\{-4, 4\}$

C **26.** $a(a - b) \neq 0$; $a \neq 0$ and $a - b \neq 0$; $a \neq 0$ and $a \neq b$

27. $c^2 + 2cd + d^2 \neq 0$; $(c + d)(c + d) \neq 0$; $c + d \neq 0$ and $c + d \neq 0$; $c \neq -d$

28. $r^2 - 2rs + s^2 \neq 0$; $(r - s)(r - s) \neq 0$; $r - s \neq 0$ and $r - s \neq 0$; $r \neq s$

Pages 284–285 · WRITTEN EXERCISES

A **1.** $\frac{30hk \cdot h}{30hk} = h$; $h \neq 0$, $k \neq 0$ **2.** $\frac{5st \cdot t}{5st \cdot 6} = \frac{t}{6}$; $s \neq 0$, $t \neq 0$

3. $\frac{-3 \cdot 7m^2n^2}{4mn \cdot 7m^2n^2} = -\frac{3}{4mn}$; $m \neq 0$, $n \neq 0$ **4.** $\frac{-5z^2 \cdot 7x^2}{9xy^3 \cdot 7x^2} = -\frac{5z^2}{9xy^3}$; $x \neq 0$, $y \neq 0$

5. $\frac{3(a+b)}{4(a+b)} = \frac{3}{4}$; $4(a + b) \neq 0$; $a + b \neq 0$; $a \neq -b$

6. $\frac{5(c-d)}{5(c+d)} = \frac{c-d}{c+d}$; $5(c + d) \neq 0$; $c + d \neq 0$; $c \neq -d$

7. $\frac{4(x+y)}{(x-y)(x+y)} = \frac{4}{x-y}$; $(x - y)(x + y) \neq 0$; $x - y \neq 0$ and $x + y \neq 0$; $x \neq y$ and $x \neq -y$ or $|x| \neq |y|$

8. $\frac{(r-5)(r+5)}{3(r+5)} = \frac{r-5}{3}$; $3(r + 5) \neq 0$; $r + 5 \neq 0$; $r \neq -5$

9. $\frac{2a+3b}{2ab}$; $a \neq 0$, $b \neq 0$ **10.** $\frac{5r+3b}{5rs}$; $r \neq 0$, $s \neq 0$

When necessary to use -1 as a factor, either the numerator or the denominator may be expressed as a product of -1. Only one of these alternatives is used in any one of the solutions given here.

11. $\frac{(x+4)(x-4)}{(-1)(x-4)} = \frac{x+4}{-1} = -x - 4$; $4 - x \neq 0$; $x \neq 4$

12. $\frac{(1+a)(1-a)}{(-1)(1-a)} = \frac{1+a}{-1} = -1 - a$; $a - 1 \neq 0$; $a \neq 1$

13. $\dfrac{(m+2)(m-2)}{(m-2)(m-2)} = \dfrac{m+2}{m-2}$; $(m-2)(m-2) \neq 0$; $m-2 \neq 0$; $m \neq 2$

14. $\dfrac{(x+4)(x-4)}{(x-4)(x-4)} = \dfrac{x+4}{x-4}$; $(x-4)(x-4) \neq 0$; $x-4 \neq 0$; $x \neq 4$

15. $\dfrac{3(a+1)}{(a+1)(a+1)} = \dfrac{3}{a+1}$; $(a+1)(a+1) \neq 0$; $a+1 \neq 0$; $a \neq -1$

16. $\dfrac{5(x+2)}{(x+2)(x+2)} = \dfrac{5}{x+2}$; $(x+2)(x+2) \neq 0$; $x+2 \neq 0$; $x \neq -2$

17. $\dfrac{s(r^2-1)}{r-1} = \dfrac{s(r-1)(r+1)}{(r-1)} = s(r+1)$; $r-1 \neq 0$; $r \neq 1$

18. $\dfrac{2(d^2-1)}{d+1} = \dfrac{2(d+1)(d-1)}{(d+1)} = 2(d-1)$; $d+1 \neq 0$; $d \neq -1$

19. $\dfrac{x^2+1}{x+1}$; $x+1 \neq 0$; $x \neq -1$

20. $\dfrac{x-4}{x^2-4}$; $(x+2)(x-2) \neq 0$; $x+2 \neq 0$ and $x-2 \neq 0$; $x \neq -2$ and $x \neq 2$ *or* $|x| \neq 2$

21. $\dfrac{6(a^2-9)}{2(a^2+4a+3)} = \dfrac{6(a+3)(a-3)}{2(a+3)(a+1)} = \dfrac{3(a-3)}{a+1}$; $2(a+3)(a+1) \neq 0$; $a+3 \neq 0$ and $a+1 \neq 0$; $a \neq -3$ and $a \neq -1$

22. $\dfrac{8(a^2+5a+4)}{2(16-a^2)} = \dfrac{8(a+1)(a+4)}{2(4-a)(4+a)} = \dfrac{4(a+1)}{4-a}$; $2(4-a)(4+a) \neq 0$; $4-a \neq 0$ and $4+a \neq 0$; $a \neq 4$ and $a \neq -4$ *or* $|a| \neq 4$

23. $\dfrac{5(c^2-9c+18)}{5(36-c^2)} = \dfrac{5(c-3)(c-6)}{5(-1)(c^2-36)} = \dfrac{5(c-3)(c-6)}{5(-1)(c+6)(c-6)} = \dfrac{c-3}{-1(c+6)} = -\dfrac{c-3}{c+6}$ *or* $\dfrac{c-3}{-c-6}$; $5(-1)(c+6)(c-6) \neq 0$; $c+6 \neq 0$ and $c-6 \neq 0$; $c \neq -6$ and $c \neq 6$ *or* $|c| \neq 6$

24. $\dfrac{3(d^2-9)}{d^2-11d+24} = \dfrac{3(d+3)(d-3)}{(d-8)(d-3)} = \dfrac{3(d+3)}{d-8}$; $(d-8)(d-3) \neq 0$; $d-8 \neq 0$ and $d-3 \neq 0$; $d \neq 8$ and $d \neq 3$

B **25.** $\dfrac{(c+2d)(c+2d)}{(c+2d)(c-2d)} = \dfrac{c+2d}{c-2d}$; $(c+2d)(c-2d) \neq 0$; $c+2d \neq 0$ and $c-2d \neq 0$; $c \neq -2d$ and $c \neq 2d$ *or* $|c| \neq 2d$

26. $\dfrac{(3a+b)(3a+b)}{(3a+b)(3a-b)} = \dfrac{3a+b}{3a-b}$; $(3a+b)(3a-b) \neq 0$; $3a+b \neq 0$ and $3a-b \neq 0$; $b \neq -3a$ and $b \neq 3a$ *or* $|b| \neq 3a$

27. $\dfrac{r(r-3)}{(r-1)(r-3)} = \dfrac{r}{r-1}$; $(r-1)(r-3) \neq 0$; $r-1 \neq 0$ and $r-3 \neq 0$; $r \neq 1$ and $r \neq 3$

28. $\dfrac{t(t-7)}{(t-1)(t-7)} = \dfrac{t}{t-1}$; $(t-1)(t-7) \neq 0$; $t-1 \neq 0$ and $t-7 \neq 0$; $t \neq 1$ and $t \neq 7$

29. $\dfrac{(a-4)(a+1)}{(a+1)(a+1)} = \dfrac{a-4}{a+1}$; $(a+1)(a+1) \neq 0$; $a+1 \neq 0$; $a \neq -1$

30. $\dfrac{(b+6)(b-1)}{(b-1)(b-1)} = \dfrac{b+6}{b-1}$; $(b-1)(b-1) \neq 0$; $b-1 \neq 0$; $b \neq 1$

31. $\dfrac{(z-3)(z-5)}{(z-3)(z+4)} = \dfrac{z-5}{z+4}$; $(z-3)(z+4) \neq 0$; $z-3 \neq 0$ and $z+4 \neq 0$; $z \neq 3$ and $z \neq -4$

32. $\dfrac{(m-4)(m+2)}{(m-3)(m+2)} = \dfrac{m-4}{m-3}$; $(m-3)(m+2) \neq 0$; $m-3 \neq 0$ and $m+2 \neq 0$; $m \neq 3$ and $m \neq -2$

33. $\dfrac{(x-2)(x-2)}{-1(x^2-4)} = \dfrac{(x-2)(x-2)}{-1(x+2)(x-2)} = \dfrac{x-2}{-1(x+2)} = \dfrac{x-2}{-x-2}$ *or* $-\dfrac{x-2}{x+2}$; $-1(x+2)(x-2) \neq 0$; $x+2 \neq 0$ and $x-2 \neq 0$; $x \neq -2$ and $x \neq 2$ *or* $|x| \neq 2$

34. Irreducible; x is not a factor of the numerator

35. Irreducible; r and 5 are not factors of the numerator or the denominator

36. Omitted second term; $\dfrac{y(a+1)}{y} = \dfrac{a+1}{1} = a+1$

37. Irreducible; b is not a factor of the numerator or the denominator

38. Irreducible; 3 and t are not factors of the numerator or the denominator

39. Irreducible; x is not a factor of the numerator

C **40.** $\frac{(3n+1)(2n-5)}{(3n-1)(2n-5)} = \frac{3n+1}{3n-1}$; $(3n-1)(2n-5) \neq 0$; $3n-1 \neq 0$ and $2n-5 \neq 0$; $n \neq \frac{1}{3}$ and $n \neq \frac{5}{2}$

41. $\frac{(4n-1)(2n+3)}{(4n-1)(2n-3)} = \frac{2n+3}{2n-3}$; $(4n-1)(2n-3) \neq 0$; $4n-1 \neq 0$ and $2n-3 \neq 0$; $n \neq \frac{1}{4}$ and $n \neq \frac{3}{2}$

42. $\frac{3m(2+m-m^2)}{6m(m^2+3m+2)} = \frac{3m(-1)(m^2-m-2)}{6m(m+2)(m+1)} = \frac{3m(-1)(m-2)(m+1)}{6m(m+2)(m+1)} = \frac{(-1)(m-2)}{2(m+2)} = \frac{-m+2}{2(m+2)}$; $6m(m+2)(m+1) \neq 0$; $m \neq 0$ and $m+2 \neq 0$ and $m+1 \neq 0$; $m \neq 0$ and $m \neq -2$ and $m \neq -1$

43. $\frac{5t(2t^2-3t-2)}{4t(1-4t^2)} = \frac{5t(2t+1)(t-2)}{4t(1+2t)(1-2t)} = \frac{5(t-2)}{4(1-2t)}$; $4t(1+2t)(1-2t) \neq 0$; $t \neq 0$ and $1+2t \neq 0$ and $1-2t \neq 0$; $t \neq 0$ and $t \neq -\frac{1}{2}$ and $t \neq \frac{1}{2}$ or $t \neq 0$ and $|t| \neq \frac{1}{2}$

44. $\frac{2x(x^2+10x+25)}{4x(x^2+25)} = \frac{2x(x+5)(x+5)}{4x(x^2+25)} = \frac{(x+5)(x+5)}{2(x^2+25)}$; $4x(x^2+25) \neq 0$; $x \neq 0$ and $x^2+25 \neq 0$; $x \neq 0$

45. $\frac{5p(p^2+4p+4)}{25p(p^2+4)} = \frac{(p+2)^2}{5(p^2+4)}$; $25p(p^2+4) \neq 0$; $p \neq 0$ and $p^2+4 \neq 0$; $p \neq 0$

46. $\frac{6z^2(z^2+9xz+20x^2)}{3(z^2+xz-12x^2)} = \frac{6z^2(z+4x)(z+5x)}{3(z+4x)(z-3x)} = \frac{2z^2(z+5x)}{z-3x}$; $3(z+4x)(z-3x) \neq 0$; $z+4x \neq 0$ and $z-3x \neq 0$; $z \neq -4x$ and $z \neq 3x$

47. $\frac{4mn(m^2-2mn-8n^2)}{6m^2(m^2-mn-6n^2)} = \frac{4mn(m-4n)(m+2n)}{6m^2(m-3n)(m+2n)} = \frac{2n(m-4n)}{3m(m-3n)}$; $6m^2(m-3n)(m+2n) \neq 0$; $m^2 \neq 0$ and $m-3n \neq 0$ and $m+2n \neq 0$; $m \neq 0$ and $m \neq 3n$ and $m \neq -2n$

Pages 287–288 · **WRITTEN EXERCISES**

A **1.** $\frac{8 \cdot 12}{4 \cdot 36} = \frac{4 \cdot 2 \cdot 12}{4 \cdot 3 \cdot 12} = \frac{2}{3}$

2. $\frac{6 \cdot 9}{9 \cdot 12} = \frac{6 \cdot 9}{9 \cdot 2 \cdot 6} = \frac{1}{2}$

3. 1 ft = 12 in; 2 ft = 24 in; $\frac{12 \cdot 24}{8 \cdot 20} = \frac{4 \cdot 3 \cdot 2 \cdot 4 \cdot 3}{4 \cdot 2 \cdot 5 \cdot 4} = \frac{9}{5}$

4. 1.5 ft = 1.5(12) = 18 in; $\frac{9^2}{18^2} = \frac{9 \cdot 9}{9 \cdot 2 \cdot 9 \cdot 2} = \frac{1}{4}$

5. $\frac{25}{100} = \frac{25}{4 \cdot 25} = \frac{1}{4}$

6. $\frac{15}{25} = \frac{5 \cdot 3}{5 \cdot 5} = \frac{3}{5}$

7. $\frac{7{,}250{,}000}{2{,}500{,}000} = \frac{25 \cdot 29 \cdot 10{,}000}{25 \cdot 10 \cdot 10{,}000} = \frac{29}{10}$

8. $\frac{360}{1200} = \frac{12 \cdot 3 \cdot 10}{12 \cdot 10 \cdot 10} = \frac{3}{10}$

9. No. of men = 10,500 − 3500 = 7000; $\frac{7000}{3500} = \frac{3500 \cdot 2}{3500} = \frac{2}{1}$

10. No. of lb of chlorine = 290 − 175 = 115; $\frac{175}{115} = \frac{5 \cdot 35}{5 \cdot 23} = \frac{35}{23}$

11. $\frac{\frac{1}{2}(12 \cdot 6)}{\frac{1}{2}(18 \cdot 5)} = \frac{3 \cdot 4 \cdot 6}{3 \cdot 6 \cdot 5} = \frac{4}{5}$

12. $\frac{\frac{1}{2}(14 \cdot 20)}{\frac{1}{2}(28 \cdot 5)} = \frac{14 \cdot 2 \cdot 5 \cdot 2}{14 \cdot 2 \cdot 5} = \frac{2}{1}$

13. 1 ft = 12 in; $1\frac{1}{2}$ ft = 18 in; $\frac{\frac{1}{2}(12 \cdot 9)}{\frac{1}{2}(18 \cdot 4)} = \frac{4 \cdot 3 \cdot 9}{2 \cdot 9 \cdot 4} = \frac{3}{2}$

14. 4 yd = 4(3) = 12 ft; 3 yd = 3(3) = 9 ft; $\frac{\frac{1}{2}(12 \cdot 2)}{\frac{1}{2}(9 \cdot 4)} = \frac{3 \cdot 4 \cdot 2}{3 \cdot 3 \cdot 4} = \frac{2}{3}$

15. $x = \frac{3y}{4}$; $\frac{x}{y} = \frac{3}{4}$

16. $x = \frac{5y}{2}$; $\frac{x}{y} = \frac{5}{2}$

17. $x = 1y$; $\frac{x}{y} = \frac{1}{1}$

18. $\frac{x}{y} = \frac{7}{1}$

19. $3x = 2y$; $x = \frac{2y}{3}$; $\frac{x}{y} = \frac{2}{3}$

20. $6y = 9x$; $2y = 3x$; $x = \frac{2y}{3}$; $\frac{x}{y} = \frac{2}{3}$

B **21.** $6y\left(\frac{3x+2y}{2y}\right) = 6y\left(\frac{4}{3}\right)$; $9x + 6y = 8y$; $9x = 2y$; $x = \frac{2y}{9}$; $\frac{x}{y} = \frac{2}{9}$

22. $4x\left(\frac{2x-y}{2x}\right) = 4x\left(-\frac{3}{4}\right)$; $4x - 2y = -3x$; $7x = 2y$; $x = \frac{2y}{7}$; $\frac{x}{y} = \frac{2}{7}$

23. $6y\left(\frac{9x-4y}{6y}\right) = 6y\left(\frac{16}{3}\right)$; $9x - 4y = 32y$;

24. $3y\left(\frac{4x+6y}{3y}\right) = 3y\left(\frac{10}{3}\right)$; $4x + 6y = 10y$;

$9x = 36y$; $x = 4y$; $\frac{x}{y} = \frac{4}{1}$

25. $y^2\left(\frac{x^2 - 2y^2}{y^2}\right) = y^2\left(\frac{2x - 3y}{y}\right)$;
$x^2 - 2y^2 = 2xy - 3y^2$; $x^2 - 2xy + y^2 = 0$;
$(x - y)(x - y) = 0$; $x - y = 0$ and
$x - y = 0$; $x = y$; $\frac{x}{y} = \frac{1}{1}$

$4x = 4y$; $x = y$; $\frac{x}{y} = \frac{1}{1}$

26. $y^2\left(\frac{x^2 + 7y^2}{y^2}\right) = y^2\left(\frac{3y - 4x}{y}\right)$;
$x^2 + 7y^2 = 3y^2 - 4xy$; $x^2 + 4xy + 4y^2 = 0$;
$(x + 2y)(x + 2y) = 0$; $x + 2y = 0$ and
$x + 2y = 0$; $x = -2y$; $\frac{x}{y} = \frac{-2}{1}$

Pages 288–289 · **PROBLEMS**

A

1. Let $5n$ = larger number; $2n$ = smaller number; $5n - 2n = 56$; $3n = 56$; $n = 18\frac{2}{3}$; $5n = 93\frac{1}{3}$, *Ans.*

2. Since both numbers will be negative, n represents a negative number. Thus, let $5n$ = smaller number; $3n$ = larger number; $5n + 3n = -24$; $8n = -24$; $n = -3$; $5n = -15$, *Ans.*

3. Let $2x$ = number from one city; $5x$ = number from the other city; $2x + 5x = 35$; $7x = 35$; $x = 5$; $2x = 10$, $5x = 25$, *Ans.*

4. Let $12x$ = weight of tin; x = weight of silver; $12x + x = 78$; $13x = 78$; $x = 6$ lb, *Ans.*

5. Let $8x$ = land height; $5x$ = sky height; $8x + 5x = 104$; $13x = 104$; $x = 8$; $8x = 64$ in, *Ans.*

6. Let $3x$ = cups of whole wheat flour; x = cups of white flour; $3x + x = 8$; $4x = 8$; $x = 2$ cups, *Ans*

7. Let x = weight of copper; $9x$ = weight of silver; $x + 9x = 45$; $10x = 45$; $x = 4\frac{1}{2}$ lb, $9x = 40\frac{1}{2}$ lb, *Ans.*

8. Let $500x$ = amount Ross invests; $600x$ = amount Morgan invests; $500x + 600x = 24{,}200$; $1100x = 24{,}200$; $x = 22$; $500x = \$11{,}000$, $600x = \$13{,}200$, *Ans.*

9. For Mr. A: $\frac{105}{21} = \frac{5}{1} = 5$; for Mr. B: $\frac{88}{16} = \frac{11}{2} = 5\frac{1}{2}$; $5 < 5\frac{1}{2}$. \$1.05 for 21 pounds is better buy.

10. First worker: $\frac{34}{90} = \frac{17}{45} = \frac{221}{585}$; second worker: $\frac{40}{104} = \frac{5}{13} = \frac{225}{585}$; $\frac{221}{585} < \frac{225}{585}$. Second worker is faster

11. Let $2x$ = shorter part; $3x$ = longer part; $2x + 3x = 30$; $5x = 30$; $x = 6$; $2x = 12$ cm, $3x = 18$ cm, *Ans*

12. Let $3x$ = shorter part; $4x$ = longer part; $3x + 4x = 21$; $7x = 21$; $x = 3$; $3x = 9$ cm, $4x = 12$ cm, *Ans.*

B

13. Let $2x$ = number built after 1950; $5x$ = number built before 1950; $2x + 5x = 1{,}343{,}790$; $7x = 1{,}343{,}790$; $x = 191{,}970$; $5x = 959{,}850$, *Ans.*

14. Let $3x$ = number of primary miles; $14x$ = number of secondary miles; $3x + 14x = 41{,}140$; $17x = 41{,}140$; $x = 2420$; $14x = 33{,}880$ mi, *Ans.*

***15.** Let x = weight of hydrogen: $16x$ = weight of sulfur; $32x$ = weight of oxygen; $x + 16x + 32x = 490$; $x = 10$; $16x = 160$; $32x = 320$. 10 lb hydrogen, 160 lb sulfur, 320 lb oxygen, *Ans.*

16. Let $9x$ = weight of clay; $2x$ = weight of sand; $9x$ = weight of silt; $9x + 2x + 9x = 500$; $20x = 500$; $x = 25$; $9x = 225$ lb; $2x = 50$ lb. 50 lb sand, 225 lb clay, 225 lb silt, *Ans.*

17. Let $3x$ = smallest part; $5x$ = middle part; $7x$ = largest part; $3x + 5x + 7x = 1200$; $15x = 1200$; $x = 80$; $7x = \$560$, *Ans.*

18. Let $2x$ = Jack's share; $3x$ = Walt's share; $4x$ = Randy's share; $2x + 3x + 4x = 6.75$; $9x = 6.75$; $x = .75$; $4x = \$3.00$, *Ans.*

Pages 290–291 · **WRITTEN EXERCISES**

A

1. $p = .07(250)$; $p = 17.5$ **2.** $p = .02(2.5)$; $p = .05$ **3.** $p = .32(12.5)$; $p = 4$

4. $p = .90(1000)$; $p = 90$ **5.** $p = .75(6)$; $p = 4.5$ **6.** $p = 1.00(72)$; $p = 72$

7. $p = 2.00(12)$; $p = 24$ **8.** $p = 1.50(38)$; $p = 57$ **9.** $p = 3.00(1)$; $p = 3$

10. $p = .0225(16)$; $p = .36$ **11.** $p = .002(635)$; $p = 1.27$ **12.** $p = .0003(1000)$; $p = .3$

13. $24 = .30b$; $30b = 2400$; $b = 80$ **14.** $17 = .25b$; $25b = 1700$; $b = 68$

15. $2.1 = .05b$; $5b = 210$; $b = 42$ **16.** $5.2 = .80b$; $80b = 520$; $b = 6.5$

17. $195 = 1.00b$; $b = 195$ **18.** $63 = 1.50b$; $150b = 6300$; $b = 42$

19. $.72 = .00\frac{1}{3}b$; $\frac{1}{3}b = 72$; $b = 216$ **20.** $20.4 = .012b$; $12b = 20400$; $b = 1700$

21. $39 = r \cdot 52$; $r = \frac{39}{52} = \frac{3 \cdot 13}{4 \cdot 13} = \frac{3}{4}$; $r = 75\%$ **22.** $21 = r \cdot 35$; $r = \frac{21}{35} = \frac{3}{5}$; $r = 60\%$

23. $12 = r \cdot 3$; $r = \frac{12}{3} = \frac{4}{1}$; $r = 400\%$ **24.** $15 = r \cdot 6$; $r = \frac{15}{6} = \frac{5}{2}$; $r = 250\%$

25. $1 = r \cdot 200$; $r = \frac{1}{200} = \frac{.5}{100}$; $r = .5\%$ *or* $r = \frac{1}{2}\%$

26. $3 = r \cdot 240$; $r = \frac{3}{240} = \frac{1}{80}$; $r = 1.25\%$ *or* $r = 1\frac{1}{4}\%$

27. $60 = r \cdot 5$; $r = \frac{60}{5} = \frac{12}{1}$; $r = 1200\%$ **28.** $500 = r \cdot 25$; $r = \frac{500}{25} = \frac{20}{1}$; $r = 2000\%$

*For problem concerning sulfuric acid.

Page 291 · **PROBLEMS**

A

1. Let d = defective output; $d = .02(1500)$; $d = 30$; good output $= 1500 - 30 = 1470$, *Ans.*

2. Let x = weight of copper; $x = .05(25)$; $x = 1.25$ lb, *Ans.*

3. Let x = amount of hydrogen chloride; $x = .40(1)$; $x = .4$ qt, *Ans.*

4. Let x = increase in weight; $x = .08(140)$; $x = 11.2$;
World War II average $= 140 + 11.2 = 151.2$ lb, *Ans.*

5. Let x = total monthly income; $116 = .20x$; $20x = 11{,}600$; $x = \$580$, *Ans.*

6. Let x = price of car; $47.50 = .05x$; $5x = 4750$; $x = \$950$, *Ans.*

7. Let x = cost of suit; $42.40 = .80x$; $80x = 4240$; $x = \$53$, *Ans.*

8. Let x = list price; $7.50 = .60x$; $60x = 750$; $x = \$12.50$, *Ans.*

9. Let x = cost of car for Ben; $1120 = 1.60x$; $160x = 112{,}000$; $x = \$700$, *Ans.*

10. Let x = cost of coat; $105 = 1.75x$; $175x = 10{,}500$; $x = \$60$, *Ans.*

11. Let r = rate of commission; $62.50 = r \cdot 1250$; $r = \dfrac{62.50}{1250} = \dfrac{6250}{125{,}000} = \dfrac{6250}{(6250)(20)} = \dfrac{1}{20}$;
$r = 5\%$, *Ans.*

12. Let r = tax rate; $1960 = r \cdot 8000$; $r = \dfrac{1960}{8000} = \dfrac{40 \cdot 49}{40 \cdot 200} = \dfrac{49}{200}$; $r = 24.5\%$ *or* $24\frac{1}{2}\%$, *Ans.*

B

13. Let r = rate of depreciation; $2500 - 2000 = 500$ = amount of depreciation; $500 = r \cdot 2500$;
$r = \dfrac{500}{2500} = \dfrac{500}{500 \cdot 5} = \dfrac{1}{5}$; $r = 20\%$, *Ans.*

14. Let r = per cent of increase; $13{,}800 - 12{,}000 = 1800$ = amount of increase; $1800 = r \cdot 12{,}000$;
$r = \dfrac{1800}{12{,}000} = \dfrac{600 \cdot 3}{600 \cdot 20} = \dfrac{3}{20}$; $r = 15\%$, *Ans.*

15. Let x = original price; actual cost is 70% of original price; $11.55 = .70x$; $70x = 1155$; $x = \$16.50$, *Ans.*

16. Let x = weight of rice; $\frac{1}{4} = .05x$; $.25 = .05x$; $5x = 25$; $x = 5$ lb, *Ans.*

17. Let x = selling price; $100\% - 15\% - 30\% = 55\%$ is per cent of selling price which cost is;
$330 = .55x$; $55x = 33{,}000$; $x = \$600$, *Ans.*

18. Let x = amount of profit; article will sell for $100\% - 15\% = 85\%$ of marked price;
$x = .85(13) - 8$; $x = 11.05 - 8$; $x = \$3.05$, *Ans.*

Pages 293–294 · **WRITTEN EXERCISES**

A

1. $\frac{15}{196}$ **2.** $-\frac{21}{80}$ **3.** $\frac{12}{72} = \frac{1}{6}$ **4.** $\frac{70}{175} = \frac{2}{5}$ **5.** $\frac{60}{120} = \frac{1}{2}$ **6.** $\frac{252}{1260} = \frac{1}{5}$

7. $\dfrac{5c^5}{12}$ **8.** $\dfrac{7x^4}{16}$ **9.** $\dfrac{abc}{a^2bc} = \dfrac{1}{a}$ **10.** $\dfrac{rst}{r^2st} = \dfrac{1}{r}$ **11.** $\dfrac{462xy}{165xy} = \dfrac{14}{5}$ **12.** $\dfrac{252bc}{567bc} = \dfrac{4}{9}$

13. $\dfrac{105abc^2d^2}{140a^2b^2cd} = \dfrac{3cd}{4ab}$ **14.** $\dfrac{90r^2s^2tu}{126rst^2u^2} = \dfrac{5rs}{7tu}$ **15.** $\dfrac{t^2 - v^2}{t^2 - 4v^2}$ **16.** $\dfrac{9r^2 - w^2}{r^2 - w^2}$

17. $\dfrac{12(a+b)(a-b)}{4(a+b)} = \dfrac{3(a-b)}{1} = 3a - 3b$ **18.** $\dfrac{2(1-x)}{2(1+x)(1-x)} = \dfrac{1}{1+x}$

19. $\dfrac{7(2+y)(2-y)}{14(2-y)} = \dfrac{2+y}{2}$ **20.** $\dfrac{3(m+n)(m-n)}{6(m+n)} = \dfrac{m-n}{2}$ **21.** $\dfrac{10 \cdot 2(u+v)}{5(u+v)} = \dfrac{4}{1} = 4$

B

22. $\dfrac{(r+s)(r-s)(m+n)}{(m+n)(m-n)(r-s)} = \dfrac{(r-s)(m+n)(r+s)}{(r-s)(m+n)(m-n)} = \dfrac{r+s}{m-n}$

23. $\dfrac{(a+7)(a-7)(b-5)}{(b+5)(b-5)(a+7)} = \dfrac{(a+7)(b-5)(a-7)}{(a+7)(b-5)(b+5)} = \dfrac{a-7}{b+5}$

24. $\dfrac{(y-1)^2(x)}{(3x)(x)(y-1)} = \dfrac{(y-1)(x)(y-1)}{(y-1)(x)(3x)} = \dfrac{y-1}{3x}$

25. $\dfrac{(b+1)^2(a)}{(a)(b^2)(b+1)} = \dfrac{(b+1)(b+1)(a)}{(b+1)(b^2)(a)} = \dfrac{b+1}{b^2}$

26. $\dfrac{k(k+1)(3)(k-1)}{k(k)(k+1)(k-1)} = \dfrac{k(k+1)(k-1)(3)}{k(k+1)(k-1)(k)} = \dfrac{3}{k}$

27. $\dfrac{3(2)(m+n)(m)(m-n)}{(m+n)(m-n)(2)(m)} = \dfrac{(2)(m+n)(m)(m-n)(3)}{(2)(m+n)(m)(m-n)(1)} = 3$

28. $\dfrac{(3x+1)(3x-1)(2)}{(2)(3x+1)(3x-1)} = \dfrac{(3x+1)(3x-1)(2)}{(3x+1)(3x-1)(2)} = 1$

29. $\dfrac{(3)(2a-b)(2a+b)}{(2a+b)(2a-b)(3)} = \dfrac{(3)(2a-b)(2a+b)}{(3)(2a-b)(2a+b)} = 1$

30. $\dfrac{(s-t)(2)(2)(s+t)}{(2)(s+t)^2} = \dfrac{(2)(s+t)(s-t)(2)}{(2)(s+t)(s+t)(1)} = \dfrac{2s-2t}{s+t}$

31. $\dfrac{(x+1)^2(5)(3)}{5(x-1)(x+1)} = \dfrac{(x+1)(x+1)(5)(3)}{(x+1)(x-1)(5)(1)} = \dfrac{3x+3}{x-1}$

32. $\dfrac{(8r+1)(8r-1)(8r+1)}{(8r+1)^2(2)(8r-1)} = \dfrac{(8r+1)(8r-1)(8r+1)(1)}{(8r+1)(8r-1)(8r+1)(2)} = \dfrac{1}{2}$

33. $\dfrac{(x+2)(x+1)(x+3)}{(x+1)(x+3)(x+2)} = \dfrac{(x+2)(x+1)(x+3)}{(x+2)(x+1)(x+3)} = 1$

34. $\dfrac{(a+4)(a+1)(a+5)}{(a+4)(a+5)(a+1)} = \dfrac{(a+4)(a+1)(a+5)}{(a+4)(a+1)(a+5)} = 1$

35. $\dfrac{(c+1)(c-6)(c+3)}{c(c+3)(-1)(c-6)} = \dfrac{(c+1)(c-6)(c+3)(1)}{(c)(c-6)(c+3)(-1)} = \dfrac{c+1}{-c}$ *or* $-\dfrac{c+1}{c}$

C **36.** $\dfrac{(-1)(x-2)(x+2)}{(x+2)(x-2)} = \dfrac{(x-2)(x+2)(-1)}{(x-2)(x+2)(1)} = -1$

37. $\dfrac{(-1)(b-2)(b+2)(4)}{5(b-2)(b+2)} = \dfrac{(b-2)(b+2)(-1)(4)}{(b-2)(b+2)(5)} = -\dfrac{4}{5}$

38. $\dfrac{2(x+5)(x-5)(4)(x+4)}{5(x+4)(x-4)(3)(x-5)} = \dfrac{(x-5)(x+4)(2)(4)(x+5)}{(x-5)(x+4)(3)(5)(x-4)} = \dfrac{8(x+5)}{15(x-4)}$

39. $\dfrac{a(a+b)(a)(a-b)}{(-1)(a-b)(a+b)} = \dfrac{(a+b)(a-b)(a^2)}{(a+b)(a-b)(-1)} = -a^2$

40. $\dfrac{3(n+3)(n-3)(2)(-1)(n-2)(n)(n+3)}{(-1)(n+3)(n-2)(-1)(n-3)(3)(2)} = \dfrac{3(n+3)(n-3)(2)(-1)(n-2)(n)(n+3)}{3(n+3)(n-3)(2)(-1)(n-2)(-1)}$
$= \dfrac{n(n+3)}{-1} = -n(n+3)$ *or* $-n^2-3n$

41. $\dfrac{(2k+3)(2k-3)(-1)(k-2)(k)(k)}{(-1)(2k+3)(k-2)(-1)(5)(k)(2k-3)(2k+3)}$
$= \dfrac{(2k+3)(2k-3)(-1)(k-2)(k)(k)}{(2k+3)(2k-3)(-1)(k-2)(k)(-1)(5)(2k+3)} = \dfrac{k}{-5(2k+3)}$ *or* $-\dfrac{k}{5(2k+3)}$

42. $\dfrac{(2x+1)(x+3)(x)(x-3)}{(x+3)(x-3)(2x+1)(x+5)} = \dfrac{(2x+1)(x+3)(x-3)(x)}{(2x+1)(x+3)(x-3)(x+5)} = \dfrac{x}{x+5}$

43. $\dfrac{(3n+1)(n+2)(3n-1)(n-2)}{(-1)(3n+1)(3n-1)(-1)(n+2)(n-2)} = \dfrac{(3n+1)(n+2)(3n-1)(n-2)(1)}{(3n+1)(n+2)(3n-1)(n-2)(-1)(-1)} = 1$

44. $\dfrac{(-1)(2)(2)(2x+1)(2x-1)(4x-3)(x-2)(x^2)(x^4)}{(2)(4x-3)(2x+1)(1)(-1)(2)(2)(x^2)(2x-1)}$
$= \dfrac{(-1)(2)(2)(2x+1)(2x-1)(4x-3)(x^2)(x-2)(x^4)}{(-1)(2)(2)(2x+1)(2x-1)(4x-3)(x^2)(1)(2)} = \dfrac{x^4(x-2)}{2}$

45. $\dfrac{3(2)(5a-2b)(3a+b)(a^2)(3a^2-b^2)(9a^2+b^2)}{3(5a-2b)(9a^2+b^2)(3a+b)(3a-b)(1)}$
$= \dfrac{3(5a-2b)(3a+b)(9a^2+b^2)(2)(a^2)(3a^2-b^2)}{3(5a-2b)(3a+b)(9a^2+b^2)(1)(3a-b)} = \dfrac{2a^2(3a^2-b^2)}{3a-b}$

Pages 295–296 · **WRITTEN EXERCISES**

A **1.** $\dfrac{3}{7}\cdot\dfrac{14}{11} = \dfrac{(3)(14)}{(11)(7)} = \dfrac{(3)(2)}{(11)(1)} = \dfrac{6}{11}$

2. $\dfrac{12}{11}\cdot\dfrac{7}{6} = \dfrac{(7)(12)}{(11)(6)} = \dfrac{(7)(2)}{(11)(1)} = \dfrac{14}{11}$

3. $\dfrac{a}{b}\cdot\dfrac{b^2}{a} = \dfrac{(a)(b^2)}{(a)(b)} = b$

4. $\dfrac{c^2}{d}\cdot\dfrac{d}{c} = \dfrac{(d)(c^2)}{(d)(c)} = c$

5. $\dfrac{3b}{4c^2}\cdot\dfrac{16c^3}{15b^2} = \dfrac{(3b)(16c^3)}{(15b^2)(4c^2)} = \dfrac{(1)(4c)}{(5b)(1)} = \dfrac{4c}{5b}$

6. $\dfrac{13r^2}{20a^2}\cdot\dfrac{5a}{39r^3} = \dfrac{(13r^2)(5a)}{(39r^3)(20a^2)} = \dfrac{(1)(1)}{(3r)(4a)} = \dfrac{1}{12ar}$

7. $\dfrac{3g^2}{10}\cdot\dfrac{25}{9g^2} = \dfrac{(3g^2)(25)}{(9g^2)(10)} = \dfrac{(1)(5)}{(3)(2)} = \dfrac{5}{6}$

8. $\dfrac{12x^3}{35}\cdot\dfrac{77}{48x^3} = \dfrac{(12x^3)(77)}{(48x^3)(35)} = \dfrac{(1)(11)}{(4)(5)} = \dfrac{11}{20}$

9. $\dfrac{a-b}{4}\cdot\dfrac{2}{a-b} = \dfrac{(a-b)(2)}{(a-b)(4)} = \dfrac{1}{2}$

10. $\dfrac{2x+1}{2}\cdot\dfrac{6}{2x+1} = \dfrac{(2x+1)(6)}{(2x+1)(2)} = 3$

11. $\dfrac{9}{11x}\cdot\dfrac{1}{18} = \dfrac{(9)(1)}{(18)(11x)} = \dfrac{(1)(1)}{(2)(11x)} = \dfrac{1}{22x}$

12. $\dfrac{4a^2}{7}\cdot\dfrac{1}{8a} = \dfrac{(4a^2)(1)}{(8a)(7)} = \dfrac{(a)(1)}{(2)(7)} = \dfrac{a}{14}$

13. $\frac{(m^2 - n^2)}{mn} \cdot \frac{1}{m - n} = \frac{(m + n)(m - n)(1)}{(m - n)(mn)} = \frac{m + n}{mn}$

14. $\frac{a^2 + 2ab + b^2}{ab} \cdot \frac{1}{a + b} = \frac{(a + b)^2(1)}{(a + b)(ab)} = \frac{a + b}{ab}$

15. $\frac{x}{(x - 2)^2} \cdot \frac{x - 2}{1} = \frac{(x - 2)(x)}{(x - 2)^2(1)} = \frac{x}{x - 2}$

16. $\frac{m}{m^2 + 6m + 9} \cdot \frac{m + 3}{1} = \frac{(m + 3)(m)}{(m + 3)^2(1)} = \frac{m}{m + 3}$

17. $\frac{1}{x^2 - 16} \cdot \frac{x + 4}{x - 4} = \frac{(x + 4)(1)}{(x + 4)(x - 4)(x - 4)} = \frac{1}{(x - 4)^2}$

18. $\frac{2x + 4}{3x + 9} \cdot \frac{5x + 15}{4x + 8} = \frac{2(x + 2)(5)(x + 3)}{4(x + 2)(3)(x + 3)} = \frac{(1)(5)}{(2)(3)} = \frac{5}{6}$

19. $\frac{ax - 2a}{bx - 3b} \cdot \frac{dx - 3d}{cx - 2c} = \frac{a(x - 2)(d)(x - 3)}{c(x - 2)(b)(x - 3)} = \frac{(a)(d)}{(c)(b)} = \frac{ad}{bc}$

20. $\frac{m^2 - n^2}{9m^2n^2} \cdot \frac{mn}{3m - 3n} = \frac{(m + n)(m - n)(mn)}{3(m - n)(9)(mn)^2} = \frac{m + n}{27mn}$

21. $\frac{5a^2 - 5b^2}{a^2b^2} \cdot \frac{10ab}{a + b} = \frac{5(a + b)(a - b)(10)(ab)}{(a + b)(ab)^2} = \frac{50(a - b)}{ab}$

22. $\frac{x^2 - y^2}{ax - ay} \cdot \frac{x^2 + xy}{x^2 + 2xy + y^2} = \frac{(x + y)(x - y)(x)(x + y)}{a(x - y)(x + y)^2} = \frac{x}{a}$

23. $\frac{x^2 + 4x + 3}{x^2 - 4x - 5} \cdot \frac{x - 5}{x + 3} = \frac{(x + 3)(x + 1)(x - 5)}{(x - 5)(x + 1)(x + 3)} = \frac{(x + 3)(x + 1)(x - 5)}{(x + 3)(x + 1)(x - 5)} = 1$

B **24.** $\frac{a + 2a^2}{9 - a^2} \cdot \frac{3a + a^2}{1 - 4a^2} = \frac{a(1 + 2a)(a)(3 + a)}{(3 + a)(3 - a)(1 + 2a)(1 - 2a)} = \frac{(1 + 2a)(a^2)(3 + a)}{(1 + 2a)(1 - 2a)(3 + a)(3 - a)}$
$= \frac{a^2}{(1 - 2a)(3 - a)}$

25. $\frac{r^3 - rs^2}{r^2s} \cdot \frac{3rs + 3s^2}{r^2 + rs} = \frac{r(r^2 - s^2)(3s)(r + s)}{r^2s(r)(r + s)} = \frac{3(r^2 - s^2)}{r^2}$

26. $\frac{x^4 - y^4}{3x + 3y} \cdot \frac{6}{2x^2 + 2y^2} = \frac{(x^2 + y^2)(x + y)(x - y)(3)(2)}{3(x + y)(2)(x^2 + y^2)} = \frac{(x^2 + y^2)(x + y)(3)(2)(x - y)}{(x^2 + y^2)(x + y)(3)(2)(1)} = x - y$

27. $\frac{x^2 - 3x + 2}{x^2 - 1} \cdot \frac{1 - x}{2 - x} = \frac{(x - 2)(x - 1)(1 - x)}{(x + 1)(x - 1)(2 - x)} = \frac{(-1)(2 - x)(x - 1)(1 - x)}{(2 - x)(x - 1)(x + 1)} = \frac{(-1)(1 - x)}{(x + 1)}$
$= \frac{x - 1}{x + 1}$

28. $\frac{5c^2 - 5cd}{cd + d^2} \cdot \frac{cd^2}{d^3 - dc^2} = \frac{(5)(c)(c - d)(c)(d)(d)}{d(c + d)(-1)(d)(c + d)(c - d)} = \frac{(c - d)(d)(d)(5)(c)(c)}{(c - d)(d)(d)(c + d)(-1)(c + d)}$
$= \frac{5c^2}{-1(c + d)^2}$ *or* $-\frac{5c^2}{(c + d)^2}$

29. $\frac{14}{7t^2 + 7u^2} \cdot \frac{u^4 - t^4}{2u + 2t} = \frac{(7)(2)(u^2 + t^2)(u^2 - t^2)}{7(t^2 + u^2)(2)(u + t)} = \frac{(7)(2)(u^2 + t^2)(u + t)(u - t)}{(7)(2)(u^2 + t^2)(u + t)(1)} = u - t$

Pages 296–297 · **WRITTEN EXERCISES**

A **1.** $\frac{3}{x(x^2)} \cdot \frac{x^2}{5} \cdot \frac{xy}{6} = \frac{(3)(x^2)(xy)}{(6)(x^2)(5x)} = \frac{y}{10}$

2. $\frac{a(a^3)}{4} \cdot \frac{3}{a^3} \cdot \frac{b}{6a} = \frac{(a^3)(3a)(b)}{(a^3)(6a)(4)} = \frac{b}{8}$

3. $\frac{(a^2)(b)}{c} \cdot \frac{(c^3)(d)}{a^3} \cdot \frac{1}{(b^2)(d)} = \frac{(a^2)(b)(c^3)(d)}{(a^3)(b^2)(c)(d)} = \frac{c^2}{ab}$

4. $\frac{6(r^2s^3)}{t} \cdot \frac{1}{(3)^2(r^2s^4)} \cdot \frac{3(t^4)}{s^2} = \frac{(6)(r^2s^3)(3)(t^4)(1)}{(3)(r^2s^4)(3)(t)(s^2)} = \frac{2t^3}{s^3}$

5. $\frac{x^2 - 1}{x^2} \cdot \frac{x}{x - 1} \cdot \frac{5}{x + 1} = \frac{(x + 1)(x - 1)(x)(5)}{(x + 1)(x - 1)(x)(x)} = \frac{5}{x}$

6. $\frac{3a}{3a + 2} \cdot \frac{6a + 4}{4a} \cdot \frac{2}{a + 2} = \frac{(3)(a)(2)(3a + 2)(2)}{(3a + 2)(4)(a)(a + 2)} = \frac{(a)(2)(2)(3a + 2)(3)}{(a)(4)(3a + 2)(a + 2)} = \frac{3}{a + 2}$

7. $\frac{9m + n}{9mn} \cdot \frac{3m}{3m - n} \cdot \frac{12}{3m + n} = \frac{(3m)(3)(4)(9m + n)}{(3m)(3)(n)(3m - n)(3m + n)} = \frac{4(9m + n)}{n(3m - n)(3m + n)}$

8. $\frac{2c-d}{2c}\cdot\frac{8cd}{8c+d}\cdot\frac{16c+2d}{4}=\frac{(2c-d)(8cd)(2)(8c+d)}{(2c)(8c+d)(4)}=\frac{(8cd)(8c+d)(2c-d)(2)}{(2c)(4)(8c+d)(1)}$
$=\frac{2d(2c-d)}{1}=2d(2c-d)$

9. $\frac{(a+b)(a-b)}{a^2}\cdot\frac{a+b}{a-b}\cdot\frac{ab}{(a+b)^2}=\frac{(a+b)(a-b)(a+b)(ab)}{(a+b)(a-b)(a+b)(a^2)}=\frac{b}{a}$

10. $\frac{(r-s)^2}{rs}\cdot\frac{r+s}{r-s}\cdot\frac{r^2}{(r+s)(r-s)}=\frac{(r-s)(r-s)(r+s)(r^2)}{(r-s)(r-s)(r+s)(rs)}=\frac{r}{s}$

11. $\frac{m}{m+1}\cdot\frac{2(m+3)}{2m^2}\cdot\frac{(m-3)(m+1)}{(m+3)(m-3)}=\frac{m(2)(m+3)(m-3)(m+1)(1)}{m^2(2)(m+3)(m-3)(m+1)}=\frac{1}{m}$

12. $\frac{3x-1}{3x^2}\cdot\frac{3x}{3x+1}\cdot\frac{(3x+1)^2}{(3x+1)(3x-1)}=\frac{(3x-1)(3x)(3x+1)(3x+1)(1)}{(3x-1)(3x^2)(3x+1)(3x+1)}=\frac{1}{x}$

13. $\frac{a(a^2)}{3a}\cdot\frac{b}{(ab+3)(ab-3)}\cdot\frac{3(ab-3)}{(a)(b)}=\frac{a(a^2)(b)(3)(ab-3)}{a(a)(b)(3)(ab-3)(ab+3)}=\frac{a}{ab+3}$

14. $\frac{4(t)}{(st+2)(st-2)}\cdot\frac{st}{(4)^2}\cdot\frac{4(st-2)}{(st)(t)}=\frac{(4)(t)(st)(4)(st-2)(1)}{(4)(t)(st)(4)(st-2)(st+2)}=\frac{1}{st+2}$

B 15. $\frac{(x+5)^2}{x(x+10)}\cdot\frac{10(x)}{(x+5)(x+10)}\cdot\frac{x+10}{x+5}=\frac{(x+5)(x+5)(x)(x+10)(10)}{(x+5)(x+5)(x)(x+10)(x+10)}=\frac{10}{x+10}$

16. $\frac{(a-8)(a-4)}{8(a)}\cdot\frac{a(a-8)}{(a-4)^2}\cdot\frac{a-4}{a-8}=\frac{(a-8)(a-4)(a)(a-4)(a-8)}{(a-8)(a-4)(a)(a-4)(8)}=\frac{a-8}{8}$

17. $\frac{3a^2(a)}{3a+c}\cdot\frac{(3a+c)(3a-c)}{3(3a^2)}\cdot\frac{3(ab)}{a(3a-c)}=\frac{3a^2(a)(3a+c)(3a-c)(3)(ab)}{3a^2(a)(3a+c)(3a-c)(3)}=ab$

18. $\frac{(xy)^3}{x(x-y)(x+y)}\cdot\frac{x(x-y)}{xy(z)}\cdot\frac{y(x+z)}{xy}=\frac{(xy)(xy)(x)(x-y)(xy)(y)(x+z)}{(xy)(xy)(x)(x-y)(x+y)(z)}=\frac{xy^2(x+z)}{z(x+y)}$

19. $\frac{a(b+c-d)}{a(bc)}\cdot\frac{bc(d^2)}{d(b+c-d)}\cdot\frac{1}{d^2}=\frac{a(b+c-d)(bc)(d^2)(1)}{a(b+c-d)(bc)(d^2)(d)}=\frac{1}{d}$

20. $\frac{(vw)^2(x^3)}{x(v-w-x)}\cdot\frac{u(v-w-x)}{u(vw)}\cdot\frac{1}{x^2}=\frac{(vw)(x)(x^2)(u)(v-w-x)(vw)}{(vw)(x)(x^2)(u)(v-w-x)}=vw$

Pages 298–299 · **WRITTEN EXERCISES**

A 1. $\frac{4-8-7}{15}=\frac{-11}{15}$ *or* $-\frac{11}{15}$

2. $\frac{3-29+1}{50}=\frac{-25}{50}=\frac{-1}{2}$ *or* $-\frac{1}{2}$

3. $\frac{8-1+2}{3z}=\frac{9}{3z}=\frac{3}{z}$

4. $\frac{10+15-4}{7a}=\frac{21}{7a}=\frac{3}{a}$

5. $\frac{12a-(2a+5)}{5r}=\frac{10a-5}{5r}=\frac{5(2a-1)}{5(r)}=\frac{2a-1}{r}$

6. $\frac{2b-(3-7b)}{9t}=\frac{9b-3}{9t}=\frac{3(3b-1)}{3(3t)}=\frac{3b-1}{3t}$

7. $\frac{2a+1+4-2a}{5}=\frac{5}{5}=1$

8. $\frac{3b+5+2-3b}{7}=\frac{7}{7}=1$

9. $\frac{3r+3s}{r+s}=\frac{3(r+s)}{(r+s)}=3$

10. $\frac{x-3}{x-3}=1$

11. $\frac{a^2-b^2}{a+b}=\frac{(a-b)(a+b)}{(a+b)}=a-b$

12. $\frac{z^2-16}{z-4}=\frac{(z+4)(z-4)}{(z-4)}=z+4$

13. $\frac{3x-3y-(2x+5y)+8y-x}{2xy}=\frac{0}{2xy}=0$

14. $\frac{5+1-2a-(6-2a)}{3ab}=\frac{0}{3ab}=0$

B 15. $\frac{2xy+x^2+y^2}{x+y}=\frac{(x+y)^2}{(x+y)}=x+y$

16. $\frac{m^2+n^2-2mn}{m-n}=\frac{(m-n)^2}{(m-n)}=m-n$

17. $\frac{3+x-2}{x^2-3x-4}=\frac{(x+1)}{(x+1)(x-4)}=\frac{1}{x-4}$

18. $\frac{2x-(x+2)}{x^2-x-2}=\frac{(x-2)}{(x-2)(x+1)}=\frac{1}{x+1}$

19. $\frac{1+a-(2a+a^2)}{a^2+3a-4}=\frac{1-a-a^2}{a^2+3a-4}$

20. $\frac{5-b-(b^2-3)}{b^2-16}=\frac{8-b-b^2}{b^2-16}$

Pages 301–302 · **WRITTEN EXERCISES**

A 1. $\frac{8b}{2a}-\frac{a}{2a}=\frac{8b-a}{2a}$

2. $\frac{x}{3x}+\frac{6a}{3x}=\frac{x+6a}{3x}$

3. $\frac{7}{6c}+\frac{2-6c}{6c}=\frac{9-6c}{6c}=\frac{3(3-2c)}{3(2c)}=\frac{3-2c}{2c}$

4. $\frac{1}{10a} - \frac{10a+25}{10a} = \frac{-10a-24}{10a} = \frac{-2(5a+12)}{2(5a)} = -\frac{5a+12}{5a}$

5. $\frac{4a+2}{10} - \frac{5a-15}{10} = \frac{-a+17}{10}$

6. $\frac{21n-12}{12} - \frac{12n+8}{12} = \frac{9n-20}{12}$

7. $\frac{6h+3}{12} + \frac{2h-6}{12} = \frac{8h-3}{12}$

8. $\frac{6h-10}{18} - \frac{6h+3}{18} = \frac{-13}{18}$ *or* $-\frac{13}{18}$

9. $\frac{x^2}{x^3} - \frac{2x}{x^3} + \frac{1}{x^3} = \frac{x^2-2x+1}{x^3}$

10. $\frac{3}{a^3} - \frac{2a}{a^3} + \frac{a^2}{a^3} = \frac{3-2a+a^2}{a^3}$

11. $\frac{c}{abc} + \frac{a^2-2a}{abc} = \frac{c+a^2-2a}{abc}$

12. $\frac{4t}{rst} - \frac{2s-st}{rst} = \frac{4t-2s+st}{rst}$

13. $\frac{ax}{abc} + \frac{bx+bc}{abc} - \frac{cx+bc}{abc} = \frac{ax+bx-cx}{abc}$

14. $\frac{ax}{xyz} - \frac{ay+yz}{xyz} + \frac{az+yz}{xyz} = \frac{ax-ay+az}{xyz}$

15. $\frac{3x}{x^2y^2} + \frac{2y-xy}{x^2y^2} = \frac{3x+2y-xy}{x^2y^2}$

16. $\frac{4s}{r^2s^2} - \frac{2r-r^2}{r^2s^2} = \frac{4s-2r+r^2}{r^2s^2}$

17. $\frac{(a-b)}{(a+b)(a-b)} + \frac{a}{(a+b)(a-b)} = \frac{a-b+a}{\text{L.C.D.}} = \frac{2a-b}{a^2-b^2}$

18. $\frac{d}{(c+d)(c-d)} - \frac{-(c+d)}{(c+d)(c-d)} = \frac{d+c+d}{\text{L.C.D.}} = \frac{c+2d}{c^2-d^2}$

19. $\frac{15-12k}{24} - \frac{8-12k}{24} = \frac{7}{24}$

20. $\frac{12+9k}{24} + \frac{2k-12}{24} = \frac{11k}{24}$

21. $\frac{2}{12} + \frac{9x-3}{12} - \frac{6x+12}{12} = \frac{-13+3x}{12}$

22. $\frac{4}{18} - \frac{6x-9}{18} + \frac{9x-9}{18} = \frac{4+3x}{18}$

23. $\frac{-2(2x-5)}{2(x-2)} + \frac{x}{2(x-2)} = \frac{-4x+10+x}{\text{L.C.D.}} = \frac{-3x+10}{\text{L.C.D.}} = \frac{-3x+10}{2x-4}$

24. $\frac{3a}{2(a+3)} - \frac{2(a-1)}{2(a+3)} = \frac{3a-2a+2}{\text{L.C.D.}} = \frac{a+2}{2a+6}$

B

25. $\frac{4x-7}{(x-2)(x-1)} - \frac{3(x-2)}{(x-2)(x-1)} = \frac{4x-7-3x+6}{\text{L.C.D.}} = \frac{(x-1)}{(x-2)(x-1)} = \frac{1}{x-2}$

26. $\frac{6m-13}{(m-3)(m-2)} - \frac{5(m-2)}{(m-3)(m-2)} = \frac{6m-13-5m+10}{\text{L.C.D.}} = \frac{(m-3)}{(m-3)(m-2)} = \frac{1}{m-2}$

27. $\frac{2x}{2(x-5)(x+5)} - \frac{x-5}{2(x-5)(x+5)} = \frac{2x-x+5}{\text{L.C.D.}} = \frac{x+5}{2(x-5)(x+5)} = \frac{1}{2x-10}$

28. $\frac{2(27)}{2(x+9)(x-9)} + \frac{3(x-9)}{2(x+9)(x-9)} = \frac{54+3x-27}{\text{L.C.D.}} = \frac{3x+27}{\text{L.C.D.}} = \frac{3(x+9)}{2(x+9)(x-9)} = \frac{3}{2x-18}$

29. $\frac{4b(2a+3b)}{12a^2b^2} - \frac{3a(a+2b)}{12a^2b^2} - \frac{2ab}{12a^2b^2} = \frac{8ab+12b^2-3a^2-6ab-2ab}{\text{L.C.D.}} = \frac{12b^2-3a^2}{\text{L.C.D.}} = \frac{3(4b^2-a^2)}{3(4a^2b^2)} = \frac{4b^2-a^2}{4a^2b^2}$

30. $\frac{4n^2(3)}{8m^2n^2} + \frac{n(2m)}{8m^2n^2} - \frac{2m(3m+n)}{8m^2n^2} = \frac{12n^2+2mn-6m^2-2mn}{\text{L.C.D.}} = \frac{12n^2-6m^2}{\text{L.C.D.}} = \frac{2(6n^2-3m^2)}{2(4m^2n^2)} = \frac{6n^2-3m^2}{4m^2n^2}$

31. $\frac{(-1)(2)(1+x)}{(1+x)(1-x)} - \frac{3(1-x)}{(1+x)(1-x)} - \frac{x-5}{(1+x)(1-x)} = \frac{-2-2x-3+3x-x+5}{\text{L.C.D.}} = \frac{0}{\text{L.C.D.}} = 0$

32. $\frac{3(2-n)}{(2+n)(2-n)} + \frac{(-1)(2)(2+n)}{(2+n)(2-n)} + \frac{5n-2}{(2+n)(2-n)} = \frac{6-3n-4-2n+5n-2}{\text{L.C.D.}} = \frac{0}{\text{L.C.D.}} = 0$

33. $\frac{2(3-y)}{(3+y)(3-y)} + \frac{(-1)(5)}{(3+y)(3-y)} + \frac{(3+y)(2y-1)}{(3+y)(3-y)} = \frac{6-2y-5+2y^2+5y-3}{\text{L.C.D.}} = \frac{2y^2+3y-2}{9-y^2}$

34. $\frac{3}{(z+5)(z-5)} - \frac{z-5}{(z+5)(z-5)} - \frac{(-1)(z+5)(z+1)}{(z+5)(z-5)} = \frac{3-z+5+z^2+6z+5}{\text{L.C.D.}} = \frac{z^2+5z+13}{z^2-25}$

C **35.** $\dfrac{25(a-2)}{25(a+2)^2}+\dfrac{5a(a+2)}{25(a+2)^2}-\dfrac{(a+2)^2}{25(a+2)^2}=\dfrac{25a-50+5a^2+10a-a^2-4a-4}{\text{L.C.D.}}=\dfrac{4a^2+31a-54}{25(a+2)^2}$

36. $\dfrac{(x+1)(x+1)}{(x+1)(x-1)^2}-\dfrac{x(x-1)}{(x+1)(x-1)^2}+\dfrac{(x+1)(x-1)}{(x+1)(x-1)^2}=\dfrac{x^2+2x+1-x^2+x+x^2-1}{\text{L.C.D.}}=\dfrac{x^2+3x}{(x+1)(x-1)^2}$

37. $\dfrac{a-6b}{(2a+b)(a+2b)}+\dfrac{3(a+2b)}{(2a+b)(a+2b)}-\dfrac{7(2a+b)}{(2a+b)(a+2b)}=\dfrac{a-6b+3a+6b-14a-7b}{\text{L.C.D.}}=\dfrac{-10a-7b}{(2a+b)(a+2b)}$

38. $\dfrac{(x-y)^2}{(x+y)^2(x-y)}-\dfrac{x(x+y)}{(x+y)^2(x-y)}-\dfrac{(x+y)^2}{(x+y)^2(x-y)}=\dfrac{x^2-2xy+y^2-x^2-xy-x^2-2xy-y^2}{\text{L.C.D.}}=\dfrac{-x^2-5xy}{(x+y)^2(x-y)}$

Pages 303–304 · **WRITTEN EXERCISES**

A **1.** $\dfrac{y(y+1)}{(y+1)}+\dfrac{1}{(y+1)}=\dfrac{y^2+y+1}{y+1}$

2. $\dfrac{b(a+b)}{(a+b)}+\dfrac{a}{(a+b)}=\dfrac{ab+b^2+a}{a+b}$

3. $\dfrac{(x+y)}{(x+y)}+\dfrac{(x-y)}{(x+y)}=\dfrac{x+y+x-y}{x+y}=\dfrac{2x}{x+y}$

4. $\dfrac{3(a-b)}{(a-b)}+\dfrac{(a+b)}{(a-b)}=\dfrac{3a-3b+a+b}{a-b}=\dfrac{4a-2b}{a-b}$

5. $\dfrac{(a+1)(a-1)}{(a-1)}+\dfrac{1}{(a-1)}=\dfrac{a^2-1+1}{a-1}=\dfrac{a^2}{a-1}$

6. $\dfrac{(x-2)(x+2)}{(x+2)}+\dfrac{4}{(x+2)}=\dfrac{x^2-4+4}{x+2}=\dfrac{x^2}{x+2}$

7. $\dfrac{2}{(x-1)}+\dfrac{(x-1)}{(x-1)}=\dfrac{2+x-1}{x-1}=\dfrac{x+1}{x-1}$

8. $\dfrac{4}{(x-2)}+\dfrac{(x-2)}{(x-2)}=\dfrac{4+x-2}{x-2}=\dfrac{x+2}{x-2}$

9. $\dfrac{x^2}{xy}+\dfrac{2xy}{xy}+\dfrac{y^2}{xy}=\dfrac{x^2+2xy+y^2}{xy}$

10. $\dfrac{a^2}{ab}+\dfrac{2ab}{ab}+\dfrac{b^2}{ab}=\dfrac{a^2+2ab+b^2}{ab}$

11. $\dfrac{(s-3)(s-2)}{(s-2)}+\dfrac{(5s-6)}{(s-2)}=\dfrac{s^2-5s+6+5s-6}{s-2}=\dfrac{s^2}{s-2}$

12. $\dfrac{(x+5)(x-4)}{(x-4)}-\dfrac{(x-20)}{(x-4)}=\dfrac{x^2+x-20-x+20}{x-4}=\dfrac{x^2}{x-4}$

13. $88\frac{4}{15}$

14. $199\frac{4}{13}$

15. $4x^2+\dfrac{2}{x}$

16. $\dfrac{4}{y}-y$

17. $2-\dfrac{1}{3a^3}$

18. $\dfrac{3}{2c^4}-2c^4$

19. $1+\dfrac{3}{ab}$

20. $\dfrac{2}{rs}-1$

21.
$$\begin{array}{r|l} & x+4 \\ \hline x+1 & x^2+5x+6 \\ & \underline{x^2+\ x} \\ & 4x+6 \\ & \underline{4x+4} \\ & 2 \end{array}$$

$\dfrac{x^2+5x+6}{x+1}=x+4+\dfrac{2}{x+1}$

22.
$$\begin{array}{r|l} & a+6 \\ \hline a+1 & a^2+7a+8 \\ & \underline{a^2+\ a} \\ & 6a+8 \\ & \underline{6a+6} \\ & 2 \end{array}$$

$\dfrac{a^2+7a+8}{a+1}=a+6+\dfrac{2}{a+1}$

23.
$$\begin{array}{r|l} & 6x+\ 5 \\ \hline 2x-3 & 12x^2-\ 8x-17 \\ & \underline{12x^2-18x} \\ & 10x-17 \\ & \underline{10x-15} \\ & -\ 2 \end{array}$$

$\dfrac{12x^2-8x-17}{2x-3}=6x+5+\dfrac{-2}{2x-3}$

24.
$$\begin{array}{r|l} & 3a+\ 2 \\ \hline 4a+2 & 12a^2+14a-17 \\ & \underline{12a^2+\ 6a} \\ & 8a-17 \\ & \underline{8a+\ 4} \\ & -21 \end{array}$$

$\dfrac{12a^2+14a-17}{4a+2}=3a+2+\dfrac{-21}{4a+2}$

Page 305 · **WRITTEN EXERCISES**

A **1.** $\frac{\frac{7}{8}(16)}{\frac{21}{16}(16)} = \frac{14}{21} = \frac{2}{3}$ **2.** $\frac{\frac{15}{16}(64)}{\frac{45}{64}(64)} = \frac{60}{45} = \frac{4}{3}$ **3.** $\frac{\frac{a}{b}(b^2)}{\frac{a^2}{b^2}(b^2)} = \frac{ab}{a^2} = \frac{b}{a}$ **4.** $\frac{\frac{x^2}{y^2}(y^2)}{\frac{x}{y}(y^2)} = \frac{x^2}{xy} = \frac{x}{y}$

5. $\frac{\frac{8a^2b}{9c}(9bc)}{\frac{4ac}{3b}(9bc)} = \frac{8a^2b^2}{12ac^2} = \frac{4a(2ab^2)}{4a(3c^2)} = \frac{2ab^2}{3c^2}$ **6.** $\frac{\frac{22r}{7s^2t}(21rs^2t)}{\frac{11s}{21rt}(21rs^2t)} = \frac{66r^2}{11s^3} = \frac{11(6r^2)}{11(s^3)} = \frac{6r^2}{s^3}$

7. $\frac{\frac{k-m}{m}(3m)}{\frac{k+m}{3m}(3m)} = \frac{3(k-m)}{k+m}$ **8.** $\frac{\frac{x+2}{2x}(2x)}{\frac{x-2}{x}(2x)} = \frac{x+2}{2(x-2)}$

9. $\frac{\left(\frac{x^2-16}{x}\right)x}{(x-4)x} = \frac{x^2-16}{x(x-4)} = \frac{(x+4)(x-4)}{x(x-4)} = \frac{x+4}{x}$

10. $\frac{(a^2-25)a}{\left(\frac{a+5}{a}\right)a} = \frac{a(a^2-25)}{a+5} = \frac{a(a+5)(a-5)}{a+5} = a(a-5)$

11. $\frac{(\frac{5}{6}+\frac{1}{2})30}{(\frac{1}{5}-4)30} = \frac{25+15}{6-120} = \frac{40}{-114} = -\frac{2(20)}{2(57)} = -\frac{20}{57}$

12. $\frac{(\frac{3}{4}+\frac{2}{5})20}{(\frac{7}{10}-2)20} = \frac{15+8}{14-40} = \frac{23}{-26} = -\frac{23}{26}$

13. $\frac{\frac{m+2}{m-2}[(m+2)(m-2)]}{\frac{2m+4}{(m+2)(m-2)}[(m+2)(m-2)]} = \frac{(m+2)^2}{2m+4} = \frac{(m+2)(m+2)}{2(m+2)} = \frac{m+2}{2}$

14. $\frac{\frac{ax+ab}{(x+b)(x-b)}[(x+b)(x-b)]}{\frac{x+b}{x-b}[(x+b)(x-b)]} = \frac{ax+ab}{(x+b)^2} = \frac{a(x+b)}{(x+b)(x+b)} = \frac{a}{x+b}$

15. $\frac{\left(\frac{m}{n}-1\right)n}{\left(\frac{m}{n}+1\right)n} = \frac{m-n}{m+n}$ **16.** $\frac{\left(\frac{t}{p}+1\right)p}{\left(\frac{t}{p}-1\right)p} = \frac{t+p}{t-p}$

B **17.** $\frac{\left(\frac{a^2+b^2}{ab}+2\right)2ab}{\left(\frac{a^2-b^2}{2ab}\right)2ab} = \frac{2a^2+2b^2+4ab}{a^2-b^2} = \frac{2(a^2+2ab+b^2)}{(a+b)(a-b)} = \frac{2(a+b)(a+b)}{(a+b)(a-b)} = \frac{2(a+b)}{a-b}$

18. $\frac{\left(\frac{x^2-9y^2}{6xy}\right)6xy}{\left(\frac{x^2+3y^2}{2xy}-2\right)6xy} = \frac{x^2-9y^2}{3x^2+9y^2-12xy} = \frac{(x+3y)(x-3y)}{3(x^2-4xy+3y^2)} = \frac{(x+3y)(x-3y)}{3(x-y)(x-3y)} = \frac{x+3y}{3(x-y)}$

19. $\frac{\left(\frac{p+q}{p^2+q^2}-\frac{1}{q}\right)[pq(p^2+q^2)]}{\left(\frac{p+q}{p^2+q^2}-\frac{1}{p}\right)[pq(p^2+q^2)]} = \frac{pq(p+q)-p(p^2+q^2)}{pq(p+q)-q(p^2+q^2)} = \frac{p^2q+pq^2-p^3-pq^2}{p^2q+pq^2-p^2q-q^3} =$

$\frac{p^2q-p^3}{pq^2-q^3} = \frac{p^2(q-p)}{(-1)(q^2)(q-p)} = -\frac{p^2}{q^2}$

20. $\frac{\left(\frac{r-s}{r^2+s^2}-\frac{1}{r}\right)[rs(r^2+s^2)]}{\left(\frac{r-s}{r^2+s^2}-\frac{1}{s}\right)[rs(r^2+s^2)]} = \frac{rs(r-s)-s(r^2+s^2)}{rs(r-s)-r(r^2+s^2)} = \frac{r^2s-rs^2-r^2s-s^3}{r^2s-rs^2-r^3-rs^2} =$

$\frac{-rs^2-s^3}{r^2s-2rs^2-r^3}$ *or* $\frac{(-1)(rs^2+s^3)}{(-1)(r^3-r^2s+2rs^2)} = \frac{rs^2+s^3}{r^3-r^2s+2rs^2}$

21. $\dfrac{\left(\dfrac{2}{a-1}+\dfrac{a-1}{a+1}\right)[(a+1)(a-1)]}{\left(1+\dfrac{a+1}{a-1}\right)[(a+1)(a-1)]} = \dfrac{2(a+1)+(a-1)^2}{(a+1)(a-1)+(a+1)^2} = \dfrac{2a+2+a^2-2a+1}{a^2-1+a^2+2a+1} = \dfrac{a^2+3}{2a^2+2a}$

22. $\dfrac{\left(\dfrac{b-3}{b+3}-1\right)[(b+3)(b-3)]}{\left(\dfrac{b+3}{b-3}+\dfrac{2}{b+3}\right)[(b+3)(b-3)]} = \dfrac{(b-3)^2-(b+3)(b-3)}{(b+3)^2+2(b-3)} = \dfrac{b^2-6b+9-b^2+9}{b^2+6b+9+2b-6} = \dfrac{-6b+18}{b^2+8b+3}$

C **23.** $1+\dfrac{(1)3}{(2+\frac{1}{3})3} = 1+\dfrac{3}{6+1} = 1+\frac{3}{7} = \frac{10}{7}$

24. $1+\dfrac{1}{a+\dfrac{(1)a}{\left(a+\frac{1}{a}\right)a}} = 1+\dfrac{1}{a+\frac{a}{a^2+1}} = 1+\dfrac{(1)(a^2+1)}{\left(a+\frac{a}{a^2+1}\right)(a^2+1)} = 1+\dfrac{a^2+1}{a^3+a+a} =$

$1+\dfrac{a^2+1}{a^3+2a} = \dfrac{a^3+2a}{a^3+2a}+\dfrac{a^2+1}{a^3+2a} = \dfrac{a^3+a^2+2a+1}{a^3+2a}$

25. $1+\dfrac{1}{1+\dfrac{(1)b}{\left(b+\frac{1}{b}\right)b}} = 1+\dfrac{1}{1+\frac{b}{b^2+1}} = 1+\dfrac{(1)(b^2+1)}{\left(1+\frac{b}{b^2+1}\right)(b^2+1)} = 1+\dfrac{b^2+1}{b^2+1+b} =$

$\dfrac{b^2+b+1}{b^2+b+1}+\dfrac{b^2+1}{b^2+b+1} = \dfrac{2b^2+b+2}{b^2+b+1}$

26. $1+\dfrac{\left(2+\frac{6}{x}\right)x}{(x+3)x} = \dfrac{1}{x}$; $1+\dfrac{2x+6}{x(x+3)} = \dfrac{1}{x}$; $1+\dfrac{2(x+3)}{x(x+3)} = \dfrac{1}{x}$; $1+\dfrac{2}{x} = \dfrac{1}{x}$; $\dfrac{1}{x} = -1$; $x = -1$. $\{-1\}$

27. $1+\dfrac{\left(4-\frac{8}{y}\right)y}{(y-2)y} = \dfrac{1}{y}$; $1+\dfrac{4y-8}{y(y-2)} = \dfrac{1}{y}$; $1+\dfrac{4(y-2)}{y(y-2)} = \dfrac{1}{y}$; $1+\dfrac{4}{y} = \dfrac{1}{y}$; $\dfrac{3}{y} = -1$; $\dfrac{y}{3} = -1$; $y = -3$. $\{-3\}$

Pages 307–308 · **WRITTEN EXERCISES**

A **1.** $2b-b=1$; $b=1$. $\{1\}$ **2.** $2k-k=3$; $k=3$. $\{3\}$

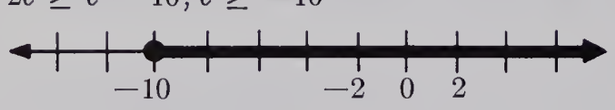
3. $12(\frac{1}{3}y+\frac{1}{4}y) > 12(\frac{7}{2})$; $4y+3y>42$; $7y>42$; $y>6$

4. $20(\frac{1}{4}x+\frac{1}{5}x) < 20(\frac{9}{2})$; $5x+4x<90$; $9x<90$; $x<10$

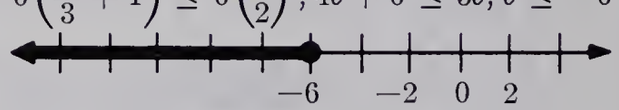
5. $15(\frac{2}{3}g-\frac{3}{5}g) = 15(1)$; $10g-9g=15$; $g=15$. $\{15\}$

6. $21(\frac{3}{7}g-\frac{1}{3}g) = 21(4)$; $9g-7g=84$; $2g=84$; $g=42$. $\{42\}$

7. $2c \geq c-10$; $c \geq -10$



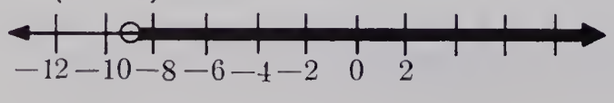
8. $6\left(\dfrac{2c}{3}+1\right) \leq 6\left(\dfrac{c}{2}\right)$; $4c+6 \leq 3c$; $c \leq -6$



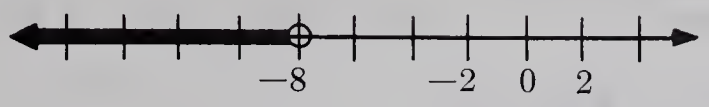
9. $3(3z-\frac{7}{3}z) = 3(1)$; $9z-7z=3$; $2z=3$; $z=\frac{3}{2}$. $\{\frac{3}{2}\}$

10. $5s-14s=9$; $-9s=9$; $s=-1$. $\{-1\}$

11. $15\left(\dfrac{n}{5}-\dfrac{n}{3}\right) < 15(\frac{6}{5})$; $3n-5n<18$; $-2n<18$; $n>-9$



12. $28\left(\frac{x}{7}-\frac{x}{4}\right) > 28(\frac{6}{7})$; $4x - 7x > 24$; $-3x > 24$; $x < -8$



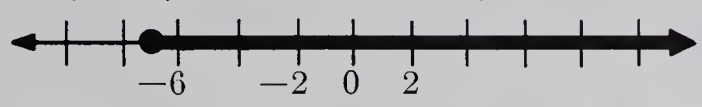
13. $6(\frac{4}{3}h) = 6(\frac{1}{2}h + 45)$; $8h = 3h + 270$; $5h = 270$; $h = 54$. $\{54\}$

14. $12(\frac{3}{4}n) = 12(26 - \frac{1}{3}n)$; $9n = 312 - 4n$; $13n = 312$; $n = 24$. $\{24\}$

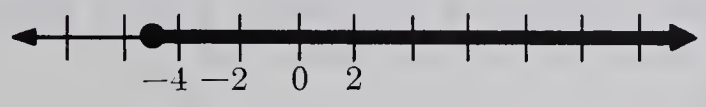
B **15.** $6\left(\frac{3m-5}{2}-\frac{m}{3}\right) = 6(8)$; $9m - 15 - 2m = 48$; $7m = 63$; $m = 9$. $\{9\}$

16. $14\left(\frac{3m}{2}+\frac{8-4m}{7}\right) = 14(3)$; $21m + 16 - 8m = 42$; $13m = 26$; $m = 2$. $\{2\}$

17. $10\left(\frac{n+3}{2}\right) \geq 10\left(\frac{n-8}{5}+1\right)$; $5n + 15 \geq 2n - 16 + 10$; $3n \geq -21$; $n \geq -7$



18. $20\left(\frac{n-3}{4}\right) \leq 20\left(\frac{2n-5}{5}+1\right)$; $5n - 15 \leq 8n - 20 + 20$; $-3n \leq 15$; $n \geq -5$



19. $8[\frac{3}{4}(2r + 5) - \frac{5}{8}(3r - 1)] = 8(1)$; $6(2r + 5) - 5(3r - 1) = 8$; $12r + 30 - 15r + 5 = 8$; $-3r + 35 = 8$; $-3r = -27$; $r = 9$. $\{9\}$

20. $9[\frac{2}{9}(s - 3) - 3] = 9[\frac{1}{3}(2s + 5)]$; $2(s - 3) - 27 = 3(2s + 5)$; $2s - 6 - 27 = 6s + 15$; $2s - 33 = 6s + 15$; $-4s = 48$; $s = -12$. $\{-12\}$

21. $24\left[\frac{n+3}{8}-\frac{n-2}{6}\right] = 24[1]$; $3(n + 3) - 4(n - 2) = 24$; $3n + 9 - 4n + 8 = 24$; $-n + 17 = 24$; $-n = 7$; $n = -7$. $\{-7\}$

22. $24\left[\frac{n+5}{12}-\frac{n+3}{8}\right] = 24[1]$; $2(n + 5) - 3(n + 3) = 24$; $2n + 10 - 3n - 9 = 24$; $-n + 1 = 24$; $-n = 23$; $n = -23$. $\{-23\}$

23. $100[.04x + .06(20{,}000 - x)] = 100(960)$; $4x + 6(20{,}000 - x) = 96{,}000$; $4x + 120{,}000 - 6x = 96{,}000$; $-2x = -24{,}000$; $x = 12{,}000$. $\{12{,}000\}$

24. $100[.03k + .05(1000 - k)] = 100(34)$; $3k + 5(1000 - k) = 3400$; $3k + 5000 - 5k = 3400$; $-2k = -1600$; $k = 800$. $\{800\}$

C **25.** $8\left(\frac{2w+5}{4}-\frac{10w+13}{8}\right) = 8(2w + 1)$; $4w + 10 - 10w - 13 = 16w + 8$; $-6w - 3 = 16w + 8$; $-22w = 11$; $w = -\frac{1}{2}$. $\{-\frac{1}{2}\}$

26. $6\left(\frac{3w+5}{3}\right) = 6\left(3w + 2 - \frac{3w-4}{6}\right)$; $6w + 10 = 18w + 12 - 3w + 4$; $6w + 10 = 15w + 16$; $-9w = 6$; $w = -\frac{6}{9} = -\frac{2}{3}$. $\{-\frac{2}{3}\}$

27. $100[.15(y - 5) - .02(4y - 3) + .9] = 100(0)$; $15(y - 5) - 2(4y - 3) + 90 = 0$; $15y - 75 - 8y + 6 + 90 = 0$; $7y + 21 = 0$; $7y = -21$; $y = -3$. $\{-3\}$

28. $100[.08(4y + 5) - .03(2y - 3)] = 100(.36)$; $8(4y + 5) - 3(2y - 3) = 36$; $32y + 40 - 6y + 9 = 36$; $26y + 49 = 36$; $26y = -13$; $y = -\frac{13}{26} = -\frac{1}{2}$. $\{-\frac{1}{2}\}$

29. $18\left(\frac{2t+3}{9}-\frac{4t-1}{6}-\frac{9-8t}{18}\right) = 18(0)$; $4t + 6 - 12t + 3 - 9 + 8t = 0$; $0 = 0$. {directed numbers}

30. $60\left(\frac{1-6t}{10}-\frac{2t+3}{6}-\frac{13+6t}{4}\right) = 60(0)$; $6 - 36t - 20t - 30 - 195 - 90t = 0$; $-146t - 219 = 0$; $-146t = 219$; $t = -\frac{219}{146} = -\frac{3(73)}{2(73)} = -\frac{3}{2}$. $\left\{-\frac{3}{2}\right\}$

Pages 309–310 · **PROBLEMS**

A **1.** $i = (500)(.04)(2) = \$40$, *Ans.*

2. $i = (600)(.05)(3) = \$90$, *Ans.*

3. $20 = (p)(.04)(\frac{1}{2})$; $.02p = 20$; $2p = 2000$; $p = \$1000$, *Ans.*

4. $360 = (p)(.06)(5)$; $.3p = 360$; $3p = 3600$; $p = \$1200$, *Ans.*
5. $a = 1200[1 + (.06)(2)]$; $a = 1200(1.12)$; $a = \$1344$, *Ans.*
6. $a = 750[1 + (.03)(4)]$; $a = 750(1.12)$; $a = \$840$, *Ans.*
7. $.04b + .06b = 40$; $.1b = 40$; $b = \$400$, *Ans.*
8. $.03b + .04b = 189$; $.07b = 189$; $7b = 18{,}900$; $b = \$2700$, *Ans.*
9. $b(.07) + 2b(.03) = 390$; $.07b + .06b = 390$; $.13b = 390$; $13b = 39{,}000$; $b = \$3000$, $2b = \$6000$, *Ans.*
10. $b(.09) + 7b(.03) = 750$; $.09b + .21b = 750$; $.3b = 750$; $3b = 7500$; $b = \$2500$, $7b = \$17{,}500$, *Ans.*
11. $b(.05) + (b + 2000)(.05) = 250$; $.05b + .05b + 100 = 250$; $.1b = 150$; $b = \$1500$, $b + 2000 = \$3500$, *Ans.*
12. $b(.02) + (b + 100)(.02) = 12$; $.02b + .02b + 2 = 12$; $.04b = 10$; $4b = 1000$; $b = \$250$, $b + 100 = \$350$, *Ans.*
13. $b(.04) + (10{,}000 - b)(.06) = 500$; $.04b + 600 - .06b = 500$; $-.02b = -100$; $2b = 10{,}000$; $b = \$5000$, $10{,}000 - b = \$5000$, *Ans.*
14. $b(.05) + (b - 800)(.035) = 210$; $.05b + .035b - 28 = 210$; $.085b = 238$; $85b = 238{,}000$; $b = \$2800$, $b - 800 = \$2000$, *Ans.*

B

15. $35{,}000(.08) + b(.10) = 5000$; $2800 + .10b = 5000$; $.1b = 2200$; $b = \$22{,}000$, *Ans.*
16. $7000(.04) + b(.06) = 1000$; $280 + .06b = 1000$; $.06b = 720$; $6b = 72{,}000$; $b = \$12{,}000$, *Ans.*
17. $(b + 500).05 = b(.04) + 33$; $.05b + 25 = .04b + 33$; $.01b = 8$; $b = \$800$, $b + 500 = \$1300$, *Ans.*
18. $b(.035) = (6000 - b)(.06) + 67.50$; $.035b = 360 - .06b + 67.50$; $.095b = 427.50$; $95b = 427{,}500$; $b = \$4500$, $6000 - b = \$1500$, *Ans.*
19. Let b = no. of dollars in the fund; $.30b$ = no. of dollars invested at 5%; $.70b$ = no. of dollars invested at 4%; $(.05)(.30b) + .04(.70b) = 860$; $.015b + .028b = 860$; $.043b = 860$; $43b = 860{,}000$; $b = 20{,}000$; $.30b = \$6{,}000$, $.70b = \$14{,}000$, *Ans.*
20. Let b = no. of dollars in estate; $.60b$ = no. of dollars left to wife; $.40b$ = no. of dollars left to son; $(.045)(.40b) = 5400$; $.018b = 5400$; $18b = 5{,}400{,}000$; $b = 300{,}000$; $(.60)(300{,}000) = 180{,}000$; $(.055)(180{,}000) = \$9900$, *Ans.*

Page 311 · PROBLEMS

A

1. $2(.30) = (2 + x)(.20)$; $.6 = .4 + .2x$; $.2x = .2$; $x = 1$ qt, *Ans.*
2. $16(.25) = (16 + x)(.10)$; $4 = 1.6 + .1x$; $.1x = 2.4$; $x = 24$ oz, *Ans.*
3. $48(.10) = (48 + x)(.06)$; $4.80 = 2.88 + .06x$; $.06x = 1.92$; $6x = 192$; $x = 32$ lb, *Ans.*
4. $100(.04) = (100 - x)(.05)$; $4 = 5 - .05x$; $.05x = 1$; $5x = 100$; $x = 20$ lb, *Ans.*
5. $200(.03) = (200 - x)(.036)$; $6 = 7.2 - .036x$; $.036x = 1.2$; $36x = 1200$; $x = 33\frac{1}{3}$ lb, *Ans.*
6. $200(.048) = (200 + x)(.032)$; $9.6 = 6.4 + .032x$; $.032x = 3.2$; $32x = 3200$; $x = 100$ lb, *Ans.*
7. $1(.20) = (1 + x)(.10)$; $.2 = .1 + .1x$; $.1x = .1$; $x = 1$ pt, *Ans.*
8. $1(.1) = (1 + x)(.02)$; $.1 = .02 + .02x$; $.02x = .08$; $2x = 8$; $x = 4$ qt, *Ans.*

B

9. Amount of original solution $= 200 + 40 = 240$ g; $(240 + x)(.50) = 40 + x$; $120 + .5x = 40 + x$; $.5x = 80$; $5x = 800$; $x = 160$ g, *Ans.*
10. Amount of original solution $= 45 + 3 = 48$ lb; $(48 + x)(.10) = 3 + x$; $4.8 + .1x = 3 + x$; $.9x = 1.8$; $9x = 18$; $x = 2$ lb, *Ans.*
11. $(24 + x)(.90) = 14 + x$; $21.60 + .90x = 14 + x$; $.1x = 7.6$; $x = 76$ oz, *Ans.*
12. $(185 + x)(.925) = 185$; $171.125 + .925x = 185$; $.925x = 13.875$; $925x = 13{,}875$; $x = 15$ oz, *Ans.*
13. Let $.50x$ = amount of pure acid from one solution; $.80(300 - x)$ = amount of pure acid from other solution; $.72(300)$ = amount of pure acid in final solution; $.50x + .80(300 - x) = .72(300)$; $50x + 80(300 - x) = 72(300)$; $50x + 24{,}000 - 80x = 21{,}600$; $-30x = -2{,}400$; $x = 80$ oz, $300 - x = 220$ oz, *Ans.*
14. $(.15)(16) - (.15)x + .90x = (.65)(16)$; $2.4 + .75x = 10.4$; $.75x = 8$; $75x = 800$; $x = \frac{32}{3} = 10\frac{2}{3}$ qt, *Ans.*

Pages 313–314 · WRITTEN EXERCISES

A

1. $(s^2 - 4) \cdot \dfrac{12}{s + 2} = (s^2 - 4) \cdot \dfrac{4}{s - 2}$; $12(s - 2) = 4(s + 2)$; $12s - 24 = 4s + 8$; $8s = 32$; $s = 4$. $\{4\}$

2. $(t-3)(t+2)\cdot\frac{7}{t-3}=(t-3)(t+2)\cdot\frac{2}{t+2}$; $7(t+2)=2(t-3)$; $7t+14=2t-6$; $5t=-20$; $t=-4$. $\{-4\}$

3. $2x\cdot\frac{5}{x}=2x\cdot\frac{x-3}{2}$; $10=x(x-3)$; $10=x^2-3x$; $x^2-3x-10=0$; $(x-5)(x+2)=0$; $x-5=0$ or $x+2=0$; $x=5$ or $x=-2$. $\{5,-2\}$

4. $2(z-1)\cdot\frac{6}{z-1}=2(z-1)\cdot\frac{z}{2}$; $12=z(z-1)$; $12=z^2-z$; $z^2-z-12=0$; $(z-4)(z+3)=0$; $z-4=0$ or $z+3=0$; $z=4$ or $z=-3$. $\{4,-3\}$

5. $6(t-4)\cdot\frac{t}{t-4}=6(t-4)\cdot\frac{t+4}{6}$; $6t=(t-4)(t+4)$; $6t=t^2-16$; $t^2-6t-16=0$; $(t-8)(t+2)=0$; $t-8=0$ or $t+2=0$; $t=8$ or $t=-2$. $\{8,-2\}$

6. $v(v+6)\cdot\frac{5}{v+6}=v(v+6)\cdot\frac{v-6}{v}$; $5v=(v+6)(v-6)$; $5v=v^2-36$; $v^2-5v-36=0$; $(v-9)(v+4)=0$; $v-9=0$ or $v+4=0$; $v=9$ or $v=-4$. $\{9,-4\}$

7. $8x\left(\frac{3}{4x}+\frac{1}{x}\right)=8x(\frac{7}{8})$; $6+8=7x$; $7x=14$; $x=2$. $\{2\}$

8. $9x\left(\frac{2}{3x}+\frac{1}{x}\right)=9x(\frac{5}{9})$; $6+9=5x$; $5x=15$; $x=3$. $\{3\}$

9. $12n\left(\frac{n+5}{2n}-\frac{7}{3n}\right)=12n(\frac{5}{12})$; $6n+30-28=5n$; $6n+2=5n$; $n=-2$. $\{-2\}$

10. $6n\left(\frac{4}{3n}-\frac{n+4}{6n}\right)=6n(2)$; $8-n-4=12n$; $4-n=12n$; $13n=4$; $n=\frac{4}{13}$. $\{\frac{4}{13}\}$

11. $a-\frac{2}{a-3}=\frac{-1(a-1)}{-1(3-a)}$; $a-\frac{2}{a-3}=\frac{1-a}{a-3}$; $(a-3)\left(a-\frac{2}{a-3}\right)=(a-3)\left(\frac{1-a}{a-3}\right)$; $a(a-3)-2=1-a$; $a^2-3a-2=1-a$; $a^2-2a-3=0$; $(a-3)(a+1)=0$; $a-3=0$ or $a+1=0$; $a=3$ (reject, gives denominator 0); $a=-1$. $\{-1\}$

12. $c-\frac{-1(c)}{-1(1-c)}=\frac{2-c}{c-1}$; $c+\frac{c}{c-1}=\frac{2-c}{c-1}$; $(c-1)\left(c+\frac{c}{c-1}\right)=(c-1)\left(\frac{2-c}{c-1}\right)$; $(c-1)c+c=2-c$; $c^2-c+c=2-c$; $c^2+c-2=0$; $(c+2)(c-1)=0$; $c+2=0$ or $c-1=0$; $c=-2$ or $c=1$ (reject, gives denominator 0). $\{-2\}$

13. $3t(3t-2)\left(\frac{4}{3t-2}+\frac{7}{3t}-\frac{1}{t}\right)=3t(3t-2)(0)$; $12t+7(3t-2)-3(3t-2)=0$; $12t+21t-14-9t+6=0$; $24t-8=0$; $24t=8$; $t=\frac{1}{3}$. $\{\frac{1}{3}\}$

14. $4k(3k-1)\left(\frac{3}{4k}+\frac{4}{3k-1}-\frac{2}{k}\right)=4k(3k-1)(0)$; $3(3k-1)+16k-8(3k-1)=0$; $9k-3+16k-24k+8=0$; $k+5=0$; $k=-5$. $\{-5\}$

B **15.** $(x+2)(x-2)\left(\frac{2x}{x+2}-2\right)=(x+2)(x-2)\left(\frac{x-8}{x-2}\right)$; $2x(x-2)-2(x+2)(x-2)=(x+2)(x-8)$; $2x^2-4x-2(x^2-4)=x^2-6x-16$; $2x^2-4x-2x^2+8=x^2-6x-16$; $-4x+8=x^2-6x-16$; $x^2-2x-24=0$; $(x-6)(x+4)=0$; $x-6=0$ or $x+4=0$; $x=6$ or $x=-4$. $\{6,-4\}$

16. $(y+3)(y-3)\left(\frac{4y}{y-3}-3\right)=(y+3)(y-3)\left(\frac{3y-1}{y+3}\right)$; $4y(y+3)-3(y+3)(y-3)=(y-3)(3y-1)$; $4y^2+12y-3(y^2-9)=3y^2-10y+3$; $4y^2+12y-3y^2+27=3y^2-10y+3$; $y^2+12y+27=3y^2-10y+3$; $2y^2-22y-24=0$; $y^2-11y-12=0$; $(y-12)(y+1)=0$; $y-12=0$ or $y+1=0$; $y=12$ or $y=-1$. $\{12,-1\}$

17. $5c-1+6=0$; $5c+5=0$; $5c=-5$; $c=-1$ (reject, gives denominator 0). $\emptyset$

18. $4k-1=3$; $4k=4$; $k=1$ (reject, gives denominator 0). $\emptyset$

19. $\frac{p+2}{3(p-2)}-\frac{2}{3(p+2)}+\frac{7}{9}=0$; $9(p-2)(p+2)\left[\frac{p+2}{3(p-2)}-\frac{2}{3(p+2)}+\frac{7}{9}\right]=9(p-2)(p+2)[0]$; $3(p+2)^2-6(p-2)+7(p-2)(p+2)=0$; $3(p^2+4p+4)-6p+12+7(p^2-4)=0$; $3p^2+12p+12-6p+12+7p^2-28=0$; $10p^2+6p-4=0$; $5p^2+3p-2=0$; $(5p-2)(p+1)=0$; $5p-2=0$ or $p+1=0$; $5p=2$ or $p=-1$; $p=\frac{2}{5}$ or $p=-1$. $\{\frac{2}{5},-1\}$

20. $15(n+6)\left(\frac{3n-2}{15}-\frac{16-3n}{n+6}\right)=15(n+6)\left(\frac{n+3}{5}\right)$; $(n+6)(3n-2)-15(16-3n)=3(n+6)(n+3)$; $3n^2+16n-12-240+45n=3(n^2+9n+18)$; $3n^2+61n-252=3n^2+27n+54$; $34n=306$; $n=9$. $\{9\}$

C **21.** $\frac{3}{2(m-3)}-\frac{2m-3}{(2m+1)(m-3)}+\frac{1}{2(2m+1)}=0$;
$2(m-3)(2m+1)\left(\frac{3}{2(m-3)}-\frac{2m-3}{(2m+1)(m-3)}+\frac{1}{2(2m+1)}\right)=2(m-3)(2m+1)(0)$;
$3(2m+1)-2(2m-3)+(m-3)=0$; $6m+3-4m+6+m-3=0$; $3m+6=0$; $3m=-6$; $m=-2$. $\{-2\}$

22. $\frac{2}{3(m+4)}-\frac{1}{3(3m-1)}-\frac{m-2}{(3m-1)(m+4)}=0$;
$3(m+4)(3m-1)\left(\frac{2}{3(m+4)}-\frac{1}{3(3m-1)}-\frac{m-2}{(3m-1)(m+4}\right)=3(m+4)(3m-1)(0)$;
$2(3m-1)-(m+4)-3(m-2)=0$; $6m-2-m-4-3m+6=0$; $2m=0$; $m=0$. $\{0\}$

23. $\frac{2t+1}{2t-3}-1=\frac{t-4}{2t+3}-\frac{7t}{(-1)(4t^2-9)}$;
$(2t-3)(2t+3)\left(\frac{2t+1}{2t-3}-1\right)=(2t-3)(2t+3)\left(\frac{t-4}{2t+3}+\frac{7t}{(2t+3)(2t-3)}\right)$;
$(2t+3)(2t+1)-(2t-3)(2t+3)=(2t-3)(t-4)+(7t)$;
$4t^2+8t+3-4t^2+9=2t^2-11t+12+7t$; $8t+12=2t^2-4t+12$; $2t^2-12t=0$;
$t^2-6t=0$; $t(t-6)=0$; $t=0$ or $t-6=0$; $t=0$ or $t=6$. $\{0,6\}$

24. $\frac{2t+3}{t-1}-4-\frac{2}{t+3}=\frac{5-6t}{(t-1)(t+3)}$;
$(t-1)(t+3)\left(\frac{2t+3}{t-1}-4-\frac{2}{t+3}\right)=(t-1)(t+3)\left(\frac{5-6t}{(t-1)(t+3)}\right)$;
$(t+3)(2t+3)-4(t-1)(t+3)-2(t-1)=5-6t$;
$2t^2+9t+9-4(t^2+2t-3)-2t+2=5-6t$; $2t^2+9t+9-4t^2-8t+12-2t+2=5-6t$;
$-2t^2-t+23=5-6t$; $2t^2-5t-18=0$; $(2t-9)(t+2)=0$; $2t-9=0$ or $t+2=0$;
$2t=9$ or $t=-2$; $t=\frac{9}{2}$ or $t=-2$; $\{\frac{9}{2},-2\}$

25. $\frac{1}{3s-2}+\frac{1}{3s+4}=\frac{6}{(3s-2)(3s+4)}$;
$(3s-2)(3s+4)\left(\frac{1}{3s-2}+\frac{1}{3s+4}\right)=(3s-2)(3s+4)\left(\frac{6}{(3s-2)(3s+4)}\right)$;
$3s+4+3s-2=6$; $6s+2=6$; $6s=4$; $s=\frac{2}{3}$ (reject, gives denominator 0). $\emptyset$

26. $\frac{1}{2d+3}+\frac{1}{2d+1}=\frac{2}{(2d+3)(2d+1)}$;
$(2d+3)(2d+1)\left(\frac{1}{2d+3}+\frac{1}{2d+1}\right)=(2d+3)(2d+1)\left(\frac{2}{(2d+3)(2d+1)}\right)$;
$2d+1+2d+3=2$; $4d+4=2$; $4d=-2$; $d=-\frac{1}{2}$ (reject, gives denominator 0). $\emptyset$

Pages 315–316 · **PROBLEMS**

A **1.** Let x = no. of hours needed to fill tank together;
$1=\frac{1}{5}x+\frac{1}{3}x$; $15(1)=15(\frac{1}{5}x+\frac{1}{3}x)$; $15=3x+5x$; $8x=15$; $x=1\frac{7}{8}$ hr, *Ans.*

2. Let x = no. of hours needed for presses to do job together;
$1=\frac{1}{4}x+\frac{1}{2}x$; $4(1)=4(\frac{1}{4}x+\frac{1}{2}x)$; $4=x+2x$; $3x=4$; $x=1\frac{1}{3}$ hr, *Ans.*

3. Let x = no. of hours needed for slower to do job alone;
$1=\frac{1}{10}(6)+\frac{1}{x}(6)$; $10x(1)=10x\left(\frac{6}{10}+\frac{6}{x}\right)$; $10x=6x+60$; $4x=60$; $x=15$ hr, *Ans.*

4. Let x = no. of minutes needed for second device to do job alone;
$1=\frac{1}{12}(4)+\frac{1}{x}(4)$; $12x(1)=12x\left(\frac{4}{12}+\frac{4}{x}\right)$; $12x=4x+48$; $8x=48$; $x=6$ min, *Ans.*

5. Let x = no. of hours needed for lathes to do job together;
$1=\frac{1}{3}x+\frac{1}{3}x+\frac{1}{4}x$; $12(1)=12\left(\frac{x}{3}+\frac{x}{3}+\frac{x}{4}\right)$; $12=4x+4x+3x$; $11x=12$; $x=1\frac{1}{11}$ hr, *Ans.*

6. Let x = no. of hours needed for third incinerator to do job alone; $1 = \frac{1}{20}(6) + \frac{1}{20}(6) + \frac{1}{x}(6)$;
$20x(1) = 20x\left(\frac{6}{20} + \frac{6}{20} + \frac{6}{x}\right)$; $20x = 6x + 6x + 120$; $8x = 120$; $x = 15$ hr, *Ans.*

7. Let $x =$ no. of hours needed for larger bulldozer to do job alone; then $2x$ = no. of hours needed for smaller bulldozer to do job alone; $1 = \frac{1}{x}(1\frac{1}{2}) + \frac{1}{2x}(1\frac{1}{2})$; $4x(1) = 4x\left(\frac{3}{2x} + \frac{3}{4x}\right)$; $4x = 6 + 3$; $4x = 9$; $x = 2\frac{1}{4}$ hr, *Ans.*

8. Let x = no. of minutes needed for larger pump to do job alone; then $2x$ = no. of min needed for smaller pump to do job alone. $1 = \frac{1}{x}(16) + \frac{1}{2x}(16)$; $2x(1) = 2x\left(\frac{16}{x} + \frac{16}{2x}\right)$; $2x = 32 + 16$; $2x = 48$; $x = 24$ min, *Ans.*

B **9.** Let x = no. of hours needed to fill pool; $1 = \frac{1}{3}x + \frac{1}{6}x - \frac{1}{4}x$;
$12(1) = 12\left(\frac{x}{3} + \frac{x}{6} - \frac{x}{4}\right)$; $12 = 4x + 2x - 3x$; $3x = 12$; $x = 4$ hr, *Ans.*

10. Let x = no. of minutes needed to fill tub; $1 = \frac{1}{40}x + \frac{1}{30}x - \frac{1}{20}x$;
$120(1) = 120\left(\frac{x}{40} + \frac{x}{30} - \frac{x}{20}\right)$; $120 = 3x + 4x - 6x$; $x = 120$ min or 2 hr, *Ans.*

11. Let x = no. of hours needed for painter to do job alone;
$1 = \frac{1}{8}(1) + \frac{1}{8}(3) + \frac{1}{x}(3)$; $8x(1) = 8x\left(\frac{1}{8} + \frac{3}{8} + \frac{3}{x}\right)$; $8x = x + 3x + 24$; $4x = 24$; $x = 6$ hr, *Ans.*

12. Let x = no. of hours slower machine works; $x + 1$ = no. of hours faster machine works;
$8200 = 1200(x + 1) + 900x$; $8200 = 1200x + 1200 + 900x$; $2100x = 7000$; $x = 3\frac{1}{3}$ hr;
$x + 1 = 4\frac{1}{3}$ hr, *Ans.*

C **13.** One man does $\frac{1}{8}$ of job in 3 hr; one man does $\frac{1}{24}$ of job in 1 hr; one boy does $\frac{1}{15}$ of job in 5 hr; one boy does $\frac{1}{75}$ of job in 1 hr; let x = no. of hours needed for 3 men and 25 boys to do job together;
$1 = 3 \cdot \frac{1}{24}x + 25 \cdot \frac{1}{75}x$; $1 = \frac{x}{8} + \frac{x}{3}$; $24(1) = 24\left(\frac{x}{8} + \frac{x}{3}\right)$; $24 = 3x + 8x$; $11x = 24$; $x = 2\frac{2}{11}$ hr, *Ans.*

14. One man does $\frac{1}{60}$ of job in 72 days; one man does $\frac{1}{4320}$ of job in 1 day; let x = no. of days the 30 men work; $1 = 50 \cdot \frac{1}{4320}(72) + 30 \cdot \frac{1}{4320}x$; $1 = \frac{3600}{4320} + \frac{30x}{4320}$; $4320(1) = 4320\left(\frac{3600}{4320} + \frac{30x}{4320}\right)$;
$4320 = 3600 + 30x$; $30x = 720$; $x = 24$ days, *Ans.*

15. Let x = no. of minutes needed for each son to do job alone;
$1 = \frac{1}{x - 10}(12) + 2 \cdot \frac{1}{x}(12)$; $1 = \frac{12}{x - 10} + \frac{24}{x}$; $x(x - 10)(1) = x(x - 10)\left(\frac{12}{x - 10} + \frac{24}{x}\right)$;
$x^2 - 10x = 12x + 24x - 240$; $x^2 - 10x = 36x - 240$; $x^2 - 46x + 240 = 0$;
$(x - 6)(x - 40) = 0$; $x - 6 = 0$ or $x - 40 = 0$; $x = 6$ (reject) or $x = 40$ min. 20 min, *Ans.*

16. Let x = no. of hours needed for third man to do job alone; then $x + 6$ = no. of hours needed for second man to do job alone; and $2(x + 6)$ = no. of hours needed for first (slowest) man to do job alone;
$1 = \frac{1}{2(x + 6)}(6) + \frac{1}{x + 6}(6) + \frac{1}{x}(6)$; $1 = \frac{3}{x + 6} + \frac{6}{x + 6} + \frac{6}{x}$;
$x(x + 6)(1) = x(x + 6)\left(\frac{3}{x + 6} + \frac{6}{x + 6} + \frac{6}{x}\right)$; $x^2 + 6x = 3x + 6x + 6x + 36$;
$x^2 + 6x = 15x + 36$; $x^2 - 9x - 36 = 0$; $(x - 12)(x + 3) = 0$; $x - 12 = 0$ or $x + 3 = 0$;
$x = 12$ hr or $x = -3$ (reject); $x + 6 = 18$ hr; $2(x + 6) = 36$ hr, *Ans.*

Pages 318–319 · PROBLEMS

A Let s = rate in still water, and let c = rate of current. Then $s + c$ = rate with current, and $s - c$ = rate against current. Use the rule $d = rt$; that is, $t = d/r$.

1. $\frac{9}{s + 6} = \frac{3}{s - 6}$; $9s - 54 = 3s + 18$; $6s = 72$; $s = 12$ mph, *Ans.*

2. $\frac{25}{s + 5} = \frac{15}{s - 5}$; $25s - 125 = 15s + 75$; $10s = 200$; $s = 20$ mph, *Ans.*

3. $\frac{12}{4 + c} = \frac{4}{4 - c}$; $48 - 12c = 16 + 4c$; $16c = 32$; $c = 2$ mph, *Ans.*

4. $\frac{800}{180 + c} = \frac{640}{180 - c}$; $800(180 - c) = 640(180 + c)$; $5(180 - c) = 4(180 + c)$;
$900 - 5c = 720 + 4c$; $9c = 180$; $c = 20$ mph, *Ans.*

5. $\frac{36}{s+c}=\frac{24}{s-c}$; $36s-36c=24s+24c$; $12s=60c$; $s=5c$; $s=c+12$; $c+12=5c$; $4c=12$; $c=3$ mph, *Ans.*

6. $\frac{15}{s-c}+\frac{15}{s+c}=4$; $s=4c$; $\frac{15}{3c}+\frac{15}{5c}=4$; $\frac{5}{c}+\frac{3}{c}=4$; $\frac{8}{c}=4$; $8=4c$; $c=2$ mph, *Ans.*

7. $\frac{4}{s-c}=4\left(\frac{4}{s+c}\right)$; $s=5$; $\frac{4}{5-c}=\frac{16}{5+c}$; $20+4c=80-16c$; $20c=60$; $c=3$ mph, *Ans.*

8. $\frac{.25}{s-c}=3\left(\frac{.25}{s+c}\right)$; $s=2$; $\frac{.25}{2-c}=\frac{.75}{2+c}$; $25(2+c)=75(2-c)$; $50+25c=150-75c$; $100c=100$; $c=1$ mph, *Ans.*

B **9.** Let s = rate in air; $15s$ = rate in metal; $\frac{8250}{15s}+7=\frac{8250}{s}$; $15s\left(\frac{8250}{15s}+7\right)=15s\left(\frac{8250}{s}\right)$; $8250+105s=123{,}750$; $105s=115{,}500$; $s=1100$; $15s=16{,}500$ ft/sec, *Ans.*

10. Let b = no. of ohms of smaller resistance; $a=b+2$; $R=\frac{2}{3}b$; $\frac{1}{\frac{2}{3}b}=\frac{1}{b+2}+\frac{1}{b}$; $\frac{3}{2b}=\frac{1}{b+2}+\frac{1}{b}$; $2b(b+2)\left(\frac{3}{2b}\right)=2b(b+2)\left(\frac{1}{b+2}+\frac{1}{b}\right)$; $3(b+2)=2b+2(b+2)$; $3b+6=2b+2b+4$; $3b+6=4b+4$; $b=2$; $b+2=4$ ohms, *Ans.*

11. $\frac{36}{s-c}=\frac{36}{s+c}+1$; $c=3$; $\frac{36}{s-3}=\frac{36}{s+3}+1$; $(s-3)(s+3)\left(\frac{36}{s-3}\right)=(s-3)(s+3)\left(\frac{36}{s+3}+1\right)$; $36(s+3)=36(s-3)+(s-3)(s+3)$; $36s+108=36s-108+s^2-9$; $s^2-225=0$; $(s+15)(s-15)=0$; $s+15=0$ or $s-15=0$; $s=-15$ (reject); $s=15$ mph, *Ans.*

12. $\frac{1}{s_1-c}+\frac{10}{s_2+c}=1\frac{2}{3}$; $s_1=4$; $s_2=12$; $\frac{1}{4-c}+\frac{10}{12+c}=\frac{5}{3}$; $3(4-c)(12+c)\left(\frac{1}{4-c}+\frac{10}{12+c}\right)=3(4-c)(12+c)\left(\frac{5}{3}\right)$; $3(12+c)+30(4-c)=5(4-c)(12+c)$; $36+3c+120-30c=5(48-8c-c^2)$; $156-27c=240-40c-5c^2$; $5c^2+13c-84=0$; $(5c+28)(c-3)=0$; $5c+28=0$ or $c-3=0$; $5c=-28$ or $c=3$; $c=-\frac{28}{5}$ (reject) or $c=3$ mph, *Ans.*

13. Let s = rate of first rocket; $s+40$ = rate of second rocket; $\frac{6400}{s}=\frac{6400}{s+40}+8$; $s(s+40)\left(\frac{6400}{s}\right)=s(s+40)\left(\frac{6400}{s+40}+8\right)$; $6400(s+40)=6400s+8s(s+40)$; $6400s+256{,}000=6400s+8s^2+320s$; $8s^2+320s-256{,}000=0$; $s^2+40s-32{,}000=0$; $(s+200)(s-160)=0$; $s+200=0$ or $s-160=0$; $s=-200$ (reject) or $s=160$; $s+40=200$ km/min, *Ans.*

14. Let x = no. of cartons first machine handles in a minute; $x+5$ = no. of cartons second machine handles in a minute; $\frac{5500}{x}=\frac{5500}{x+5}+10$; $x(x+5)\left(\frac{5500}{x}\right)=x(x+5)\left(\frac{5500}{x+5}+10\right)$; $5500(x+5)=5500x+10x(x+5)$; $5500x+27{,}500=5500x+10x^2+50x$; $10x^2+50x-27{,}500=0$; $x^2+5x-2750=0$; $(x+55)(x-50)=0$; $x+55=0$ or $x-50=0$; $x=-55$ (reject) or $x=50$; $x+5=55$, *Ans.*

Pages 322–323 · CHAPTER TEST

1. $\frac{v^2}{v^2-16}$ is not defined when $v^2-16=0$; $v^2-16=0$; $(v+4)(v-4)=0$; $v+4=0$ or $v-4=0$. $v=-4$ or $v=4$. $\{-4, 4\}$

2. $\frac{7(3r-5s)}{7(r+2s)}=\frac{3r-5s}{r+2s}$

3. $\frac{(z+3)(z-3)}{(z+5)(z-3)}=\frac{z+3}{z+5}$

4. $\frac{2-t}{12-8t+t^2}=\frac{2-t}{(2-t)(6-t)}=\frac{1}{6-t}$ *or* $\frac{-(1)(t-2)}{t^2-8t+12}=\frac{-(t-2)}{(t-2)(t-6)}=\frac{-1}{t-6}=-\frac{1}{t-6}$

5. 2 : 3

6. Let $3x$ = length of one part in meters; $5x$ = length of second part in meters; $3x+5x=16$; $8x=16$; $x=2$; $3x=6$ m, $5x=10$ m, *Ans.*

7. Let r = rate of discount; b = \$3.75; p = percentage or markdown = \$3.75 − \$3.30 = \$.45; $\frac{p}{b} = r$; $\frac{.45}{3.75} = r$; $r = 12\%$, *Ans.*

8. $\frac{2 \cdot 5p^2}{2 \cdot 3q^2t^2} = \frac{5p^2}{3q^2t^2}$

9. $\frac{(a+5)(a-5)}{4a \cdot 2b} \cdot \frac{2b}{a+5} = \frac{(a-5)(a+5)(2b)}{4a(a+5)(2b)} = \frac{a-5}{4a}$

10. $\frac{(m-n)(m-n)}{(m+n)} \cdot \frac{m+n}{m(m-n)} = \frac{(m-n)(m-n)(m+n)}{m(m-n)(m+n)} = \frac{m-n}{m}$

11. $\frac{1}{2y^2-50} \cdot \frac{2y-10}{y+5} = \frac{1}{2(y+5)(y-5)} \cdot \frac{2(y-5)}{y+5} = \frac{(1)\ 2(y-5)}{(y+5)^2 \cdot 2(y-5)} = \frac{1}{(y+5)^2}$

12. $\frac{6a^2b^2}{a^2-b^2} \cdot \frac{a^2-2ab+b^2}{3a^2b^2} = \frac{2 \cdot 3a^2b^2}{(a+b)(a-b)} \cdot \frac{(a-b)(a-b)}{3a^2b^2} = \frac{2 \cdot 3a^2b^2(a-b)(a-b)}{(a+b)(3a^2b^2)(a-b)} = \frac{2(a-b)}{a+b}$

13. $\frac{c+d}{c-d} \cdot \frac{c-3d}{c+3d} \cdot \frac{c^2+2cd-3d^2}{c^2-2cd-3d^2} = \frac{c+d}{c-d} \cdot \frac{c-3d}{c+3d} \cdot \frac{(c+3d)(c-d)}{(c-3d)(c+d)}$
$= \frac{(c+d)(c-3d)(c+3d)(c-d)}{(c+d)(c-3d)(c+3d)(c-d)} = 1$

14. $\frac{4y+4-(y-5)}{y+3} = \frac{4y+4-y+5}{y+3} = \frac{3y+9}{y+3} = \frac{3(y+3)}{y+3} = 3$

15. $\frac{(m+p)2m}{3mp \cdot 2m} - \frac{(2m+3)p}{6m^2 \cdot p} = \frac{2m^2+2mp-2mp-3p}{6m^2p} = \frac{2m^2-3p}{6m^2p}$

16. $\frac{(h+2)(h+2)}{(h+3)(h+2)} + \frac{(h+3)(h+3)}{(h+2)(h+3)} = \frac{(h+2)(h+2)+(h+3)(h+3)}{(h+2)(h+3)}$
$= \frac{h^2+4h+4+h^2+6h+9}{(h+2)(h+3)} = \frac{2h^2+10h+13}{(h+2)(h+3)}$

17. $\frac{x+1}{x-1} + \frac{3x^2-1}{x^2-1} = \frac{x+1}{x-1} + \frac{3x^2-1}{(x+1)(x-1)} = \frac{(x+1)(x+1)}{(x-1)(x+1)} + \frac{3x^2-1}{(x+1)(x-1)}$
$= \frac{(x+1)(x+1)+3x^2-1}{(x+1)(x-1)} = \frac{x^2+2x+1+3x^2-1}{(x+1)(x-1)} = \frac{4x^2+2x}{(x+1)(x-1)}$

18. $\frac{(w+v)(2w+v)-(wv+v^2)}{2w+v} = \frac{2w^2+3wv+v^2-wv-v^2}{2w+v} = \frac{2w^2+2wv}{2w+v}$

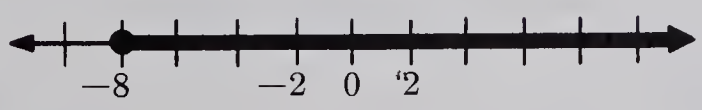
19. $\frac{4x^2}{4x} + \frac{8x}{4x} + \frac{5}{4x} = x + 2 + \frac{5}{4x}$

20. a. $\frac{\frac{y}{t^2}(t^2)}{\frac{y^2}{t}(t^2)} = \frac{y}{y^2t} = \frac{1}{yt}$

b. $\frac{\frac{2(x-a)}{(x+a)^2}}{\frac{x-a}{x+a}} = \frac{\frac{2(x-a)}{(x+a)^2}(x+a)^2}{\frac{x-a}{x+a}(x+a)^2} = \frac{2(x-a)}{(x-a)(x+a)} = \frac{2}{x+a}$

c. $\frac{\frac{5k-n}{k}}{\frac{25k^2-n^2}{k^2}} = \frac{\frac{5k-n}{k}(k^2)}{\left(\frac{25k^2-n^2}{k^2}\right)k^2} = \frac{(5k-n)k}{25k^2-n^2} = \frac{(5k-n)k}{(5k-n)(5k+n)} = \frac{k}{5k+n}$

21. $6\left(\frac{x-1}{3}\right) \le 6\left(\frac{x}{2}+1\right)$; $2(x-1) \le 3x+6$; $2x-2 \le 3x+6$; $-8 \le x$; $x \ge -8$



22. Let x = the number of dollars invested at 3%; 1000 − x = the number of dollars invested at 5%; $.03x$ = yearly income from investment at 3%; $.05(1000-x)$ = yearly income from investment at 5%; $.03x = .05(1000-x)+6$; $3x = 5(1000-x)+600$; $3x = 5000-5x+600$; $8x = 5600$; x = \$700, *Ans.*

23. Let x = number of pounds of salt to be added; $75+x$ = number of pounds of final solution; 4% of 75 = amount of salt in original solution; 10% of $(75+x)$ = amount of salt in final solution; $.04(75)+x = .10(75+x)$; $4(75)+100x = 10(75+x)$; $300+100x = 750+10x$; $90x = 450$; x = 5 lb, *Ans.*

24. $\frac{x^2-7}{(x+4)(x-4)} - \frac{x-3}{x-4} = 0$; $x^2-7-(x-3)(x+4) = 0$;
$x^2-7-(x^2+x-12) = 0$; $x^2-7-x^2-x+12 = 0$; $-x+5 = 0$; $x = 5$. {5}

25. Let x = number of minutes needed for George; $\frac{1}{x}$ = rate of George; $\frac{1}{24}$ = rate of Henry; 15 = number of minutes needed if the boys work together; $\frac{1}{x} \cdot 15 + \frac{1}{24} \cdot 15 = 1$; $\frac{15}{x} + \frac{5}{8} = 1$; $15 \cdot 8 + 5x = 8x$; $120 = 3x$; $x = 40$ min, *Ans.*

26. Let x = the rate of the boat in still water; $x + 2$ = the rate downstream; 48 = distance downstream; $x - 2$ = the rate upstream; 32 = distance upstream; $\frac{48}{x+2} = \frac{32}{x-2}$; $48(x - 2) = 32(x + 2)$; $48x - 96 = 32x + 64$; $16x = 160$; $x = 10$ mph, *Ans.*

Pages 323–326 · CHAPTER REVIEW

1. zero

2. $y^2 - 9 = 0$; $(y - 3)(y + 3) = 0$; $y - 3 = 0$ or $y + 3 = 0$; $y = 3$ or $y = -3$; $y = \pm 3$, *Ans.*

3. $x - 2 = 0$; $x = 2$, *Ans.*

4. $x = 0$, *Ans.*

5. $a^2 - 2ab + b^2 = 0$; $(a - b)(a - b) = 0$; $a - b = 0$; $a = b$, *Ans.*

6. $\frac{2xy \cdot 2x}{2xy \cdot 3y} = \frac{2x}{3y}$

7. $\frac{1(a-b)}{(a+b)(a-b)} = \frac{1}{a+b}$

8. $\frac{4x+3y}{4xy}$ is in lowest terms.

9. $a - 2$

10. $\frac{-1(a-2)}{(a-2)} = -1$

11. $\frac{(x-1)(x-1)}{-1(x-1)} = \frac{x-1}{-1} = 1 - x$

12. $\frac{(2+y)(2-y)}{(2y+3)(y-2)} = \frac{-1(2+y)(y-2)}{(2y+3)(y-2)} = \frac{-1(2+y)}{2y+3}$ or $\frac{-2-y}{2y+3}$, if $y \notin \{2, -\frac{3}{2}\}$

13. same unit

14. 3 : 1

15. 50 cm : 3 M = 50 cm : 300 cm = 1 : 6

16. $(8)^2 : (12 \cdot 6) = 64 : 72 = 8 : 9$

17. 17

18. $\frac{3}{4} \cdot 100\% = 75\%$; 75, *Ans.*

19. $\frac{3}{2} \cdot 100\% = 150\%$; 150, *Ans.*

20. .28($8000) = $2240, *Ans.*

21. $\frac{500 - 249}{500} \cdot 100\% = \frac{241}{500} \cdot 100\% = \frac{241}{5}\% = 48.2\%$, *Ans.*

22. $\frac{xu}{tv}$

23. $\frac{10a^3x^2zt}{21b^2t^2zt} = \frac{10a^3x^2}{21b^2t^2}$

24. $\frac{25}{m^2 - n^2}$

25. $\frac{(r+t)(r-t)}{t(r-t)} = \frac{r+t}{t}$

26. $\frac{6z^2 + 5z - 6}{6z^2 - 5z - 6}$

27. $\frac{(a+1)(a+1)}{(a-1)(a-1)} \cdot \frac{2(a-1)}{5(a+1)} = \frac{2(a+1)(a+1)(a-1)}{5(a-1)(a+1)(a-1)} = \frac{2(a+1)}{5(a-1)} = \frac{2a+2}{5a-5}$

28. $\frac{r^2t^2}{4} \cdot \frac{12}{r^2s^2t^2} = \frac{3 \cdot 4r^2t^2}{s^2 \cdot 4r^2t^2} = \frac{3}{s^2}$

29. $\frac{a^2-4}{2a+2} \cdot \frac{a+1}{a-2} = \frac{(a+2)(a-2)}{2(a+1)} \cdot \frac{a+1}{a-2} = \frac{(a+2)(a-2)(a+1)}{2(a-2)(a+1)} = \frac{a+2}{2}$

30. $\frac{x^2+10x+25}{x^2-12x+36} \cdot \frac{x-6}{x+5} = \frac{(x+5)(x+5)}{(x-6)(x-6)} \cdot \frac{x-6}{x+5} = \frac{(x+5)(x+5)(x-6)}{(x-6)(x+5)(x-6)} = \frac{x+5}{x-6}$

31. $\frac{(r+3k)(r-3k)}{9r} \cdot \frac{k}{3r+k} = \frac{k(r-3k)(r+3k)}{9r(3r+k)}$

32. $\frac{9s+3s}{4t} = \frac{12s}{4t} = \frac{3s \cdot 4}{t \cdot 4} = \frac{3s}{t}$

33. $\frac{6g+3}{2g+1} = \frac{3(2g+1)}{2g+1} = 3$

34. $\frac{b^2+1-2b}{b^2-1} = \frac{b^2-2b+1}{b^2-1} = \frac{(b-1)(b-1)}{(b+1)(b-1)} = \frac{b-1}{b+1}$

35. $\frac{4(a+2b)}{(2a+b)(a+2b)} - \frac{2(2a+b)}{(a+2b)(2a+b)} = \frac{4a+8b-2(2a+b)}{(2a+b)(a+2b)} = \frac{4a+8b-4a-2b}{(2a+b)(a+2b)}$
$= \frac{6b}{(2a+b)(a+2b)}$

36. $\frac{s(r+s)}{(r-s)(r+s)} + \frac{r}{r-s} = \frac{s}{r-s} + \frac{r}{r-s} = \frac{s+r}{r-s} = \frac{r+s}{r-s}$

37. $\dfrac{(x-1)(x+1)}{3(x+1)} - \dfrac{(x-1)(x+1)}{4(x+1)} = \dfrac{x-1}{3} - \dfrac{(x-1)}{4} = \dfrac{(x-1)4}{3\cdot 4} - \dfrac{(x-1)3}{4\cdot 3}$
$= \dfrac{4(x-1)-3(x-1)}{12} = \dfrac{4x-4-3x+3}{12} = \dfrac{x-1}{12}$

38. $\dfrac{2n-3}{n-5} + \dfrac{-2}{n-5} = \dfrac{2n-3-2}{n-5} = \dfrac{2n-5}{n-5}$

39. $\dfrac{a}{a+2} + \dfrac{6}{a-2} - \dfrac{8}{(a+2)(a-2)} = \dfrac{a(a-2)+6(a+2)-8}{(a+2)(a-2)} = \dfrac{a^2-2a+6a+12-8}{(a+2)(a-2)}$
$= \dfrac{a^2+4a+4}{(a+2)(a-2)} = \dfrac{(a+2)(a+2)}{(a-2)(a+2)} = \dfrac{a+2}{a-2}$

40. $\dfrac{y+1}{y+2} - \dfrac{2}{y+4} + \dfrac{4}{(y+2)(y+4)} = \dfrac{(y+1)(y+4)-2(y+2)+4}{(y+2)(y+4)}$
$= \dfrac{y^2+5y+4-2y-4+4}{(y+2)(y+4)} = \dfrac{y^2+3y+4}{(y+2)(y+4)}$

41. $\dfrac{a^2}{1}\cdot\dfrac{a}{a} - \dfrac{(a^3-1)}{a} = \dfrac{a^3-(a^3-1)}{a} = \dfrac{a^3-a^3+1}{a} = \dfrac{1}{a}$

42. $\dfrac{(x+y)(x+y)}{x+y} - \dfrac{2xy}{x+y} = \dfrac{(x+y)(x+y)-2xy}{x+y} = \dfrac{x^2+2xy+y^2-2xy}{x+y} = \dfrac{x^2+y^2}{x+y}$

43. $\dfrac{8m^2}{8m} - \dfrac{16m}{8m} + \dfrac{2}{8m} = m - 2 + \dfrac{1}{4m}$

44.
$$\begin{array}{r|l} w+2 & w^2+5w+7 \\ & w^2+2w \\ \hline & 3w+7 \\ & 3w+6 \\ \hline & 1 \end{array} \qquad w+3 : w+3+\frac{1}{w+2}$$

45. $\dfrac{\frac{n}{x}(x^2)}{\frac{n^2}{x^2}(x^2)} = \dfrac{nx}{n^2} = \dfrac{x\cdot n}{n\cdot n} = \dfrac{x}{n}$

46. $\dfrac{\left(9-\frac{a^2}{h^2}\right)h^2}{\left(3+\frac{a}{h}\right)h^2} = \dfrac{9h^2-a^2}{3h^2+ah} = \dfrac{(3h-a)(3h+a)}{h(3h+a)} = \dfrac{3h-a}{h}$

47. $5z + 3z = 45\cdot 15 - z$; $8z = 675 - z$; $9z = 675$; $z = 75$. {75}

48. $6(\frac{3}{2}n) \geq 6(\frac{4}{3}n + 9)$; $9n \geq 8n + 54$; $n \geq 54$. {nos. ≥ 54}

49. Let x = the number of dollars in each investment; $.03x$ = the income from the investment at 3%; $.05x$ = the income from the investment at 5%; $.03x + .05x = 120$; $.08x = 120$; $8x = 12{,}000$; $x = \$1500$, *Ans.*

50. Let x = the number of dollars invested at 5%; $.05x$ = the income from the investment at 5%; $.04(2500)$ = the income from the \$2500 investment at 4%; $.05x + .04(2500) = 325$; $5x + 4(2500) = 32{,}500$; $5x + 10{,}000 = 32{,}500$; $5x = 22{,}500$; $x = \$4500$, *Ans.*

51. Let x = the number of dollars invested at 5%; then $2x$ = the number of dollars invested at 6%; $.05x$ = the income from the investment at 5%; $.06(2x)$ = the income from the investment at 6%; $.05x + .06(2x) = 289$; $5x + 12x = 28{,}900$; $17x = 28{,}900$; $x = \$1700$, $2x = \$3400$, *Ans.*

52. Let x = the number of dollars invested at $3\frac{1}{2}$%; then $7000 - x$ = the number of dollars invested at 7%; $.035x$ = the income from the investment at $3\frac{1}{2}$%; $.07(7000 - x)$ = the income from the investment at 7%; $.035x + .07(7000 - x) = 350$; $35x + 70(7000 - x) = 350{,}000$; $35x + 490{,}000 - 70x = 350{,}000$; $-35x = -140{,}000$; $x = \$4000$, $7000 - x = \$3000$, *Ans.*

53. 100 **54.** 3 **55.** 3, 3 **56.** 4 **57.** $100 - x$

58. Let x = the number of milliliters of water added; $100 + x$ = the number of milliliters of 20% acid solution; 100 = the number of milliliters of acid in the original and final solutions; $.20(100 + x)$ = the number of milliliters of acid in the 20% acid solution; $100 = .20(100 + x)$; $1000 = 2(100 + x)$; $1000 = 200 + 2x$; $800 = 2x$; $x = 400$ milliliters, *Ans.*

59. $6m\left[\dfrac{3}{m} + \dfrac{(2m-3)}{6m} - \dfrac{m+1}{2m}\right] = 6m[0]$; $18 + (2m - 3) - 3(m + 1) = 0$;
$18 + 2m - 3 - 3m - 3 = 0$; $-m = -12$; $m = 12$. {12}

60. $(n^2-4)\dfrac{(n+5)}{4-n^2}+(n^2-4)\dfrac{(3)}{n-2}=(n^2-4)\dfrac{(1)}{n+2}$; $-1(n+5)+3(n+2)=n-2$; $-n-5+3n+6=n-2$; $n=-3$. $\{-3\}$

61. $\frac{1}{40}$, $\frac{1}{40}$ **62.** $\dfrac{1}{x}$ **63.** $\frac{24}{40}$ *or* $\frac{3}{5}$, $\dfrac{24}{x}$

64. Let x = the number of minutes needed by Arno; $\dfrac{1}{x}$ = rate of Arno; $\frac{1}{30}$ = rate of Joe; 18 = the number of minutes needed for both boys. $\dfrac{1}{x}\cdot 18+\dfrac{1}{30}\cdot 18=1$; $\dfrac{18}{x}+\dfrac{3}{5}=1$; $90+3x=5x$; $90=2x$; $x=45$ min, *Ans.*

65. $s+4$, $s-4$

66. Let s = rate in miles per hour of the plane in still air; $s+25$ = rate with the wind; $s-25$ = rate against the wind; $\dfrac{600}{s+25}$ = time with the wind; $\dfrac{450}{s-25}$ = time against the wind; $\dfrac{600}{s+25}=\dfrac{450}{s-25}$; $600(s-25)=450(s+25)$; $600s-15{,}000=450s+11{,}250$; $150s=26{,}250$; $s=175$ mph, *Ans.*

Pages 326–328 · CUMULATIVE REVIEW 1–8

1. multiplication (The product of two squares is also a square.)

2. $-3<y<3$ **3.** $\{-4\}$ (If $r=2$, $r-2=0$.) **4.** $a<b$

5. sometimes **6.** $-2a+k$ **7.** $10r^2$

8. $\{3,-4\}$ **9.** $a<0$ **10.** $2t=0$; $t=0$

11. $5s(t-3)\neq 0$; $s\neq 0$ and $t-3\neq 0$; $s\neq 0$ and $t\neq 3$. $\{t\neq 3\}$

12. $6s^2t^3<0$; since s^2 is never less than zero, t^3 must be less than zero which means $t<0$.

13. $\dfrac{t+1}{t+1}=1$; $t+1\neq 0$; $\therefore t\neq -1$ **14.** False; $\dfrac{34-m}{17}=\dfrac{34}{17}-\dfrac{m}{17}=2-\dfrac{m}{17}$

15. False; $5(6x^2+x)-2(4x^2-3x)=30x^2+5x-8x^2+6x=22x^2+11x$

16. True; $y^4+7y^3+10y^2=y^2(y^2+7y+10)=y^2(y+2)(y+5)$

17. False; if $A=x^3+5$ and $B=-x^3+x$, $A+B=x+5$, so that $A+B$ has degree one in this case

18. False; $2k^2+k=1$; $2k^2+k-1=0$; $(2k-1)(k+1)=0$; $2k-1=0$ or $k+1=0$; $k=-1$; $2k=1$; $k=\frac{1}{2}$, not an integer.

19. $(2+y)(2-y)^2$ **20.** If $z\notin\{-3,3\}$, $\dfrac{27(z-3)^2}{z^2-9}=\dfrac{27(z-3)(z-3)}{(z+3)(z-3)}=\dfrac{27(z-3)}{z+3}$

21. $\dfrac{12}{12+16}=\dfrac{12}{28}=3:7$

22.
$$\begin{array}{r|l} x-2 & x^2+2x+4 \\ \hline & x^3\qquad\quad -8 \\ & \underline{x^3-2x^2} \\ & \quad 2x^2 \\ & \quad \underline{2x^2-4x} \\ & \qquad\quad 4x-8 \\ & \qquad\quad \underline{4x-8} \end{array}$$

23. $\dfrac{\frac{7xy}{11ab}}{\frac{11a^2}{7x}}=\dfrac{7xy}{11ab}\div\dfrac{11a^2}{7x}=\dfrac{7xy}{11ab}\cdot\dfrac{7x}{11a^2}=\dfrac{49x^2y}{121a^3b}$ **24.** 1 and -1

25. $\dfrac{21(2)(n-3)}{7}-\dfrac{21(2-n)}{3}=\dfrac{3n-2}{21}(21)$; $6(n-3)-7(2-n)=3n-2$; $6n-18-14+7n=3n-2$; $10n=30$; $n=3$. $\{3\}$

26. $(3x-4)(x+1)-(x+4)(x-4)=2(x-4)(x+1)$; $3x^2-x-4-(x^2-16)=(2x-8)(x+1)$; $3x^2-x-4-x^2+16=2x^2-6x-8$; $5x=-20$; $x=-4$. $\{-4\}$

27. $(t-3)^2+2(t-3)=3$; $t^2-6t+9+2t-6=3$; $t^2-4t=0$; $t(t-4)=0$; $t=0$ or $t=-4$; $t=0$ or $t=4$. $\{0,4\}$

28. $y+1-y=1$; $1=1$. {all directed numbers except 0 and -1}

29. $7k^2(k^2-9)=7k^2(k-3)(k+3)$

30. $(12 - h)(5 + h)$ *or* $-1(h - 12)(h + 5)$

31. $4c(3r - 2s) - 5d(3r - 2s) = (4c - 5d)(3r - 2s)$

32. $zt(4z^2 - 20zt + 25t^2) = zt(2z - 5t)^2$

33. $[7(n - k) - 5][7(n - k) + 5] = (7n - 7k - 5)(7n - 7k + 5)$

34. $\frac{1}{6}(6x^2 - x - 1) = \frac{1}{6}(3x + 1)(2x - 1)$

35. $\dfrac{1}{y+2} - \left\{\dfrac{3y}{y^2-4} + \dfrac{2y}{2-y}\right\} = \dfrac{1}{y+2} - \dfrac{3y}{y^2-4} - \dfrac{2y}{2-y} = \dfrac{1}{y+2} - \dfrac{3y}{y^2-4} + \dfrac{2y}{y-2}$

$= \dfrac{1(y-2)}{(y+2)(y-2)} - \dfrac{3y}{(y+2)(y-2)} + \dfrac{2y(y+2)}{(y-2)(y+2)} = \dfrac{y - 2 - 3y + 2y^2 + 4y}{y^2 - 4}$

$= \dfrac{2y^2 + 2y - 2}{y^2 - 4} = \dfrac{2(y^2 + y - 1)}{y^2 - 4}$

36. $\dfrac{1}{(x-3)^2} + \dfrac{12}{(x+3)^2} - \dfrac{8}{x^2-9} = \dfrac{1}{(x-3)^2} + \dfrac{12}{(x+3)^2} - \dfrac{8}{(x+3)(x-3)}$

$= \dfrac{1(x+3)^2}{(x-3)^2(x+3)^2} + \dfrac{12(x-3)^2}{(x+3)^2(x-3)^2} - \dfrac{8(x+3)(x-3)}{(x+3)^2(x-3)^2}$

$= \dfrac{(x+3)^2 + 12(x-3)^2 - 8(x+3)(x-3)}{(x+3)^2(x-3)^2} = \dfrac{x^2 + 6x + 9 + 12(x^2 - 6x + 9) - 8(x^2 - 9)}{(x+3)^2(x-3)^2}$

$= \dfrac{x^2 + 6x + 9 + 12x^2 - 72x + 108 - 8x^2 + 72}{(x+3)^2(x-3)^2} = \dfrac{5x^2 - 66x + 189}{(x+3)^2(x-3)^2}$ *or* $\dfrac{(5x - 21)(x - 9)}{(x+3)^2(x-3)^2}$

37. $\dfrac{4a^2 - 9b^2}{a^2 - 2ab + b^2} \div \dfrac{2a^2 + 5ab + 3b^2}{a^2 - b^2} = \dfrac{4a^2 - 9b^2}{a^2 - 2ab + b^2} \cdot \dfrac{a^2 - b^2}{2a^2 + 5ab + 3b^2}$

$= \dfrac{(2a + 3b)(2a - 3b)}{(a - b)(a - b)} \cdot \dfrac{(a + b)(a - b)}{(2a + 3b)(a + b)} = \dfrac{(2a - 3b)(2a + 3b)(a + b)(a - b)}{(a - b)(2a + 3b)(a + b)(a - b)} = \dfrac{2a - 3b}{a - b}$

38. $\left(\dfrac{2q}{k}\right)^2 \cdot \dfrac{48k^2s}{(5tq^2)^2} \div \left(\dfrac{6qs}{5t^2}\right)^3 = \dfrac{4q^2}{k^2} \cdot \dfrac{48k^2s}{25t^2q^4} \div \dfrac{216q^3s^3}{125t^6} = \dfrac{4q^2}{k^2} \cdot \dfrac{48k^2s}{25t^2q^4} \cdot \dfrac{125t^6}{216q^3s^3}$

$= \dfrac{192 \cdot 125k^2q^2st^6}{216 \cdot 25k^2q^7s^3t^2} = \dfrac{8 \cdot 5t^4 \cdot 24 \cdot 25k^2q^2st^2}{9 \cdot q^5 \cdot s^2 \cdot 24 \cdot 25k^2q^2 \cdot s \cdot t^2} = \dfrac{40t^4}{9q^5s^2}$

39. $\dfrac{\dfrac{1}{t-v} - \dfrac{1}{t+v}}{\dfrac{2}{t^2 - v^2}} = \dfrac{\left(\dfrac{1}{t-v} - \dfrac{1}{t+v}\right)(t^2 - v^2)}{\dfrac{2}{t^2 - v^2}(t^2 - v^2)} = \dfrac{(t + v) - (t - v)}{2} = \dfrac{t + v - t + v}{2} = \dfrac{2v}{2} = v$

40. Let x = the list price in dollars; $.75x$ = selling price at a discount of 25% of list price; $.75x - 150$ = profit; $.75x - 150 = .20(150)$; $.75x - 150 = 30$; $.75x = 180$; $x = \$240$, *Ans.*

41. Let x = the radius in cm of the larger circle; then $x - 3$ = the radius in cm of the smaller circle; πx^2 = area of larger circle; $\pi(x - 3)^2$ = area of smaller circle; $\pi x^2 - \pi(x - 3)^2 = 36\pi$; $x^2 - (x^2 - 6x + 9) = 36$; $x^2 - x^2 + 6x - 9 = 36$; $6x = 45$; $x = 7\frac{1}{2}$ cm, *Ans.*

42. Let x = number of lb of 75% silver; then $20 - x$ = number of lb of 70% silver; $.75x$ = number of lb of silver in the 75% pure silver; $.70(20 - x)$ = number of lb of silver in the 70% pure silver; $.73(20)$ = number of lb of silver in the mixture; $.75x + .70(20 - x) = .73(20)$; $75x + 70(20 - x) = 73(20)$; $75x + 1400 - 70x = 1460$; $5x = 60$; $x = 12$; $20 - x = 8$. 12 lb 75% pure, 8 lb 70% pure, *Ans.*

43. Let x = the number; then $\frac{1}{x}$ = its reciprocal; $\frac{1}{2}\left(x + \frac{1}{x}\right) = \frac{17}{15}$; $\frac{x}{2} + \frac{1}{2x} = \frac{17}{15}$; $15x^2 + 15 = 34x$; $15x^2 - 34x + 15 = 0$; $(5x - 3)(3x - 5) = 0$; $x = \frac{3}{5}$ or $x = \frac{5}{3}$; $\frac{3}{5} < \frac{5}{3}$. $\frac{3}{5}$, *Ans.*

44. Let x = average speed in miles per hour for the last 600 miles.

	r	t	d
1st 600 miles	$x + 100$	$\dfrac{600}{x + 100}$	600
2nd 600 miles	x	$\dfrac{600}{x}$	600

(continued on p. 96)

$\frac{600}{x+100} + \frac{600}{x} = 3\frac{1}{2}$; $600(2x) + 600(2)(x+100) = 7x(x+100)$;
$1200x + 1200x + 120{,}000 = 7x^2 + 700x$; $7x^2 - 1700x - 120{,}000 = 0$; $(7x+400)(x-300) = 0$;
$x = \frac{-400}{7}$ (reject); $x = 300$ mph, *Ans.*

45. Let x = the number of minutes it takes to fill the tank when all three pipes are opened; $\frac{x}{40}$ = part that the first tank fills in x minutes; $\frac{x}{60}$ = part that the second tank fills in x minutes; $\frac{x}{30}$ = part that the third tank empties in x minutes; $\frac{x}{40} + \frac{x}{60} - \frac{x}{30} = 1$; $3x + 2x - 4x = 120$; $x = 120$ min, *Ans.*

Page 328 · EXTRA FOR EXPERTS

1. a. $5 + 2 + 0 + 8 = 15$; $15 \div 3 = 5$; $5208 \div 3$ is an integer
b. $08 \div 4 = 2$; $5208 \div 4$ is an integer
c. $8 \neq 0$; $8 \neq 5$; $5208 \div 5$ is not an integer
d. $5 + 2 + 0 + 8 = 15$; $15 \div 9$ is not an integer; $5208 \div 9$ is not an integer
e. $809 - 147 = 662$; $662 \div 7$ is not an integer; $147{,}809 \div 7$ is not an integer
f. $9 + 8 + 1 + 9 = 27$; $27 \div 9 = 3$; $9819 \div 9$ is an integer
g. $2 - 9 + 8 - 7 + 6 - 3 = -3$; $-3 \div 11$ is not an integer; $367{,}892 \div 11$ is not an integer
h. $810 - 147 = 663$; $663 \div 13 = 51$; $147{,}810 \div 13$ is an integer

2. a. $N = a_0 + 10a_1 + 10^2a_2 + 10^3a_3 + \cdots + 10^{n-1}a_{n-1} + 10^na_n$
$N = \underbrace{a_0}_{?} + \underbrace{10(a_1 + 10a_2 + 10^2a_3 + \cdots + 10^{n-2}a_{n-1} + 10^{n-1}a_n}_{\text{a multiple of 2}}$
N is divisible by 2 if, and only if, a_0 is divisible by 2.

b. From part (a), $N = \underbrace{a_0}_{?} + \underbrace{10(a_1 + 10a_2 + 10^2a_3 + \cdots + 10^{n-2}a_{n-1} + 10^{n-1}a_n)}_{\text{a multiple of 5}}$
N is divisible by 5 if, and only if, a_0 is divisible by 5, i.e., if a_0 is 0 or 5.

c. $N = a_0 + 10a_1 + 10^2a_2 + 10^3a_3 + 10^4a_4 + 10^5a_5 + \cdots$
$N = a_0 + 10a_1 + 100a_2 + (1001 - 1)a_3 + (10{,}010 - 10)a_4 + (100{,}100 - 100)a_5 + (999{,}999 + 1)a_6 + (9{,}999{,}990 + 10)a_7 + (99{,}999{,}900 + 100)a_8 + \cdots$
$N = (a_0 + 10a_1 + 100a_2) + (7 \cdot 143 - 1)a_3 + (7 \cdot 1430 - 10)a_4 + (7 \cdot 14{,}300 - 100)a_5 + (7 \cdot 142857 + 1)a_6 + (7 \cdot 1{,}428{,}570 + 10)a_7 + (7 \cdot 14{,}285{,}700 + 100)a_8 + \cdots$
$N = (a_0 + 10a_1 + 100a_2) + \underbrace{7(143a_3 + 1430a_4 + 14{,}300a_5)}_{s} - (a_3 + 10a_4 + 100a_5) + \underbrace{7(142{,}857a_6 + 1{,}428{,}570a_7 + 14{,}285{,}700a_8)}_{t} + (a_6 + 10a_7 + 100a_8) + \cdots$
$N = \underbrace{(a_0 + 10a_1 + 100a_2) - (a_3 + 10a_4 + 100a_5) + (a_6 + 10a_7 + 100a_8) - \cdots}_{?} + \underbrace{7(s + t + \cdots)}_{\text{a multiple of 7}}$
N is divisible by 7 if, and only if, $[(a_0 + 10a_1 + 100a_2) - (a_3 + 10a_4 + 100a_5) + (a_6 + 10a_7 - 100a_8) - \cdots]$ is divisible by 7.

d. $N = a_0 + 10a_1 + 10^2a_2 + 10^3a_3 + 10^4a_4 + 10^5a_5 + \cdots$
$N = a_0 + 10a_1 + 100a_2 + 1000a_3 + 10{,}000a_4 + 100{,}000a_5 + \cdots$
$N = a_0 + (11 - 1)a_1 + (99 + 1)a_2 + (1001 - 1)a_3 + (9999 + 1)a_4 + (100{,}001 - 1)a_5 + \cdots$
$N = (a_0 - a_1 + a_2 - a_3 + a_4 - a_5 + \cdots) + (11a_1 + 99a_2 + 1001a_3 + 9999a_4 + 100{,}001a_5 + \cdots$
$N = \underbrace{(a_0 - a_1 + a_2 - a_3 + a_4 - a_5 + \cdots)}_{?} + \underbrace{11(a_1 + 9a_2 + 91a_3 + 909a_4 + 9091a_5 + \cdots)}_{\text{a multiple of 11}}$
N is divisible by 11 if, and only if, $a_0 - a_1 + a_2 - a_3 + a_4 - a_5 + \cdots$ is divisible by 11.

e. $N = a_0 + 10a_1 + 10^2a_2 + 10^3a_3 + 10^4a_4 + 10^5a_5 + 10^6a_6 + 10^7a_7 + 10^8a_8 + \cdots$
$N = a_0 + 10a_1 + 100a_2 + (1001 - 1)a_3 + (10{,}010 - 10)a_4 + (100{,}100 - 100)a_5 + (999{,}999 + 1)a_6 + (9{,}999{,}990 + 10)a_7 + (99{,}999{,}900 + 100)a_8 + \cdots$

$$N = (a_0 + 10a_1 + 100a_2) + (13 \cdot 77 - 1)a_3 + (13 \cdot 770 - 10)a_4 + (13 \cdot 7700 - 100)a_5 + (13 \cdot 76{,}923 + 1)a_6 + (13 \cdot 769{,}230 + 10)a_7 + (13 \cdot 7{,}692{,}300 + 100)a_8 + \cdots$$

$$N = (a_0 + 10a_1 + 100a_2) + 13\underbrace{(77a_3 + 770a_4 + 7700a_5)}_{s} - (a_3 + 10a_4 + 100a_5) + 13\underbrace{(76{,}923a_6 + 769{,}230a_7 + 7{,}692{,}300a_8)}_{t} + (a_6 + 10a_7 + 100a_8) + \cdots$$

$$N = \underbrace{(a_0 + 10a_1 + 100a_2) - (a_3 + 10a_4 + 100a_5) + (a_6 + 10a_7 + 100a_8) - \cdots}_{?} + \underbrace{13(s + t + \cdots)}_{\text{a multiple of 13}}$$

N is divisible by 13 if, and only if, $[(a_0 + 10a_1 + 100a_2) - (a_3 + 10a_4 + 100a_5) + (a_6 + 10a_7 + 100a_8) - \cdots]$ is divisible by 13.

3. If N is divisible by prime number P_1, then $N = a \cdot P_1$ for a an integer; if N or aP_1 is also divisible by prime number P_2, then since P_1 is prime, a is divisible by P_2, or $a = b \cdot P_2$ for b an integer. Therefore $N = aP_1 = (bP_2)P_1 = b(P_1P_2)$, for b an integer, i.e., N is divisible by (P_1P_2). A number is divisible by 6 if it is divisible by 2 and by 3, i.e., if its last digit is 0, 2, 4, 6, or 8 and the sum of its digits is divisible by 3.

4. A number is divisible by 12 if it is divisible by 4 and by 3, i.e., if the number represented by its last two digits is divisible by 4 and the sum of its digits is divisible by 3. $46 \div 4$ is not an integer; $1346 \div 12$ is not an integer.

5. $N = a_0 + 10a_1 + 10^2a_2 + 10^3a_3 + 10^4a_4 + \cdots$

$N = a_0 + 10a_1 + 100a_2 + 1000a_3 + 10{,}000a_4 + \cdots$

$$N = \underbrace{a_0 + 10a_1}_{?} + \underbrace{25(4a_2 + 40a_3 + 400a_4 + \cdots)}_{\text{a multiple of 25}}$$

N is divisible by 25 if, and only if, $a_0 + 10a_1$ is divisible by 25. $75 \div 25$ is an integer; $67{,}475 \div 25$ is an integer.

6. a. From Example 5 in text,

$N = (a_0 + 10a_1) + 4(25a_2 + 250a_3 + \cdots + 2^{n-2} \cdot 5^n a_n)$

$N = (a_0 + 2a_1) + 8a_1 + 4(25a_2 + 250a_3 + \cdots + 2^{n-2} \cdot 5^n a_n)$

$$N = \underbrace{(a_0 + 2a_1)}_{?} + \underbrace{4(2a_1 + 25a_2 + 250a_3 + \cdots + 2^{n-2} \cdot 5^n a_n)}_{\text{a multiple of 4}}$$

N is divisible by 4 if, and only if, $2a_1 + a_0$ is divisible by 4.

b. $N = a_0 + 10a_1 + 10^2a_2 + 10^3a_3 + 10^4a_4 + 10^5a_5 + \cdots$

$N = a_0 + (8 + 2)a_1 + (96 + 4)a_2 + 8 \cdot 125a_3 + 8 \cdot 1250a_4 + 8 \cdot 12{,}500a_5 + \cdots$

$$N = \underbrace{a_0 + 2a_1 + 4a_2}_{?} + \underbrace{8(a_1 + 12a_2 + 125a_3 + 1250a_4 + 12{,}500a_5 + \cdots)}_{\text{a multiple of 8}}$$

N is divisible by 8 if, and only if, $4a_2 + 2a_1 + a_0$ is divisible by 8.

CHAPTER 9. Graphs

Pages 336–337 · WRITTEN EXERCISES

A **1.** $a - 11 = 12a$; $11a = -11$; $a = -1$ and $b + 8 = 3b$; $2b = 8$; $b = 4$

2. $3a = a - 6$; $2a = -6$; $a = -3$ and $2b - 1 = 3b$; $b = -1$

3. $1 - a = 5 - 7a$; $6a = 4$; $a = \frac{2}{3}$ and $3 = |b|$; $3 = b$ or $3 = -b$; $b = 3$ or $b = -3$

4. $2a + 4 = 3a + 7$; $-a = 3$; $a = -3$ and $b^2 = -b$; $b^2 + b = 0$; $b(b + 1) = 0$; $b = 0$ or $b + 1 = 0$; $b = 0$ or $b = -1$

5. $2 - a^2 = a$; $-a^2 - a + 2 = 0$; $a^2 + a - 2 = 0$; $(a + 2)(a - 1) = 0$; $a + 2 = 0$ or $a - 1 = 0$; $a = -2$ or $a = 1$ and $b^2 = 36$; $b^2 - 36 = 0$; $(b + 6)(b - 6) = 0$; $b + 6 = 0$ or $b - 6 = 0$; $b = -6$ or $b = 6$

6. $|a + 1| = 2$; $a + 1 = 2$ or $-(a + 1) = 2$; $a = 1$ or $-a - 1 = 2$; $a = 1$ or $-a = 3$; $a = 1$ or $a = -3$ and $b^3 = 0$; $(b)(b)(b) = 0$; $b = 0$ or $b = 0$ or $b = 0$; $b = 0$

7. $\{(-6, -3), (0, 3), (6, 9)\}$

8. $\{(-6, 10), (0, 4), (6, -2)\}$

9. $y = 8 - 5x$. $\{(-6, 38), (0, 8), (6, -22)\}$

10. $y = 3x + 7$. $\{(-6, -11), (0, 7), (6, 25)\}$

11. $y = \frac{7}{3}x - 10$. $\{(-6, -24), (0, -10), (6, 4)\}$ **12.** $y = -\frac{3}{2}x + 6$. $\{(-6, 15), (0, 6), (6, -3)\}$

13. $y = \frac{1}{6}x + \frac{5}{3}$; $x = -10$, $y = 0$; $x = 2$, $y = 2$. $\{(2, 2)\}$

14. $y = \frac{1}{8}x - \frac{15}{8}$; $x = -1$, $y = -2$; $x = 15$, $y = 0$. $\{(-1, -2)\}$

15. $y = -\frac{2}{5}x + 2$; $x = 0$, $y = 2$; $x = 5$, $y = 0$. $\{(5, 0)\}$

16. $y = \frac{8}{3}x - 7$; $x = 0$, $y = -7$; $x = 3$, $y = 1$. $\{(3, 1)\}$

B **17.** $y = -\frac{2}{5}x + \frac{7}{5}$; $x = -4$, $y = 3$; $x = -1$, $y = \frac{9}{5}$. $\{(-4, 3)\}$

18. $y = -\frac{5}{3}x + \frac{1}{3}$; $x = 1$, $y = -\frac{4}{3}$; $x = 2$, $y = -3$. $\{(2, -3)\}$

19. $y \geq 2x - 1$; $x = 0$, $y \geq -1$; $x = 1$, $y \geq 1$. $\{(0, -1), (0, 1), (1, 1)\}$

20. $y < 3x + 1$; $x = -1$, $y < -2$; $x = 0$, $y < 1$. $\{(-1, -3), (0, -3), (0, 0)\}$

21. $-y < 3 - |x|$; $y > |x| - 3$; $x = -1$, $y > -2$; $x = 3$, $y > 0$; $x = 4$, $y > 1$.
$\{(-1, 0), (-1, 1), (-1, 2), (3, 1), (3, 2), (4, 2)\}$

22. $-y \geq 2 - |x|$; $y \leq |x| - 2$; $x = -2$, $y \leq 0$; $x = -1$, $y \leq -1$; $x = 3$, $y \leq 1$. $\{(-2, 0), (3, 1), (3, 0)\}$

C **23.** $y = 4 - |x|$; $x = -3$, $y = 1$; $x = 0$, $y = 4$; $x = 4$, $y = 0$. $\{(-3, 1), (0, 4)\}$

24. $4y = |x| - 12$; $y = \dfrac{|x|}{4} - 3$; $x = -1$, $y = -\frac{11}{4}$; $x = 0$, $y = -3$; $x = 4$, $y = -2$. $\{(0, -3), (4, -2)\}$

25. $y^2 - 8y + 10 + 2x = 0$; $x = -5$, $y^2 - 8y + 10 - 10 = 0$; $y^2 - 8y = 0$; $y(y - 8) = 0$; $y = 0$ or $y - 8 = 0$; $y = 0$ or $y = 8$. $x = 1$, $y^2 - 8y + 10 + 2 = 0$; $y^2 - 8y + 12 = 0$; $(y - 6)(y - 2) = 0$; $y - 6 = 0$ or $y - 2 = 0$; $y = 6$ or $y = 2$. $\{(1, 2)\}$

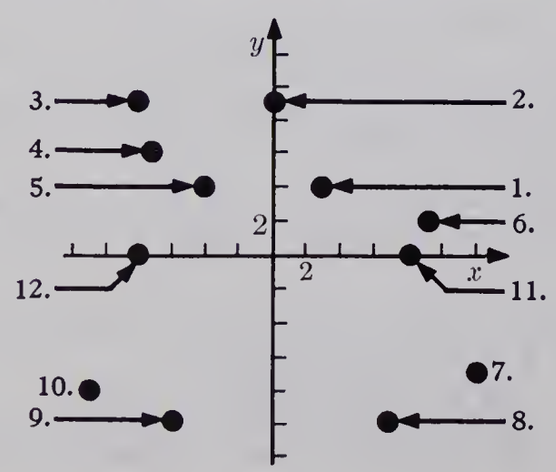
26. $y^2 - 7y + 3 + 3x = 0$; $x = -1$, $y^2 - 7y + 3 - 3 = 0$; $y^2 - 7y = 0$; $y(y - 7) = 0$; $y = 0$ or $y - 7 = 0$; $y = 0$ or $y = 7$. $x = 3$, $y^2 - 7y + 3 + 9 = 0$; $y^2 - 7y + 12 = 0$; $(y - 3)(y - 4) = 0$; $y - 3 = 0$ or $y - 4 = 0$; $y = 3$ or $y = 4$. $\{(-1, 7), (3, 3)\}$

27. Let N = no. of baseballs, $N \in \{0, 1, 2, \ldots, 33\}$; $1.5N$ = cost, in dollars, of N baseballs; P = no. of bats, $P \in \{0, 1, 2, \ldots, 14\}$; $3.5P$ = cost, in dollars, of P bats; $1.5N + 3.5P \leq 50$; $15N + 35P \leq 500$; $15N \leq 500 - 35P$; $N \leq \frac{100}{3} - \frac{7}{3}P$. $P = 0$, $N \leq \frac{100}{3}$, $P = 1$, $N \leq 31$, $\ldots$, $P = 14$, $N \leq \frac{2}{3}$; $\{(P, N)\} = \{(0, 33), (1, 31), (2, 28), (3, 26), (4, 24), (5, 21), (6, 19), (7, 17), (8, 14), (9, 12), (10, 10), (11, 7), (12, 5), (13, 3), (14, 0)\}$

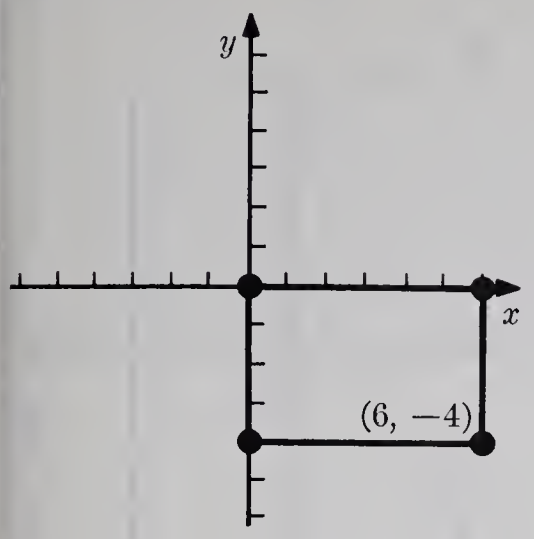
28. Let N = no. of cupcakes and no. of ice cream bars, $N \in \{20, 21, \ldots, 30\}$; $5N$ = cost of N cupcakes; $10N$ = cost of N ice cream bars; P = no. of qt of punch, $P \in \{12, 13, \ldots, 18\}$; $25P$ = cost of P qt of punch; $5N + 10N + 25P \leq 750$; $15N \leq 750 - 25P$; $N \leq 50 - \frac{5}{3}P$. $P = 12$, $N \leq 30$, $P = 13$, $N \leq \frac{85}{3}$, $\ldots$, $P = 18$, $N \leq 20$; $\{(P, N)\} = \{(12, 30), (13, 28), (14, 26), (15, 25), (16, 23), (17, 21), (18, 20)\}$

Page 340 · **WRITTEN EXERCISES**

A



B **13.**



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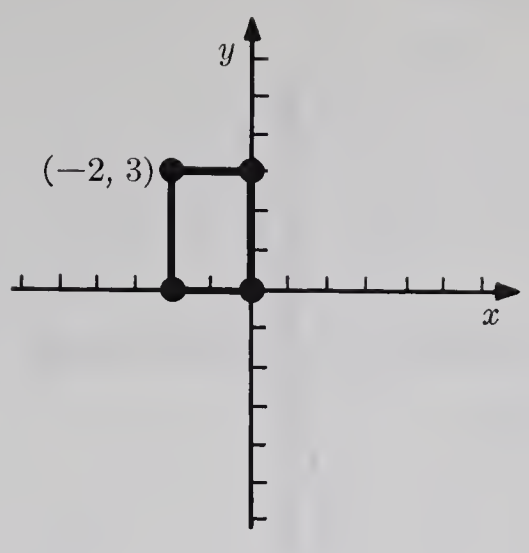

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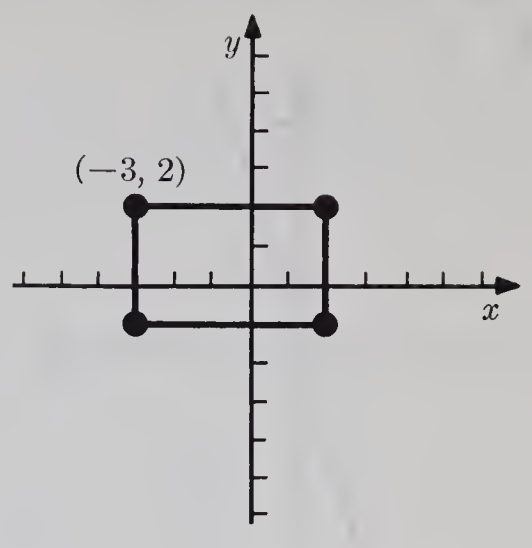

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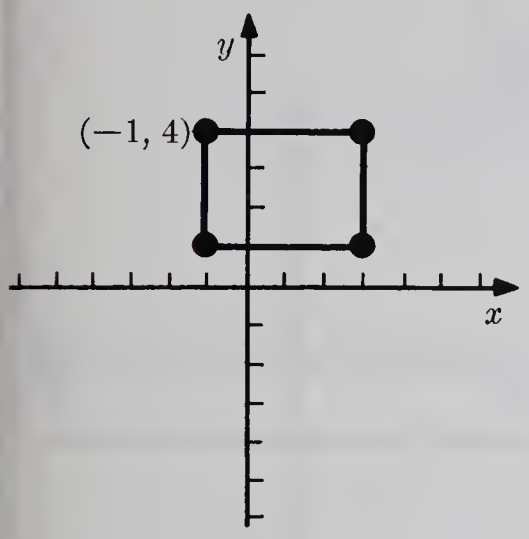

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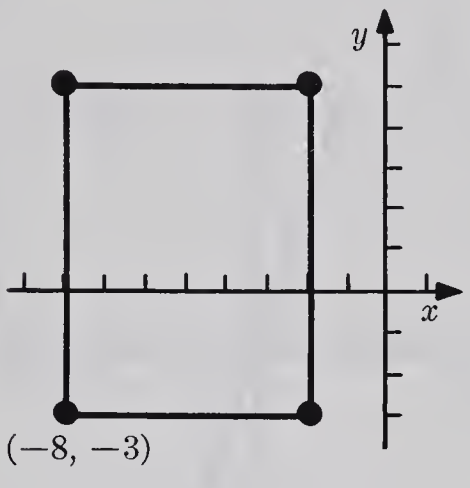

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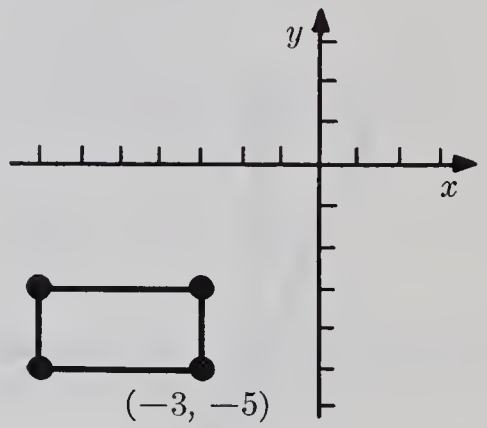
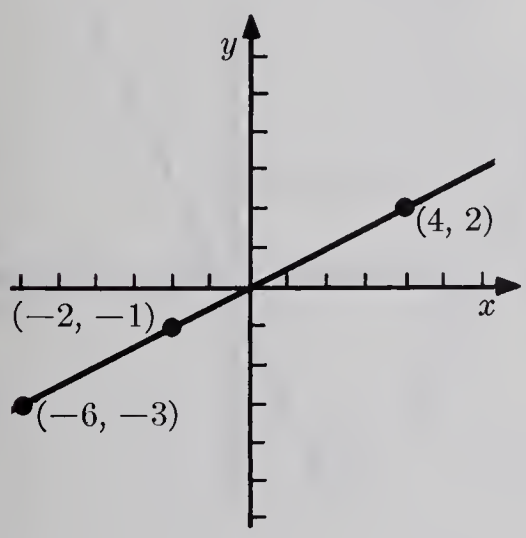


Note: In Ex. 19–22 the coordinates of any point on the lines drawn will satisfy the given requirements. We have indicated three such points on each line.

19.



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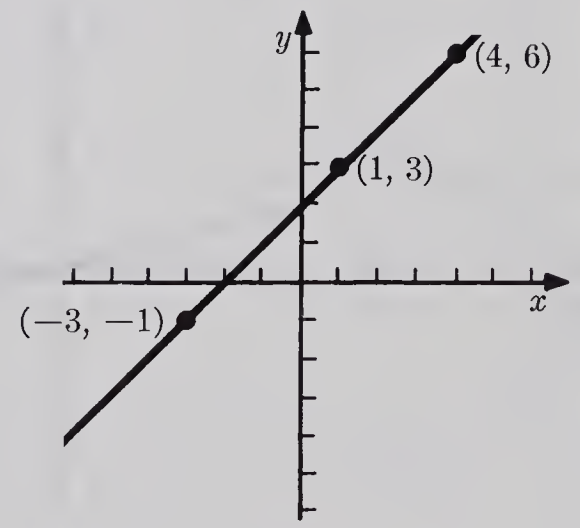

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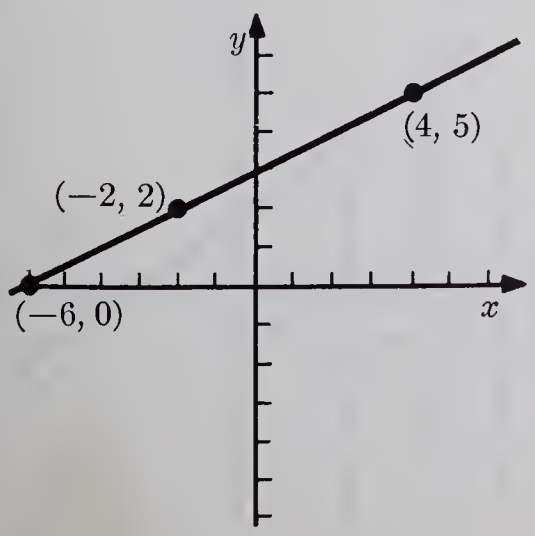

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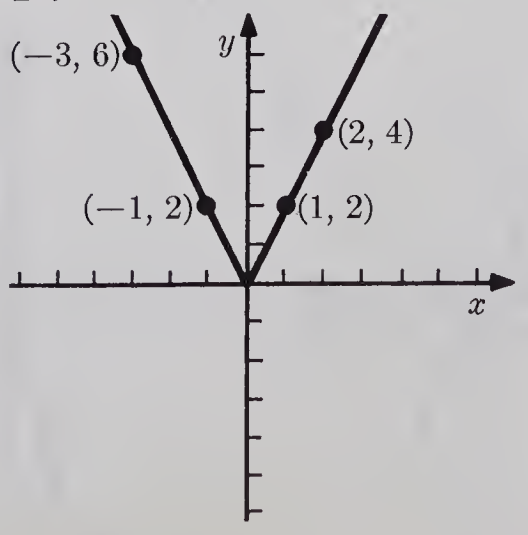
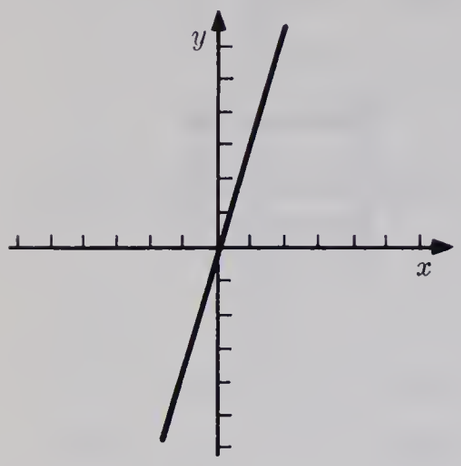


Pages 342–343 · **WRITTEN EXERCISES**

A **1.**



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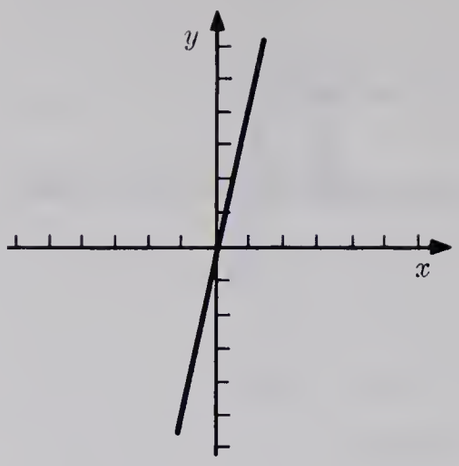

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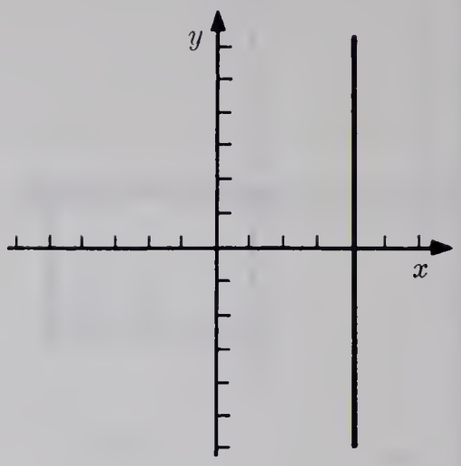

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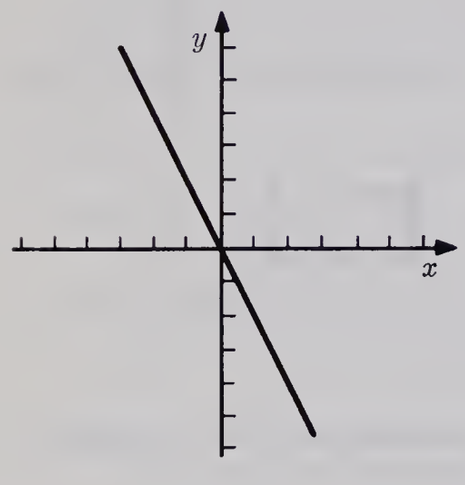

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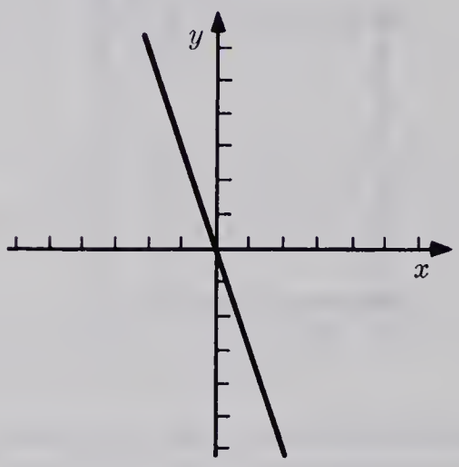

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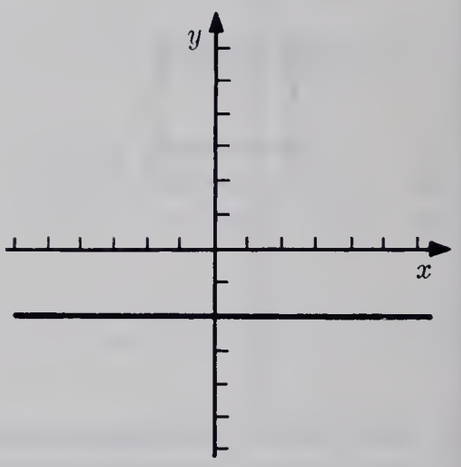

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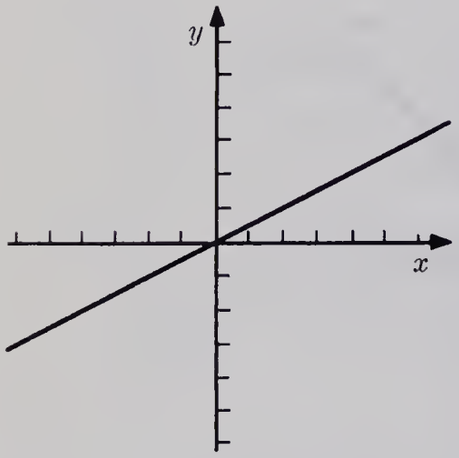

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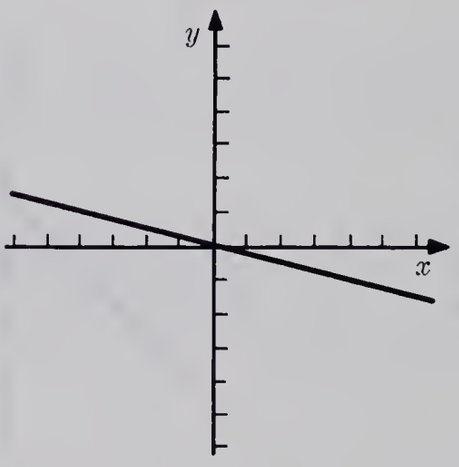

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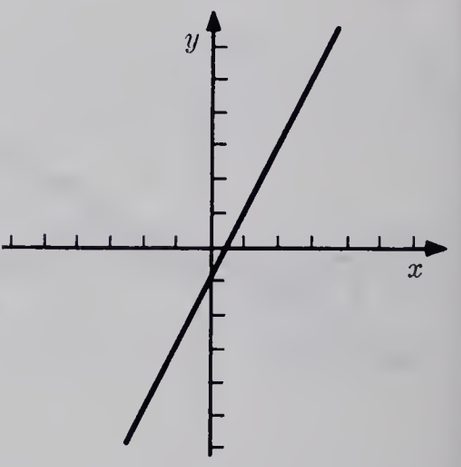

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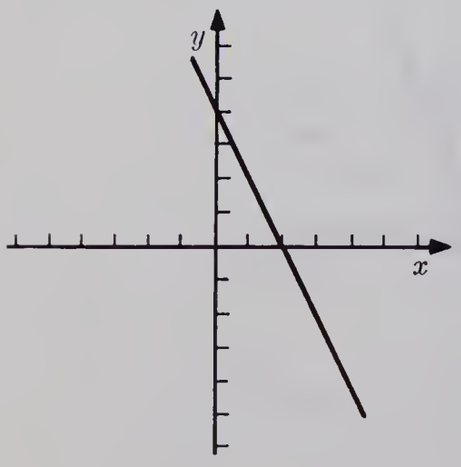

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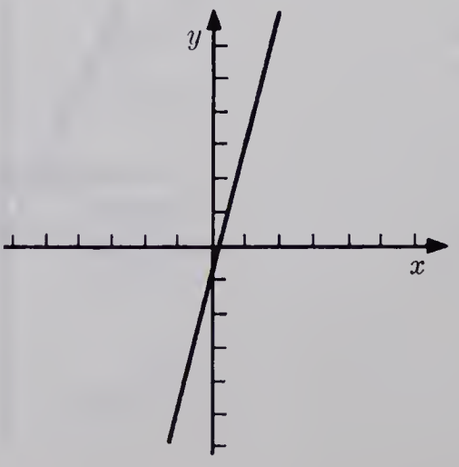

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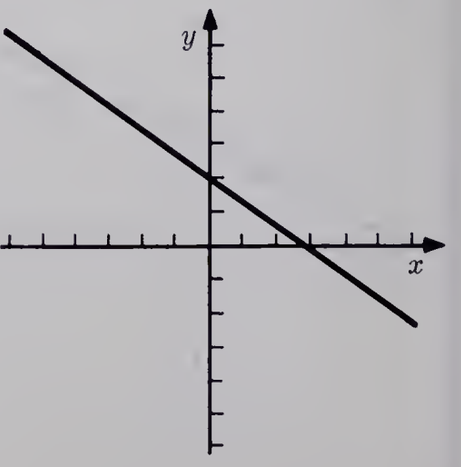


B **13.** (a) $5x - 3(0) - 30 = 0$; $5x = 30$; $x = 6$; $(6, 0)$
(b) $5(0) - 3y - 30 = 0$; $-3y = 30$; $y = -10$; $(0, -10)$

14. (a) $7(0) + 4x - 28 = 0$; $4x = 28$; $x = 7$; $(7, 0)$ (b) $7y + 4(0) - 28 = 0$; $7y = 28$; $y = 4$; $(0, 4)$

15. (a) $12x - 60 = 5(0)$; $12x = 60$; $x = 5$; $(5, 0)$ (b) $12(0) - 60 = 5y$; $5y = -60$; $y = -12$; $(0, -12)$

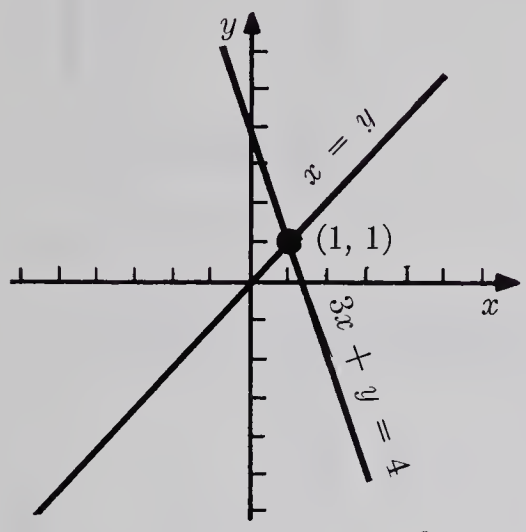
16. (a) $9(0) - 36 = 2x$; $2x = -36$; $x = -18$; $(-18, 0)$ (b) $9y - 36 = 2(0)$; $9y = 36$; $y = 4$; $(0, 4)$

17. (a) $3(0) = 7x$; $7x = 0$; $x = 0$; $(0, 0)$ (b) $3y = 7(0)$; $3y = 0$; $y = 0$; $(0, 0)$

18. (a) $(0) = |x|$; $x = 0$; $(0, 0)$ (b) $y = |0|$; $y = 0$; $(0, 0)$

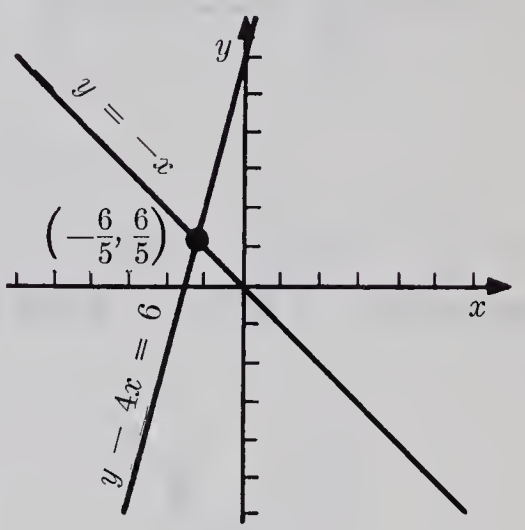
C

19.



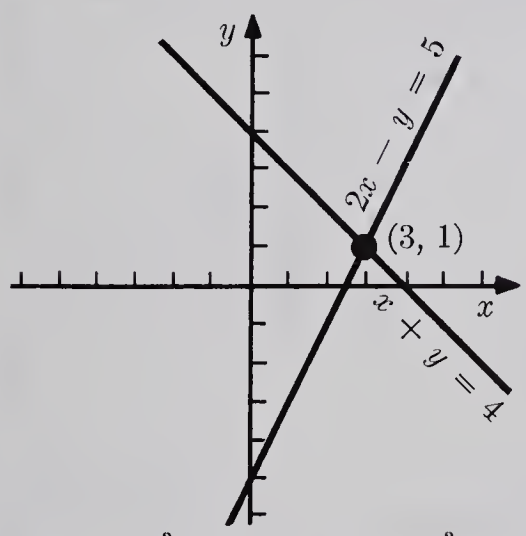
$1 = 1$ $\quad 3(1) + 1 \stackrel{?}{=} 4$
$4 = 4$

20.



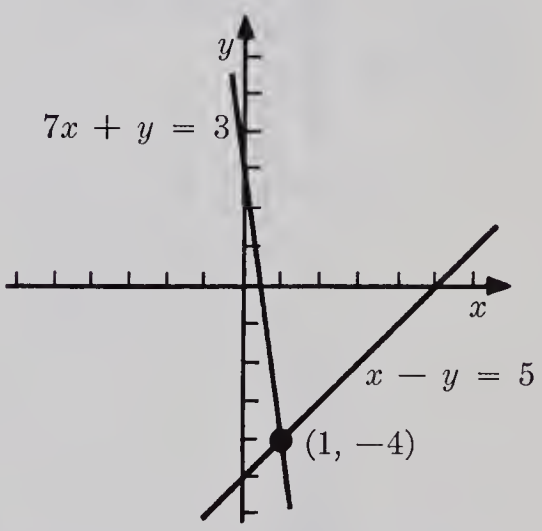
$1 \stackrel{?}{=} -(-1)$ $\quad 1 - 4(-1) \stackrel{?}{=} 5$
$1 = 1$ $\quad 1 + 4 = 5$
$5 = 5$

21.



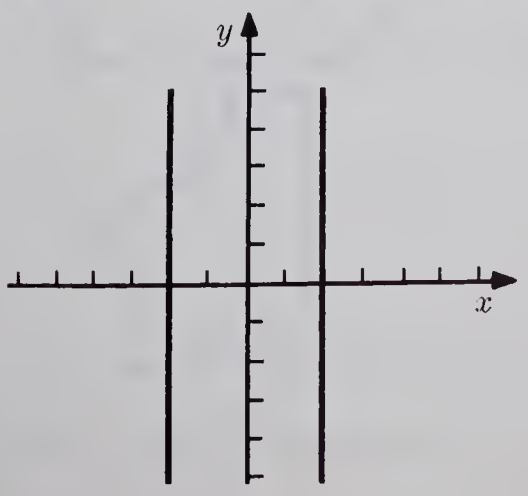
$3 + 1 \stackrel{?}{=} 4$ $\quad 2(3) - 1 \stackrel{?}{=} 5$
$4 = 4$ $\quad 5 = 5$

22.



$1 - (-4) \stackrel{?}{=} 5$ $\quad 7(1) + (-4) \stackrel{?}{=} 3$
$5 = 5$ $\quad 3 = 3$

23.



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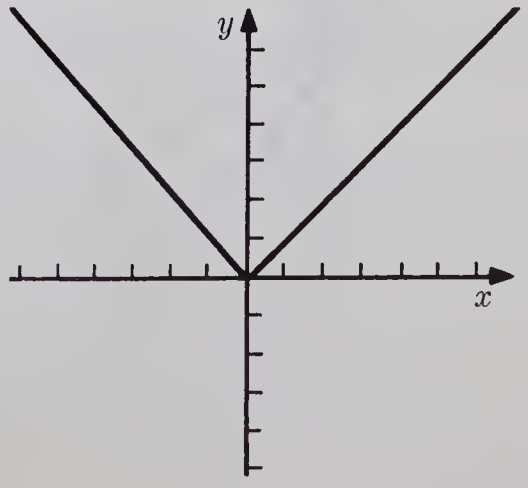

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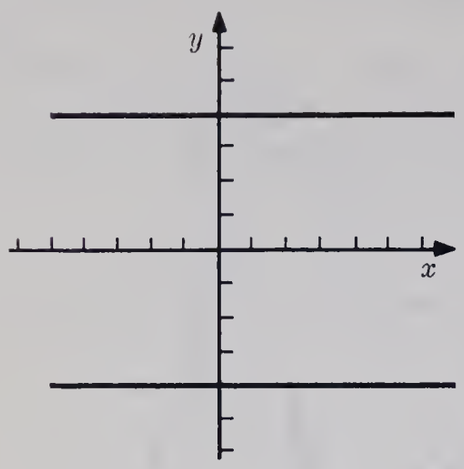

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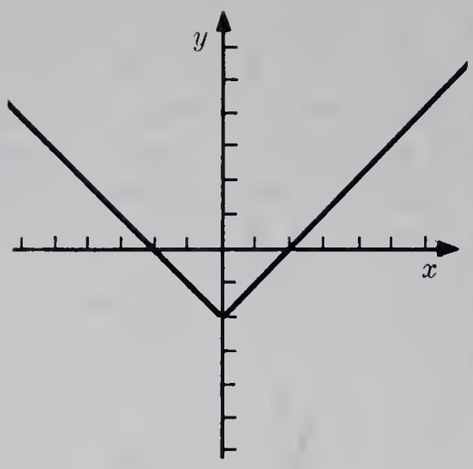
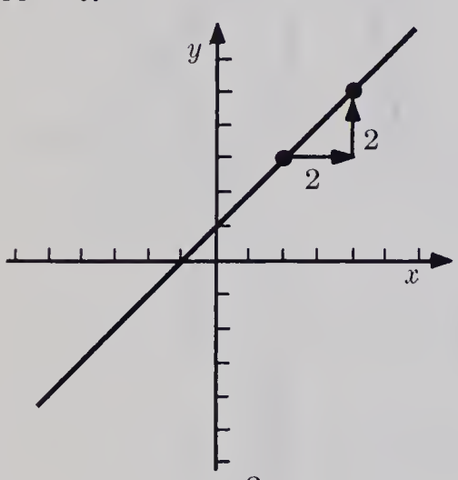


Pages 345–346 · **WRITTEN EXERCISES**

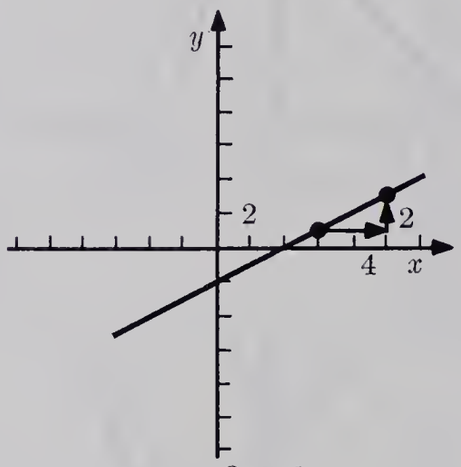
A **1.**



$$m = \frac{2}{2} = 1$$

$$m = \frac{5 - 3}{4 - 2} = \frac{2}{2} = 1$$

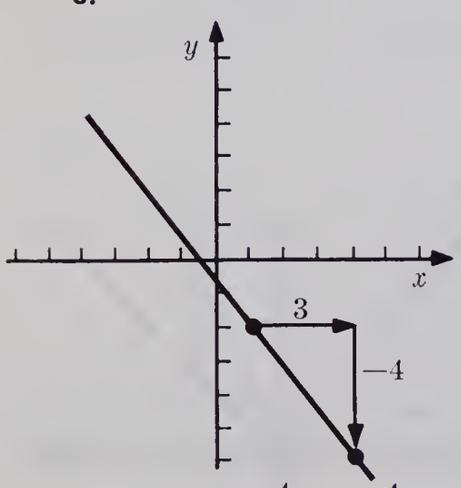
2.



$$m = \frac{2}{4} = \frac{1}{2}$$

$$m = \frac{3 - 1}{10 - 6} = \frac{2}{4} = \frac{1}{2}$$

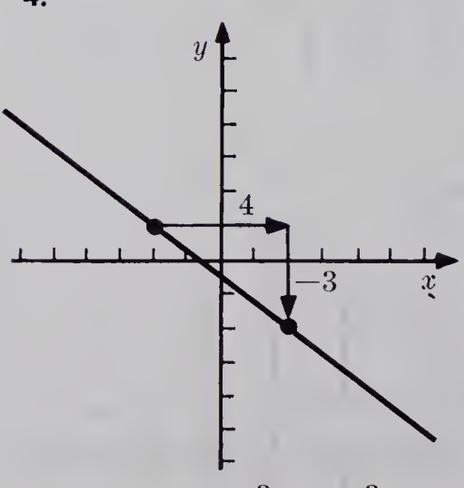
3.



$$m = \frac{-4}{3} = -\frac{4}{3}$$

$$m = \frac{-6 - (-2)}{4 - 1} = \frac{-6 + 2}{3} = -\frac{4}{3}$$

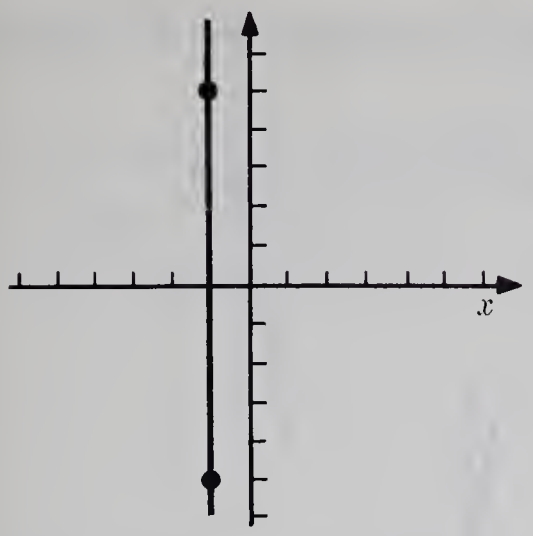
4.



$$m = \frac{-3}{4} = -\frac{3}{4}$$

$$m = \frac{-2 - 1}{2 - (-2)} = \frac{-3}{2 + 2} = -\frac{3}{4}$$

5.

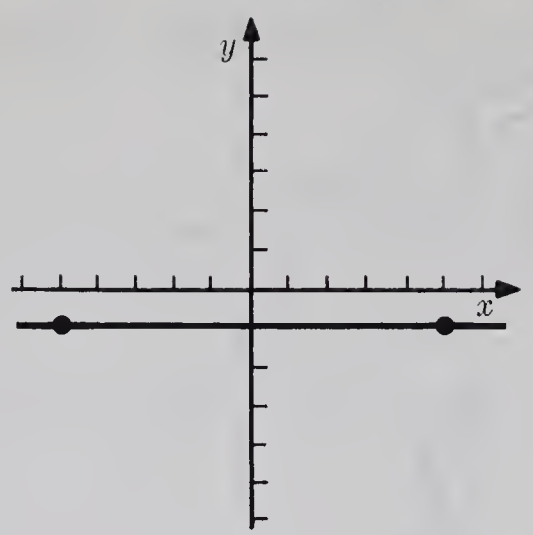


$m = \frac{10}{0}$ is undefined

$m = \frac{5 - (-5)}{-1 - (-1)}$

$= \frac{5 + 5}{-1 + 1} = \frac{10}{0}$ is undefined

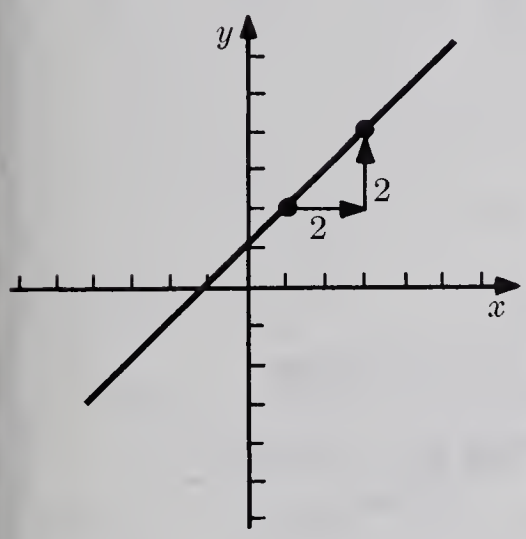
6.



$m = \frac{0}{10} = 0$

$m = \frac{-1 - (-1)}{5 - (-5)} = \frac{-1 + 1}{5 + 5} = \frac{0}{10} = 0$

7.



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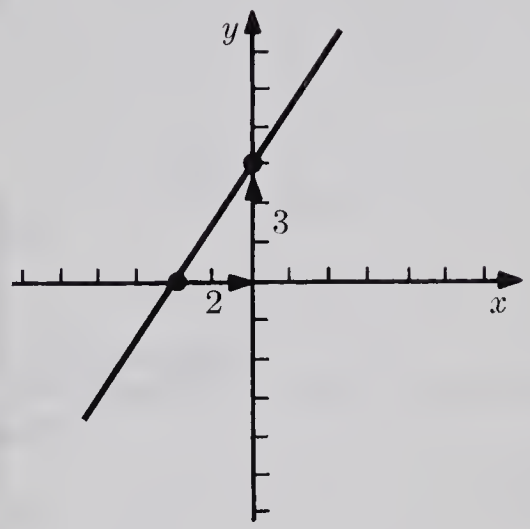

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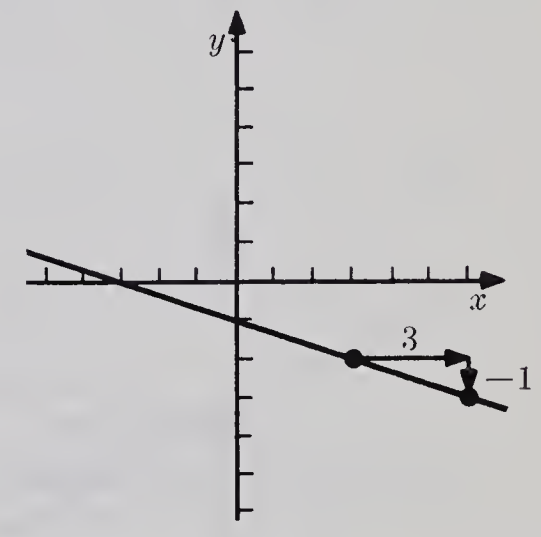

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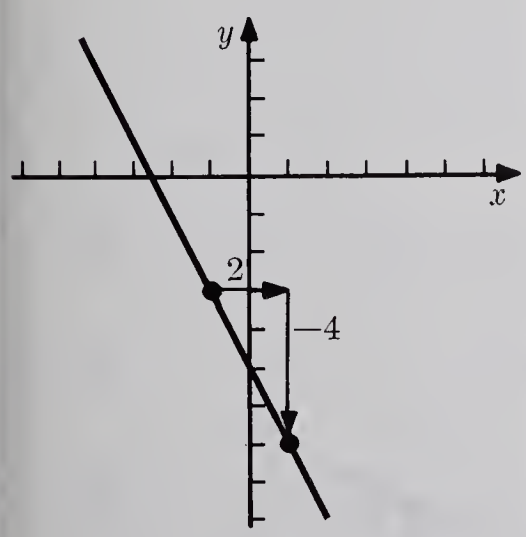

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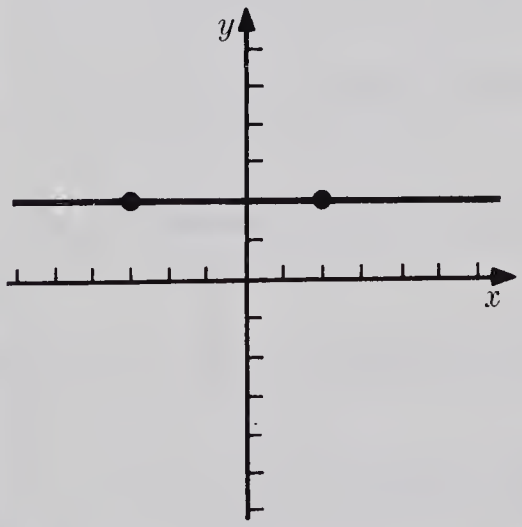

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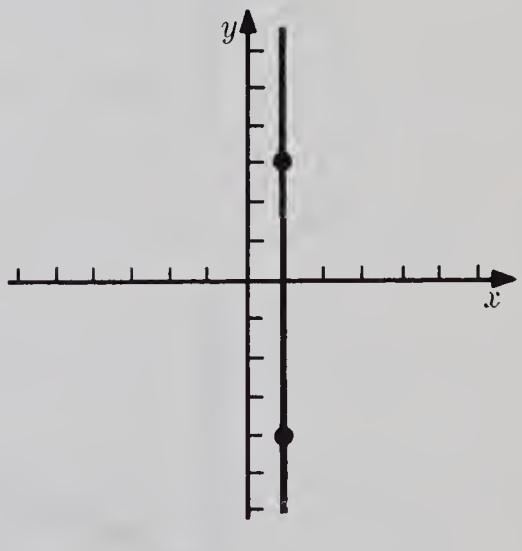


B **13.** $\frac{3a - 2a}{-1 - (-3)} = -\frac{1}{2}; \frac{a}{2} = -\frac{1}{2}; a = -1$

14. $\frac{5a - a}{-5 - 3} = \frac{1}{2}; \frac{4a}{-8} = \frac{1}{2}; \frac{a}{-2} = \frac{1}{2}; a = -1$

15. $\frac{a - 5}{4 - 3} = 2; \frac{a - 5}{1} = 2; a - 5 = 2; a = 7$

16. $\frac{a - 0}{7 - 2} = -1; \frac{a}{5} = -1; a = -5$

17. $\frac{3a - 6}{1 - (-2)} = -2; \frac{3a - 6}{3} = -2; 3a - 6 = -6; 3a = 0; a = 0$

18. $\frac{2a - (-4)}{3 - 1} = 5; \frac{2a + 4}{2} = 5; 2a + 4 = 10; 2a = 6; a = 3$

Page 348 · **WRITTEN EXERCISES**

Note: In Ex. 1–10, substitute the given values for m and b in the slope-intercept form of a linear equation: $y = mx + b$.

1. $y = 3x + 1$

2. $y = -2x + 5$

3. $y = -\frac{1}{4}x + (-2); 4y = -x - 8$

4. $y = \frac{1}{3}x + (-3); 3y = x - 9$

5. $y = -\frac{4}{7}x + 0; 7y = -4x$

6. $y = 0 \cdot x + 7; y = 7$

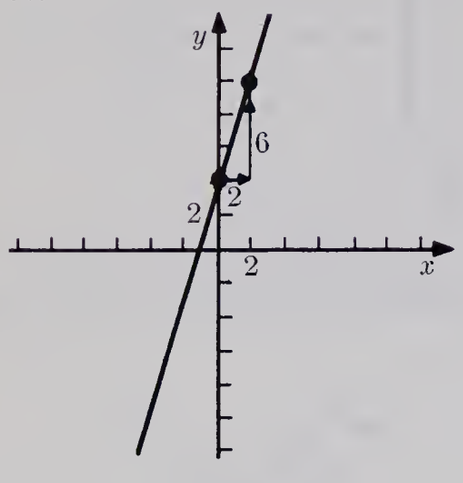
7. $y = 0 \cdot x + (-10); y = -10$

8. $y = -x + 2$

9. $y = 1 \cdot x + (-1); y = x - 1$

10. $y = 7x + 0; y = 7x$

11.



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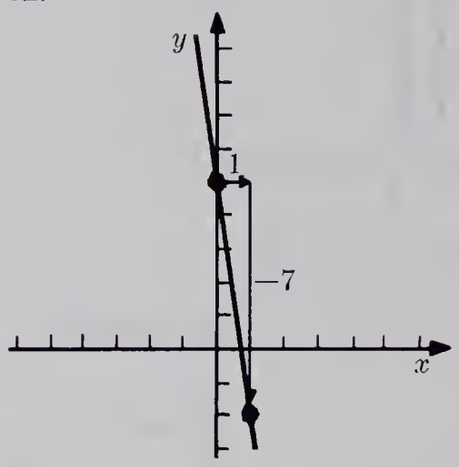

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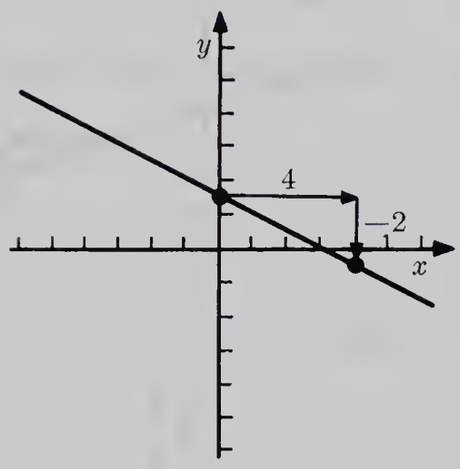
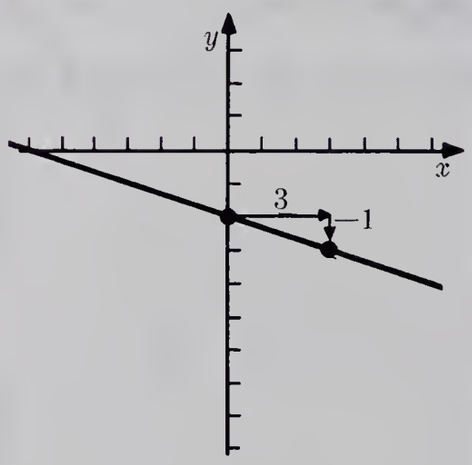
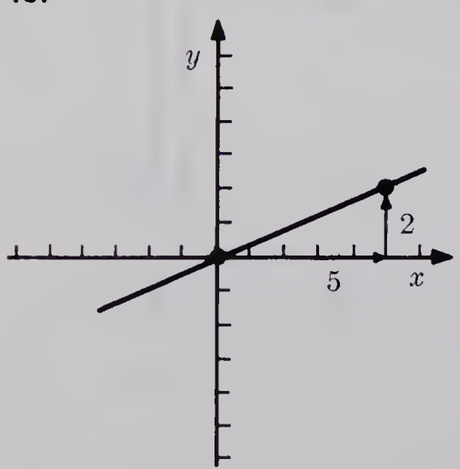


14.



15.



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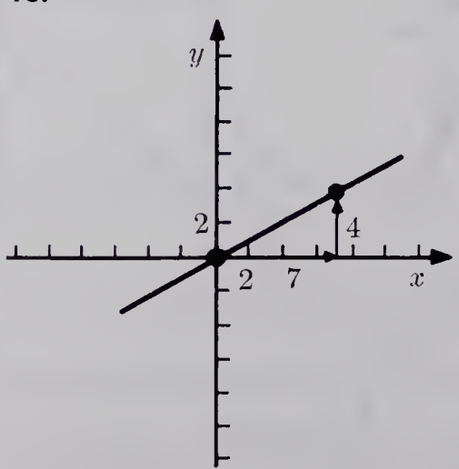

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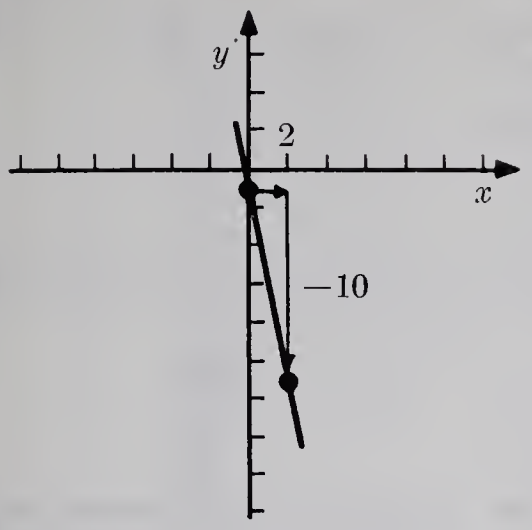
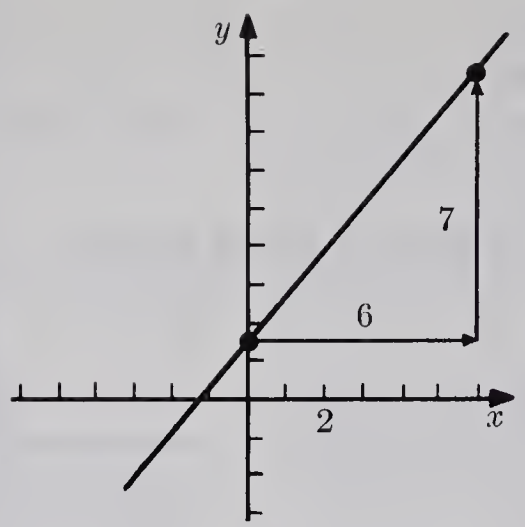


18.



Page 350 · **WRITTEN EXERCISES**

A

1. $y = 3x + b$; $(-2) = 3(-3) + b$; $-2 = -9 + b$; $b = 7$; $y = 3x + 7$

2. $y = 5x + b$; $(0) = 5(-1) + b$; $0 = -5 + b$; $b = 5$; $y = 5x + 5$

3. $y = -2x + b$; $(8) = -2(2) + b$; $8 = -4 + b$; $b = 12$; $y = -2x + 12$

4. $y = \frac{2}{7}x + b$; $(0) = \frac{2}{7}(-5) + b$; $0 = -\frac{10}{7} + b$; $b = \frac{10}{7}$; $y = \frac{2}{7}x + \frac{10}{7}$ *or* $7y = 2x + 10$

5. $y = \frac{7}{2}x + b$; $(-5) = \frac{7}{2}(-2) + b$; $-5 = -7 + b$; $b = 2$; $y = \frac{7}{2}x + 2$ *or* $2y = 7x + 4$

6. $y = -\frac{8}{3}x + b$; $(0) = -\frac{8}{3}(0) + b$; $b = 0$; $y = -\frac{8}{3}x$ *or* $3y = -8x$

7. $y = -\frac{3}{8}x + b$; $(0) = -\frac{3}{8}(0) + b$; $b = 0$; $y = -\frac{3}{8}x$ *or* $8y = -3x$

8. $y = 0 \cdot x + b$; $(-5) = 0(1) + b$; $b = -5$; $y = -5$

9. $y = 0 \cdot x + b$; $(3) = 0(-2) + b$; $b = 3$; $y = 3$

10. $m = \dfrac{6 - 3}{5 - 2} = \dfrac{3}{3} = 1$; $y = 1 \cdot x + b$; choose $(2, 3)$: $3 = 1 \cdot 2 + b$; $b = 1$; $y = x + 1$

11. $m = \dfrac{6 - 4}{3 - 1} = \dfrac{2}{2} = 1$; $y = 1 \cdot x + b$; choose $(1, 4)$: $4 = 1 \cdot 1 + b$; $b = 3$; $y = x + 3$

12. $m = \dfrac{1 - 0}{0 - 0} = \dfrac{1}{0}$; m is undefined and line is vertical through origin $(0, 0)$; $x = 0$ (the eqn. of y-axis)

13. $m = \dfrac{0 - (-1)}{0 - 0}$; same as **12**; $x = 0$

14. $m = \dfrac{2 - 0}{-1 - 0} = -2$; $y = -2x + b$; choose $(0, 0)$: $0 = -2(0) + b$; $b = 0$; $y = -2x$

15. $m = \dfrac{0 - (-3)}{0 - 5} = \dfrac{3}{-5} = -\dfrac{3}{5}$; $y = -\frac{3}{5}x + b$; choose $(0, 0)$: $0 = -\frac{3}{5}(0) + b$; $b = 0$; $y = -\frac{3}{5}x$ *or* $5y = -3x$

16. $m = \dfrac{1 - (-4)}{-2 - (-4)} = \dfrac{5}{2}$; $y = \frac{5}{2}x + b$; choose $(-2, 1)$: $1 = \frac{5}{2}(-2) + b$; $1 = -5 + b$; $b = 6$; $y = \frac{5}{2}x + 6$ *or* $2y = 5x + 12$

17. $m = \dfrac{1 - (-2)}{-5 - 0} = \dfrac{3}{-5} = -\dfrac{3}{5}$; $y = -\frac{3}{5}x + b$; choose $(0, -2)$: $(-2) = -\frac{3}{5}(0) + b$; $b = -2$; $y = -\frac{3}{5}x - 2$ *or* $5y = -3x - 10$

18. $m = \dfrac{-3 - (-1)}{4 - 0} = \dfrac{-2}{4} = -\dfrac{1}{2}$; $y = -\frac{1}{2}x + b$; choose $(0, -1)$: $(-1) = -\frac{1}{2}(0) + b$; $b = -1$; $y = -\frac{1}{2}x - 1$ *or* $2y = -x - 2$

B

19. $a(1) + 3(4) = 5$; $a + 12 = 5$; $a = -7$

20. $2(3) + a(1) = 4$; $6 + a = 4$; $a = -2$

21. $5(-1) - 2(3) + a = 0$; $-5 - 6 + a = 0$; $a = 11$

22. $4(-3) - a(-2) = 2$; $-12 + 2a = 2$; $2a = 14$; $a = 7$

23. $y = -x + 5$; $m = -1$; $y = -1 \cdot x + b$; $4 = -1(3) + b$; $4 = -3 + b$; $b = 7$; $y = -x + 7$

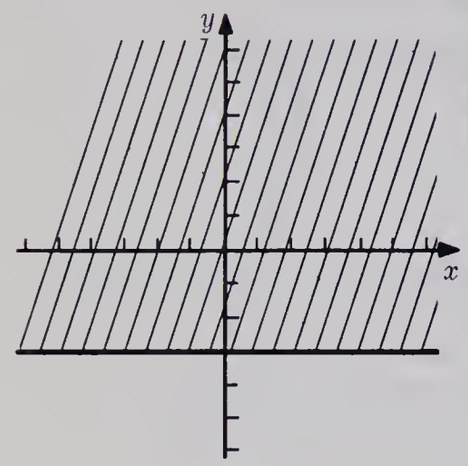
24. $y = x - 3$; $m = 1$; $y = 1 \cdot x + b$; $1 = 1(-2) + b$; $b = 3$; $y = x + 3$

25. $y = -\frac{1}{2}x + 3$; $m = -\frac{1}{2}$; $y = -\frac{1}{2}x + b$; $-3 = -\frac{1}{2}(2) + b$; $-3 = -1 + b$; $b = -2$; $y = -\frac{1}{2}x - 2$ *or* $2y = -x - 4$

26. $y = \frac{1}{3}x - 2$; $m = \frac{1}{3}$; $y = \frac{1}{3}x + b$; $-2 = \frac{1}{3}(-6) + b$; $-2 = -2 + b$; $b = 0$; $y = \frac{1}{3}x$ *or* $3y = x$
27. $5x + 7(0) - 35 = 0$; $5x = 35$; $x = 7$; $(7, 0)$ **28.** $7(0) - 2x + 14 = 0$; $-2x = -14$; $x = 7$; $(7, 0)$
29. $6(3) - 5y - 3 = 0$; $15 - 5y = 0$; $5y = 15$; $y = 3$; $(3, 3)$
30. $3(-1) - 8y + 27 = 0$; $24 - 8y = 0$; $8y = 24$; $y = 3$; $(-1, 3)$

Pages 352–353 · **WRITTEN EXERCISES**

A **1.**



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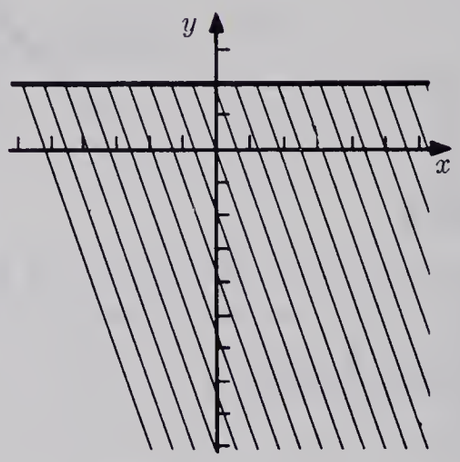

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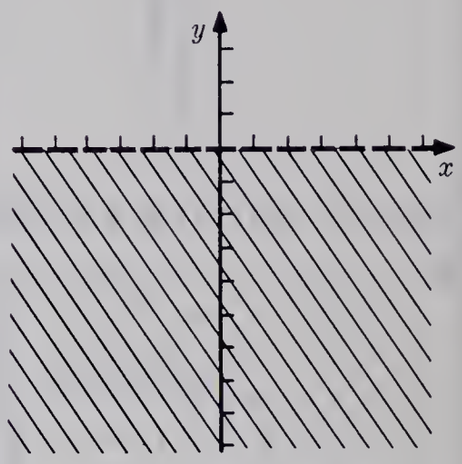

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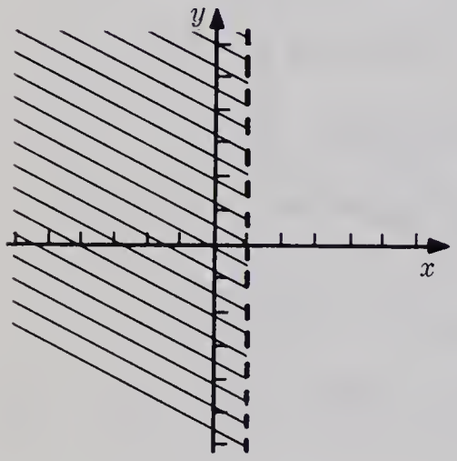

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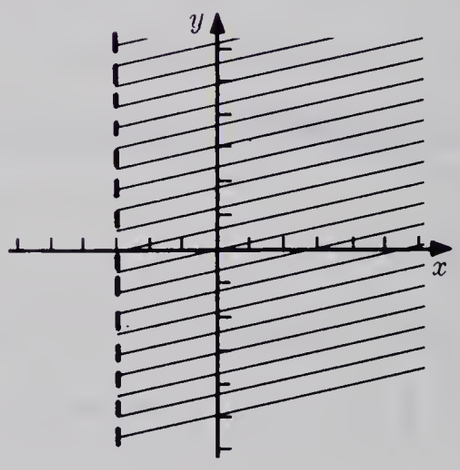

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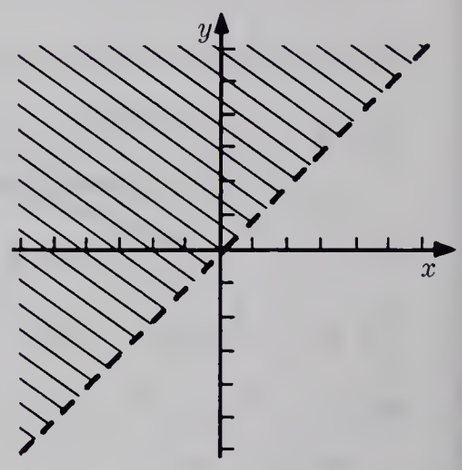

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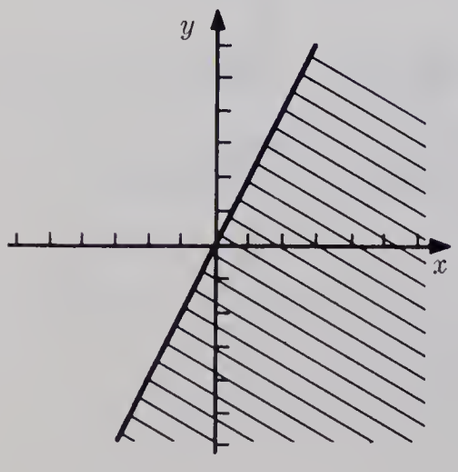

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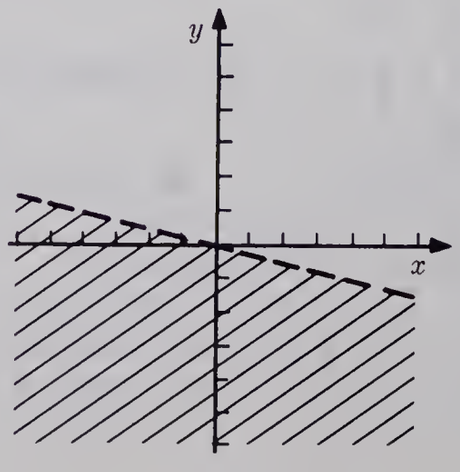

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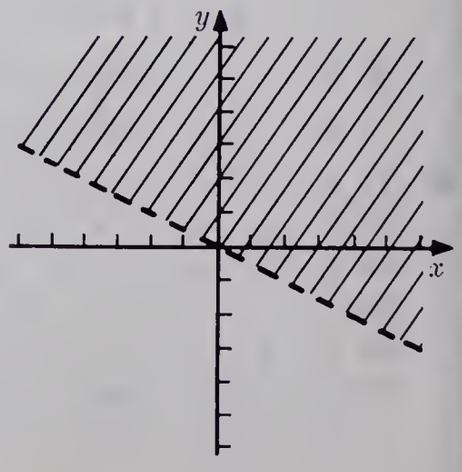

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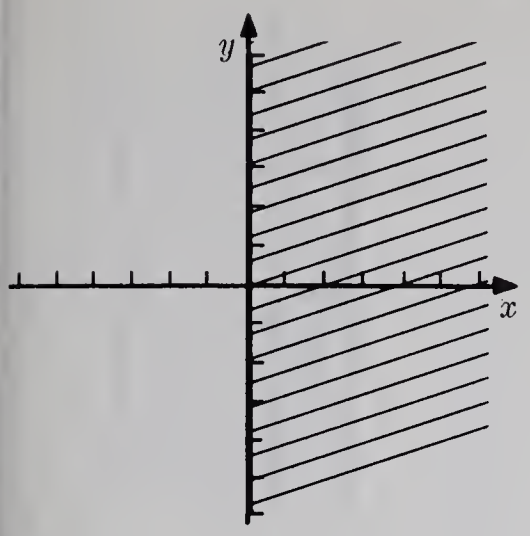

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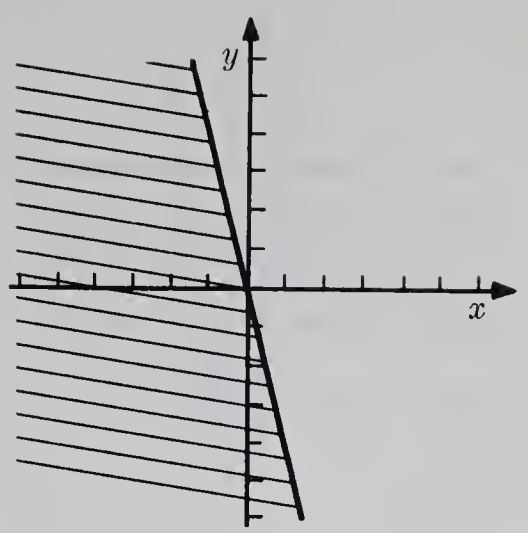

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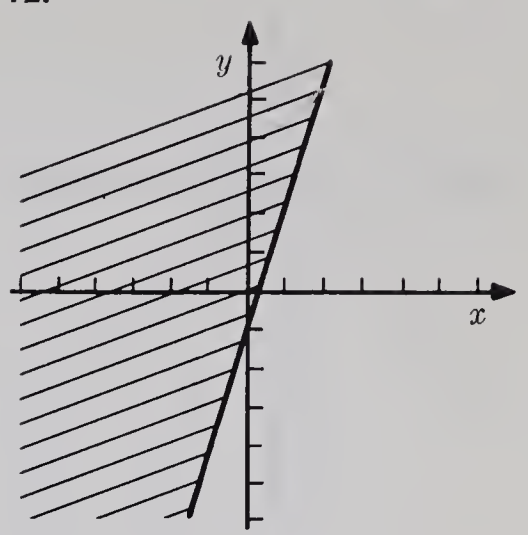

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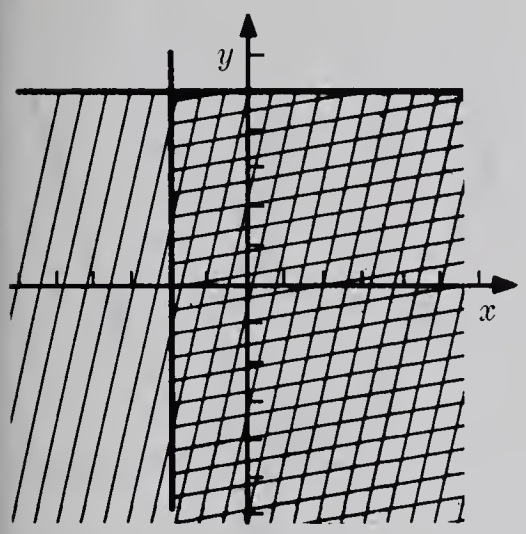

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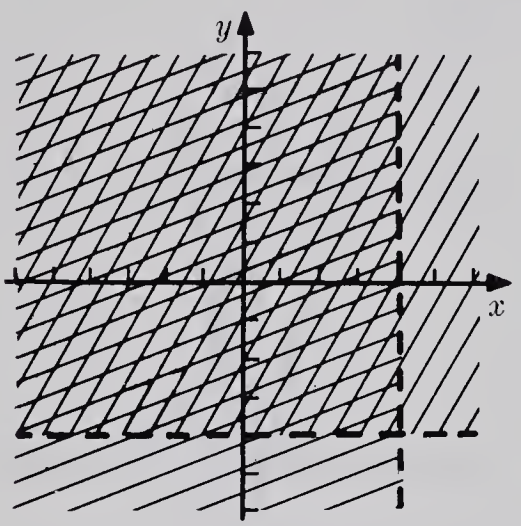

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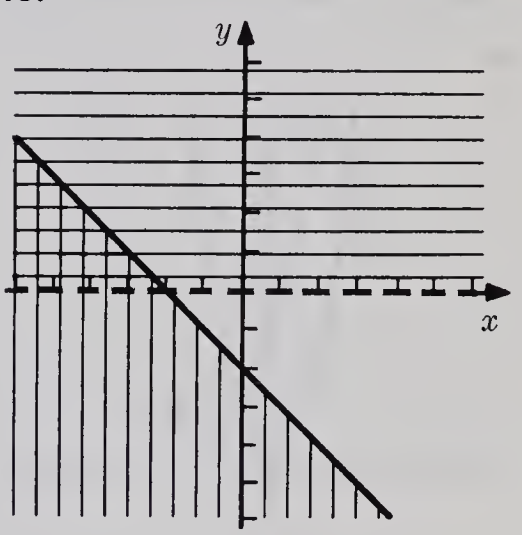

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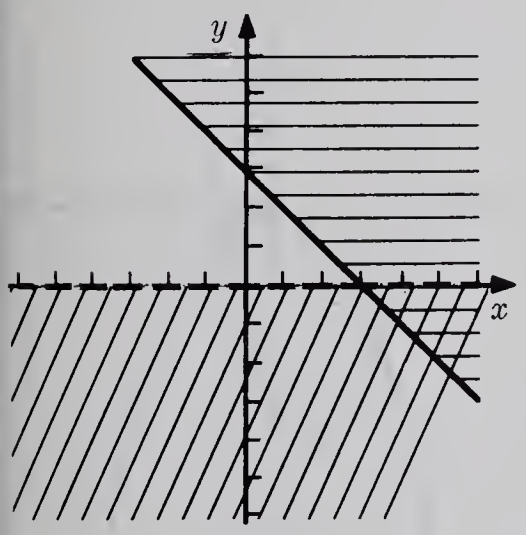

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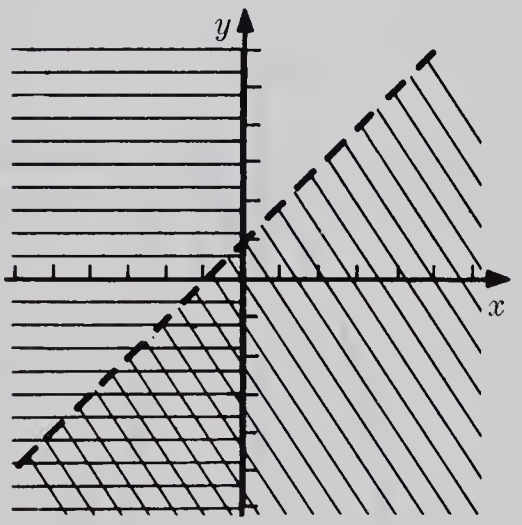

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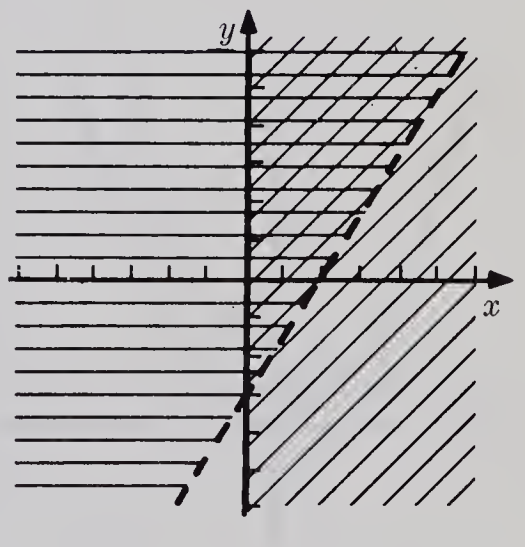

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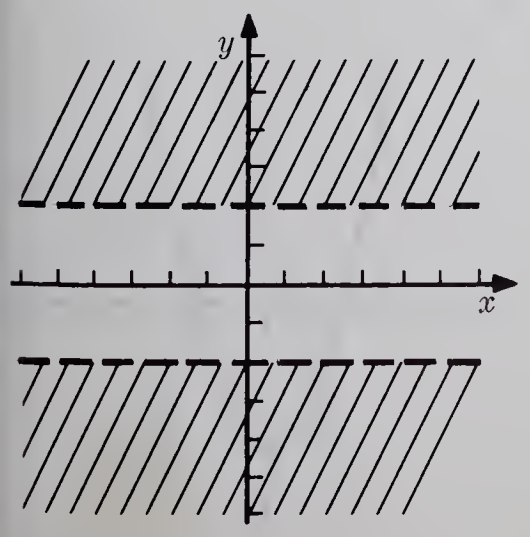

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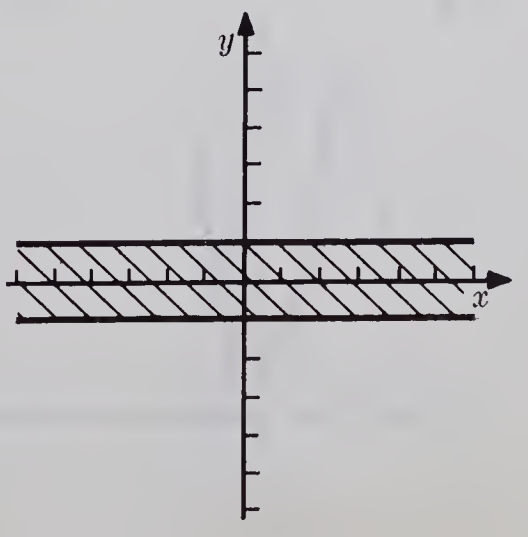

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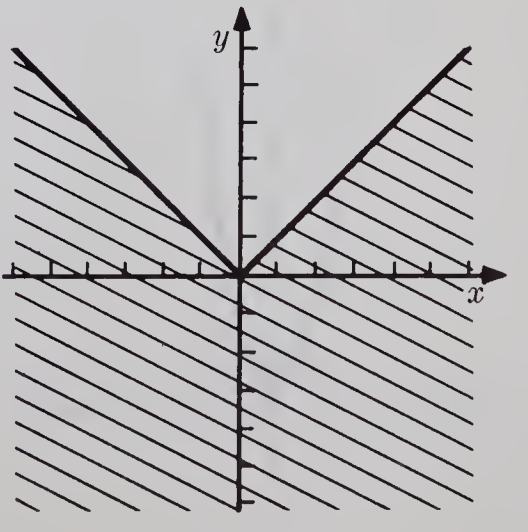

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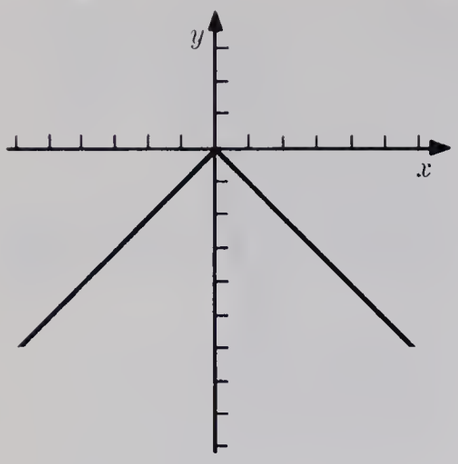

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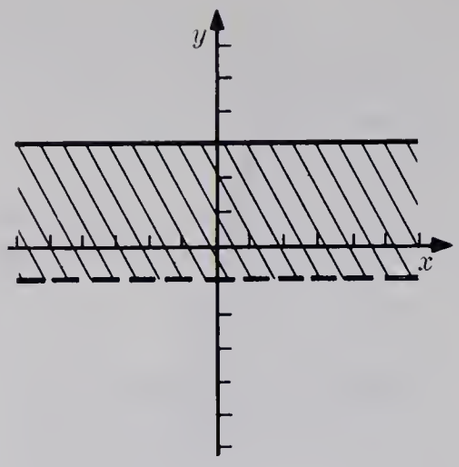

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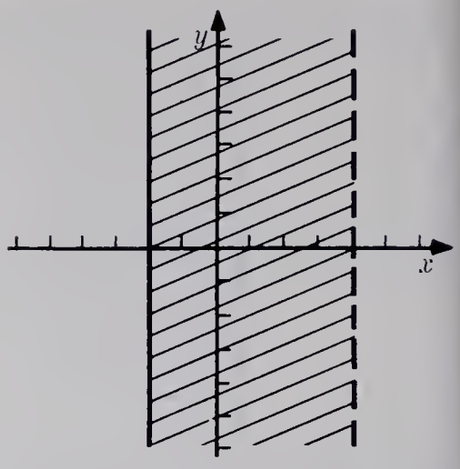

Page 354 · **WRITTEN EXERCISES**

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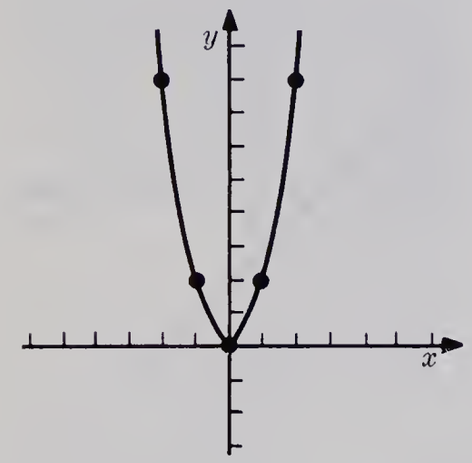

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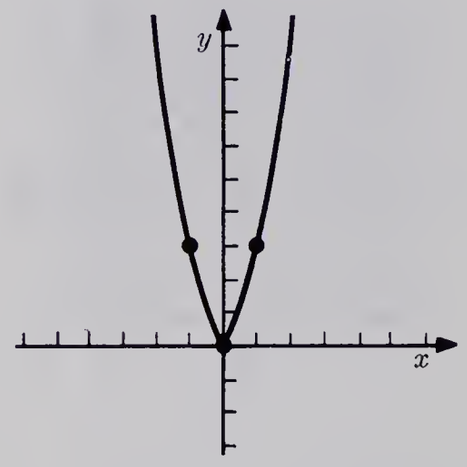

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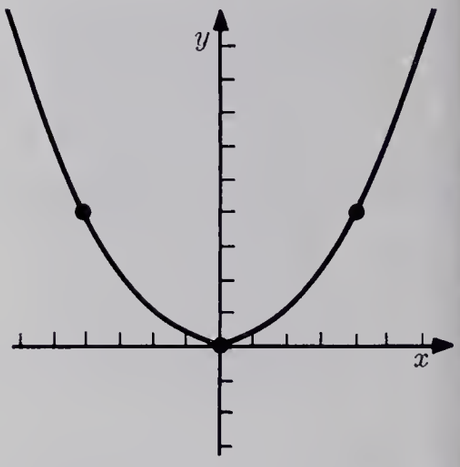

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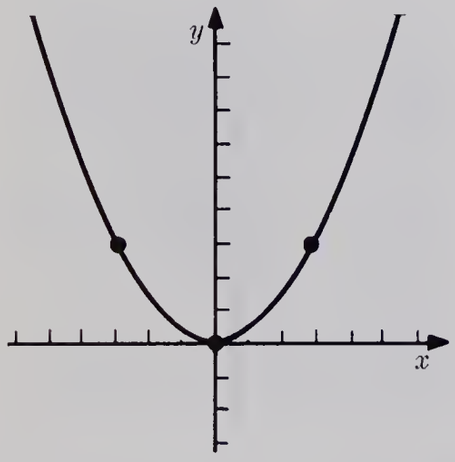

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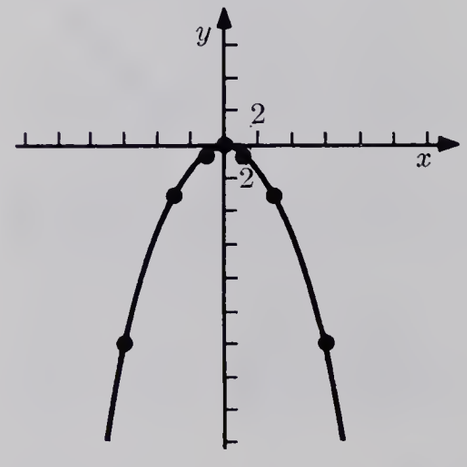

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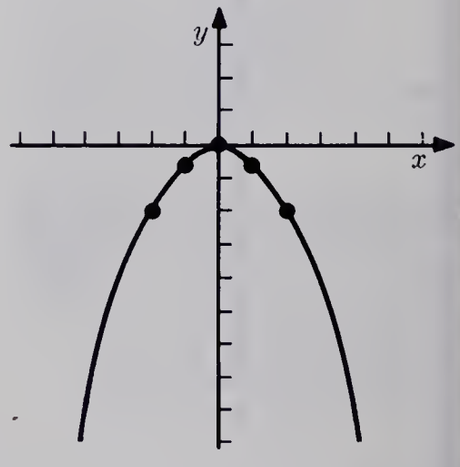

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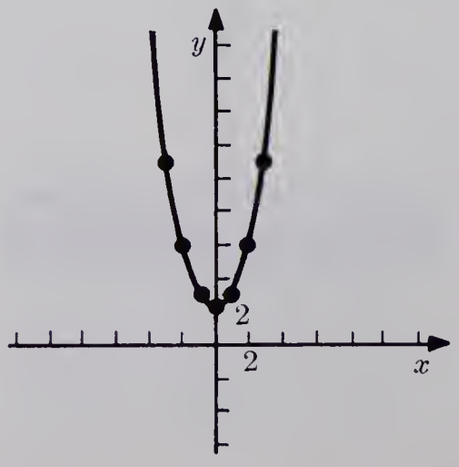

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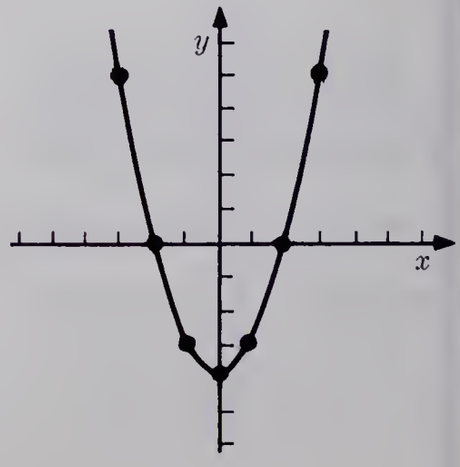

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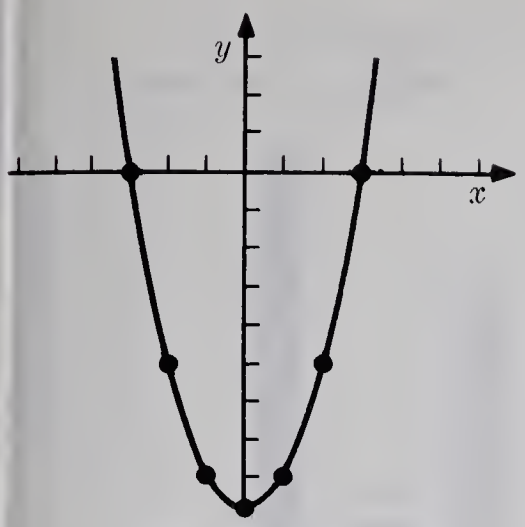

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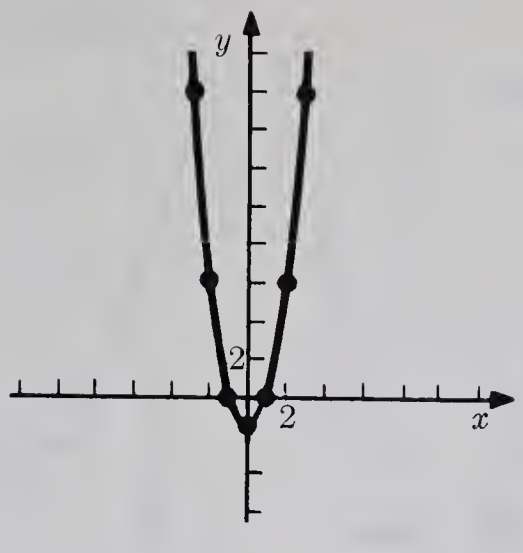

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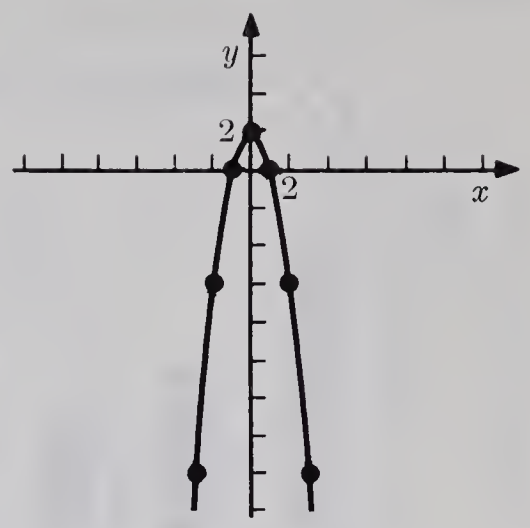

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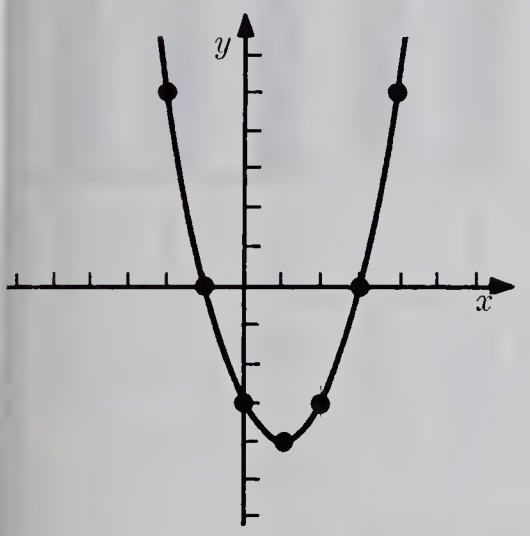

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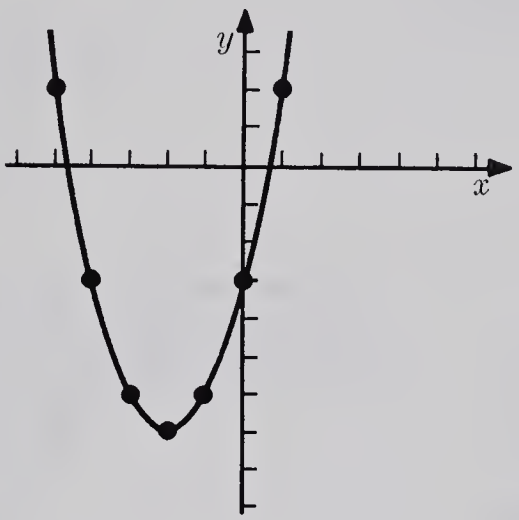

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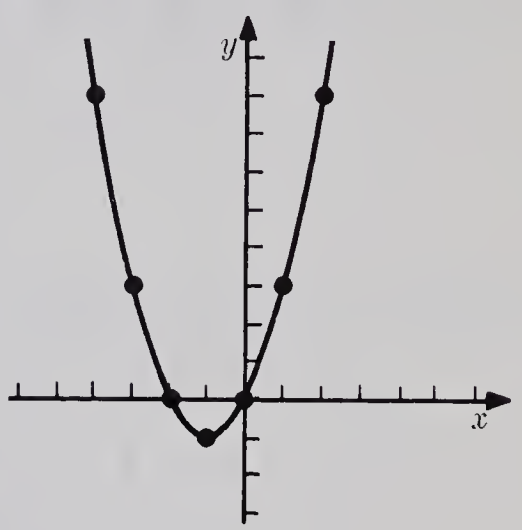

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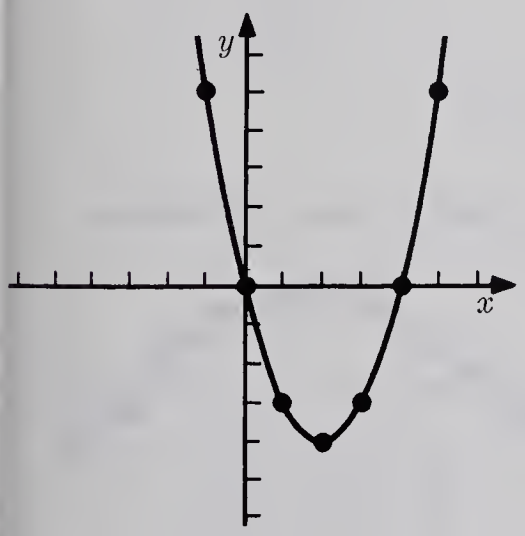

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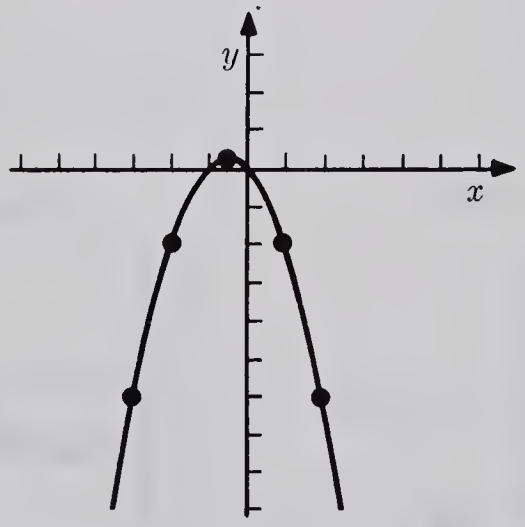

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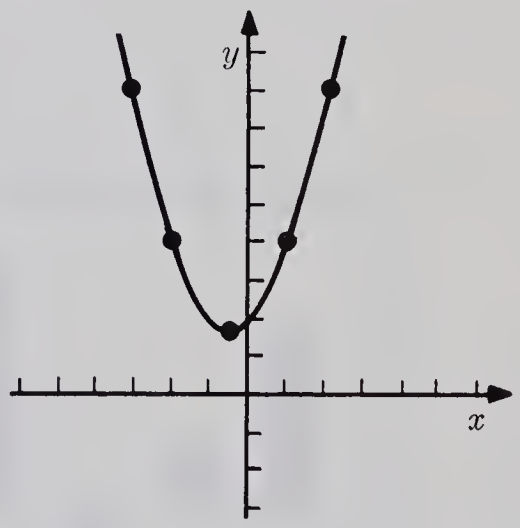

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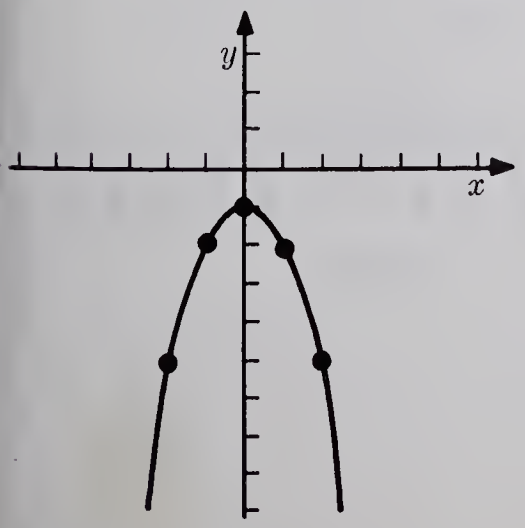

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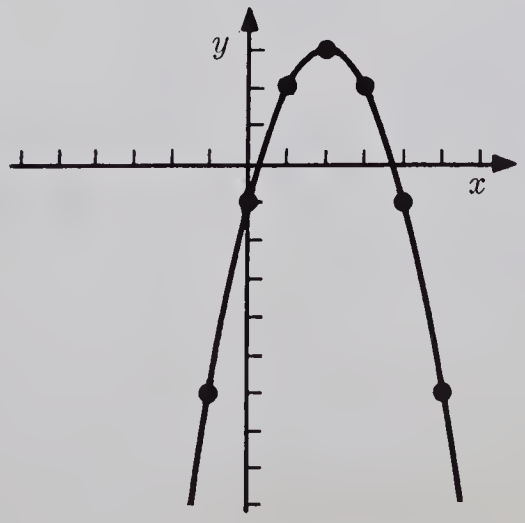
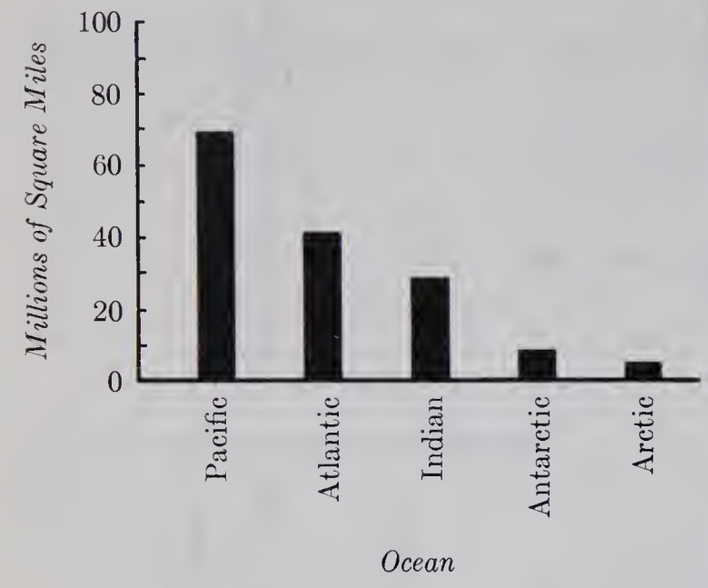


Key to Chapter 9, page 356

Pages 356–357 · **WRITTEN EXERCISES**

1.



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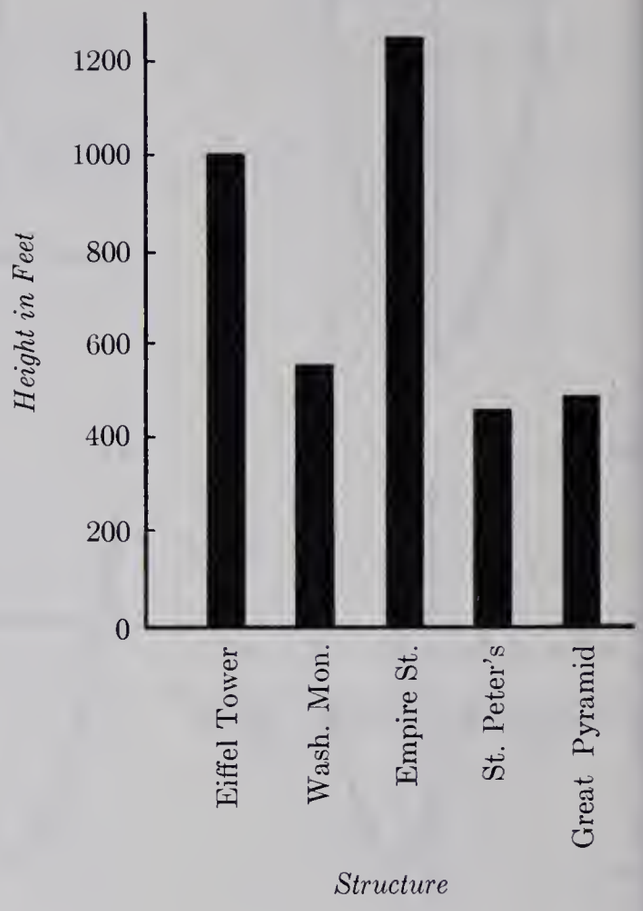

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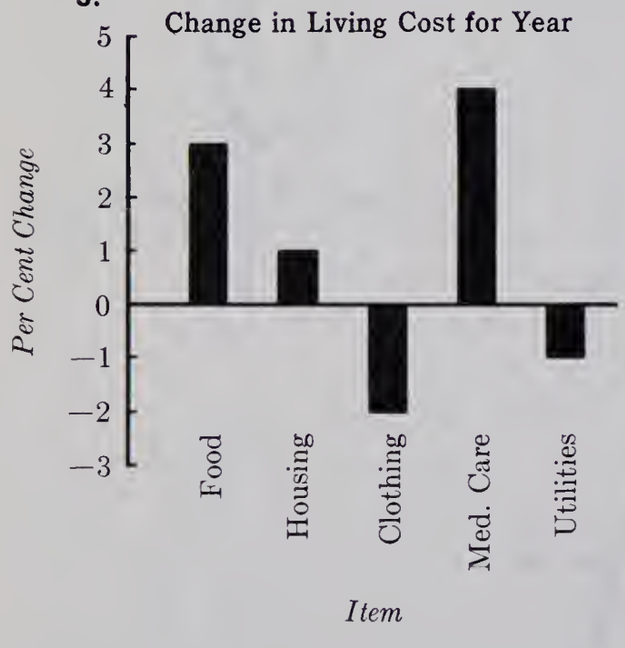

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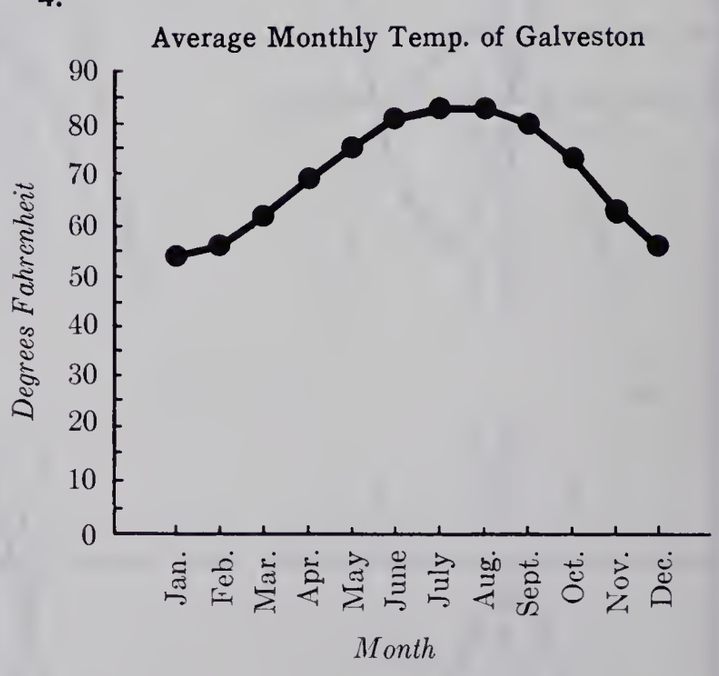

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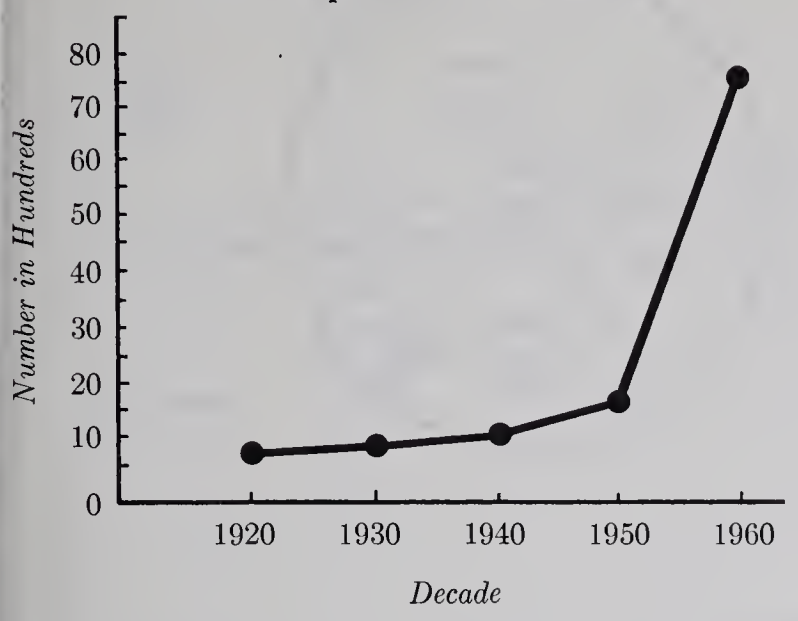

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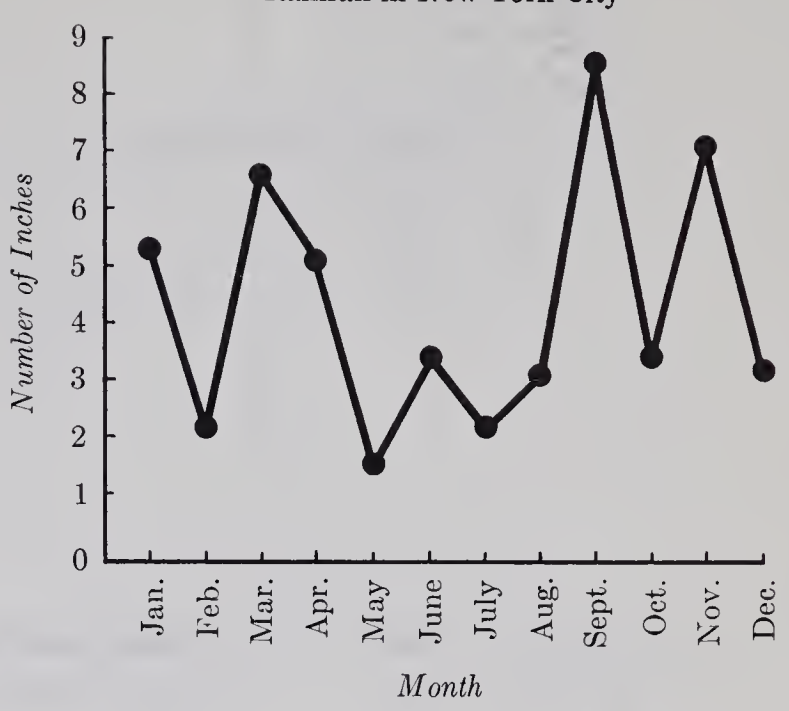

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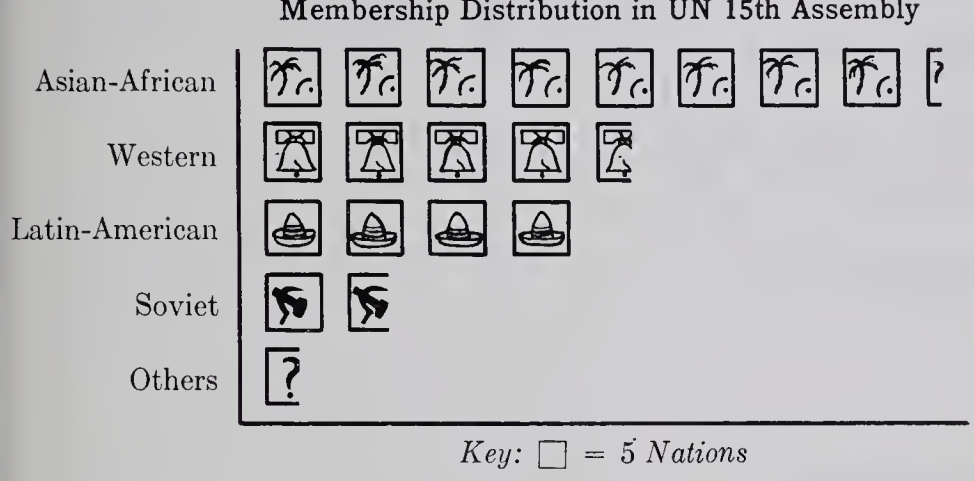

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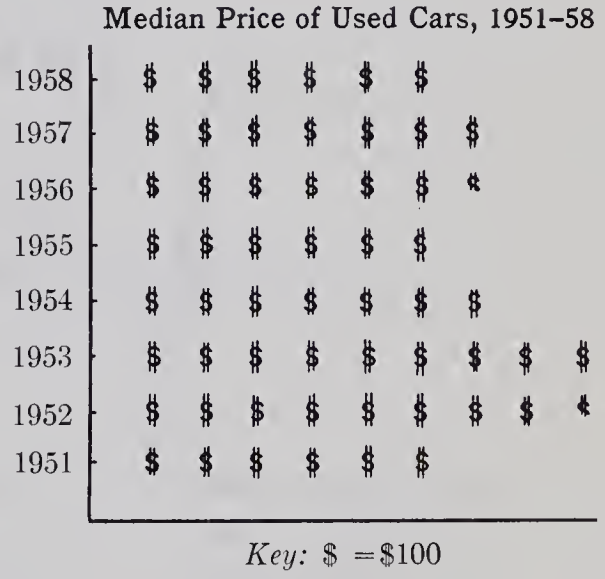
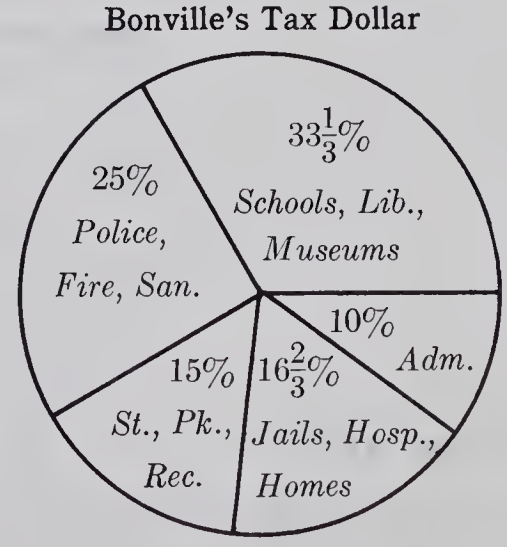


Pages 357–358 · **WRITTEN EXERCISES**

1. a) $33\frac{1}{3}\% = \frac{100}{300}$; $\frac{100}{300} \times 360 = 120°$
$\frac{25}{100} \times 360 = 90°$
$\frac{15}{100} \times 360 = 54°$
$16\frac{2}{3}\% = \frac{50}{300}$; $\frac{50}{300} \times 360 = 60°$
$\frac{10}{100} \times 360 = 36°$

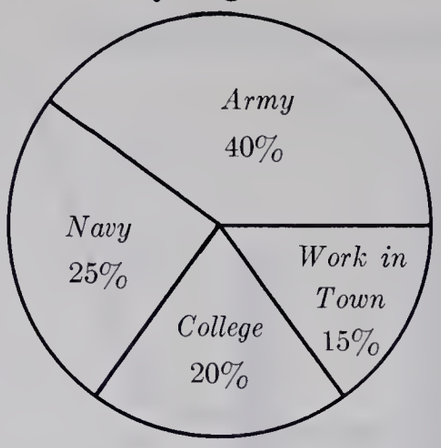
b) Let x = total budget;
$.15x = 31{,}500$;
$15x = 3{,}150{,}000$;
$x = \$210{,}000$



2. a) $\frac{40}{100} \times 360 = 144°$
$\frac{25}{100} \times 360 = 90°$
$\frac{20}{100} \times 360 = 72°$
$\frac{15}{100} \times 360 = 54°$

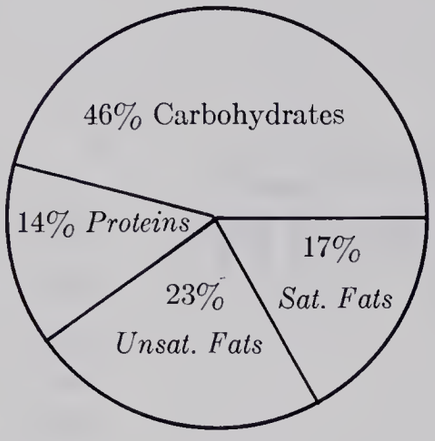
b) Let N = total no. of graduates;
$4N = 80$;
$4N = 800$;
$N = 200$

Where Midville Boys High School Graduates Go



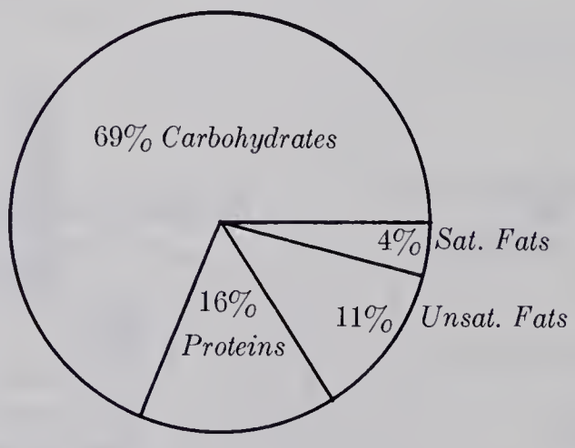
3. $\frac{46}{100} \times 360 \doteq 166°$
$\frac{14}{100} \times 360 \doteq 50°$
$\frac{23}{100} \times 360 \doteq 83°$
$\frac{17}{100} \times 360 \doteq 61°$

Average American Diet



4. $\frac{69}{100} \times 360 \doteq 248°$
$\frac{16}{100} \times 360 \doteq 58°$
$\frac{11}{100} \times 360 \doteq 40°$
$\frac{4}{100} \times 360 \doteq 14°$

An Ideal Diet



Pages 360–361 · **CHAPTER TEST**

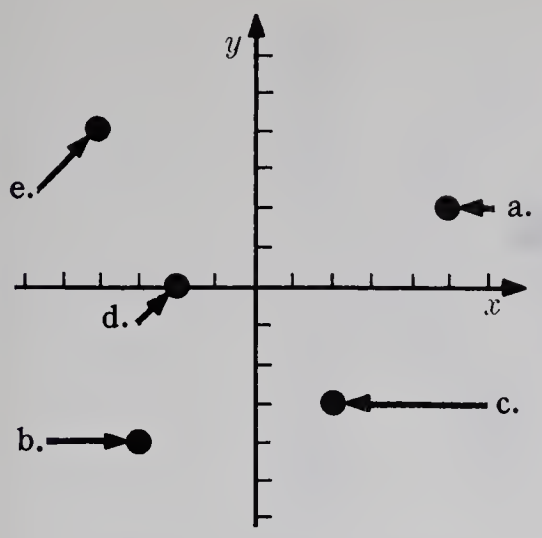
1. $y = -4$; $3x + 4(-4) = 17$; $3x - 16 = 17$; $3x = 33$; $x = 11$; $c = 11$, *Ans.*

2. $3a - 1 = 5a - 7$; $6 = 2a$; $3 = a$ and $b^2 = 2b - 1$; $b^2 - 2b + 1 = 0$; $(b - 1)(b - 1) = 0$; $b - 1 = 0$ or $b - 1 = 0$, $b = 1$. $a = 3$ and $b = 1$, *Ans.*

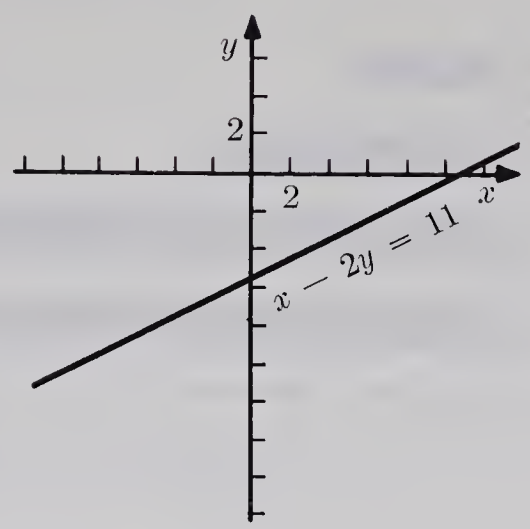
3. When $x = -1$, $2(-1) + y < 3$; $-2 + y < 3$; $y < 5$. When $x = 0$, $y < 3$. When $x = 1$, $2(1) + y < 3$, $y < 1$. $\{(-1, 0), (-1, 1), (0, 0), (0, 1), (1, 0)\}$

4. a. $(-6, 9)$ b. $(3, 5)$ c. $(0, 2)$ d. $(-3, -2)$ e. $(5, -6)$

5.



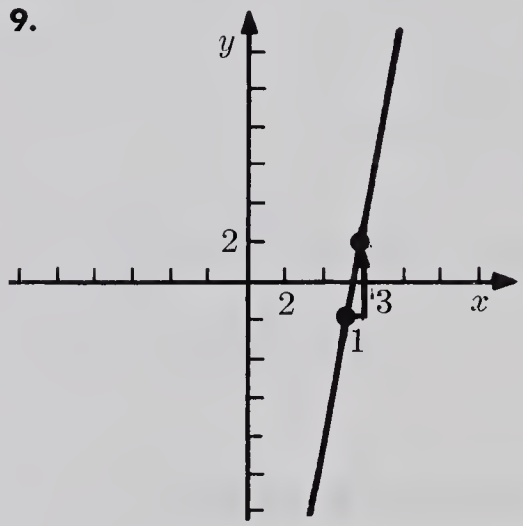
6. $x - 2y = 11; -2y = 11 - x; y = \frac{1}{2}x - \frac{11}{2}$



7. $(2, 7)$: $2 \cdot 7 - 3 \cdot 2 \stackrel{?}{=} 8$; $14 - 6 \stackrel{?}{=} 8$; $8 = 8$. $(7, 2)$: $2 \cdot 2 - 3 \cdot 7 \stackrel{?}{=} 8$; $4 - 21 \stackrel{?}{=} 8$; $-17 \neq 8$. $(2, 7)$ does belong to the graph of $2y - 3x = 8$, but $(7, 2)$ does not.

8. Slope $= \dfrac{18 - 2}{1 - (-7)} = \dfrac{16}{8} = 2.$

9.



10. $2(3) + a(-1) = 4; 6 - a = 4; a = 2$, *Ans.*

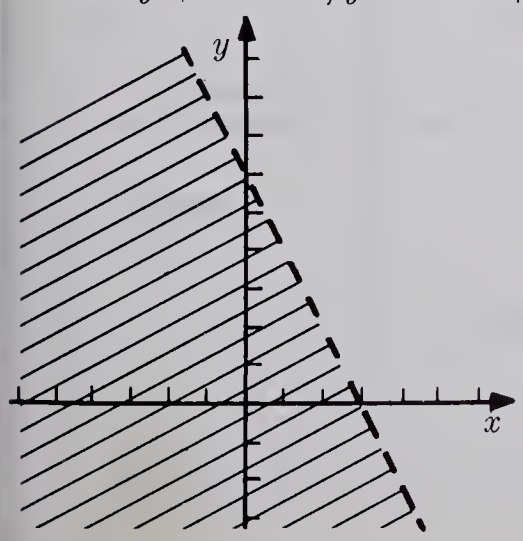
11. $5x - 3y = 24; -3y = -5x + 24; y = \frac{5}{3}x - 8$. Slope $= \frac{5}{3}$, y-intercept $= -8$, *Ans.*

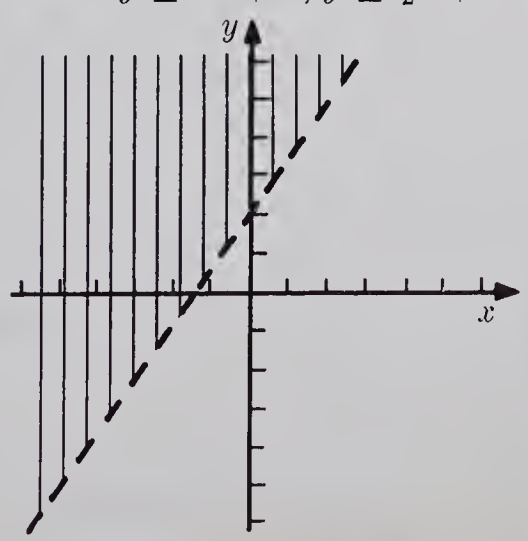
12. $y = 10x + b$; the point $(0, -4)$ gives the y-intercept, b, as -4; $y = 10x - 4$, *Ans.*

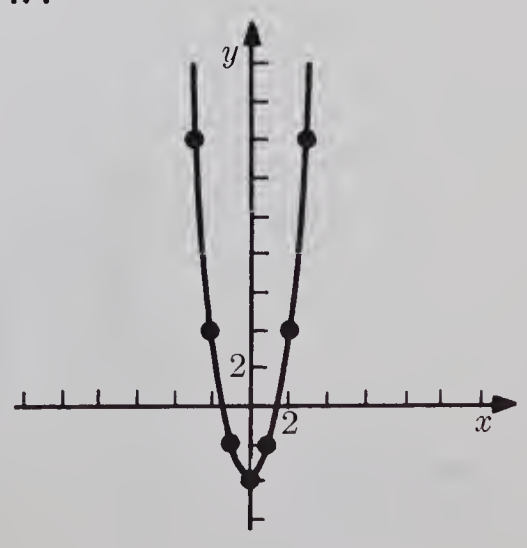
13. $y = -\frac{7}{4}x + b; 0 = -\frac{7}{4} \cdot 8 + b; b = 14; y = -\frac{7}{4}x + 14$ *or* $4y = -7x + 56$, *Ans.*

14. Slope $= \dfrac{1 - (-3)}{5 - 7} = \dfrac{4}{-2} = -2; y = -2x + b; 1 = -2(5) + b; b = 11; y = -2x + 11$, *Ans.*

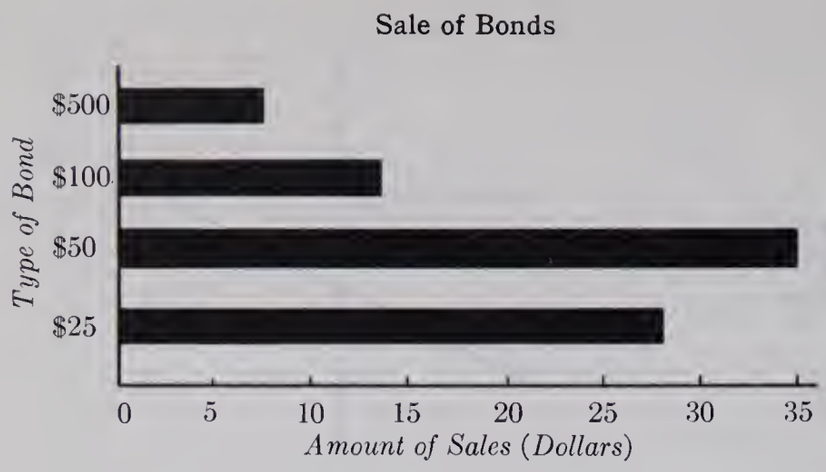
15. $y + 2x < 6; y < -2x + 6$ **16.** $2y \geq 3x + 4; y \geq \frac{3}{2}x + 2$ **17.**



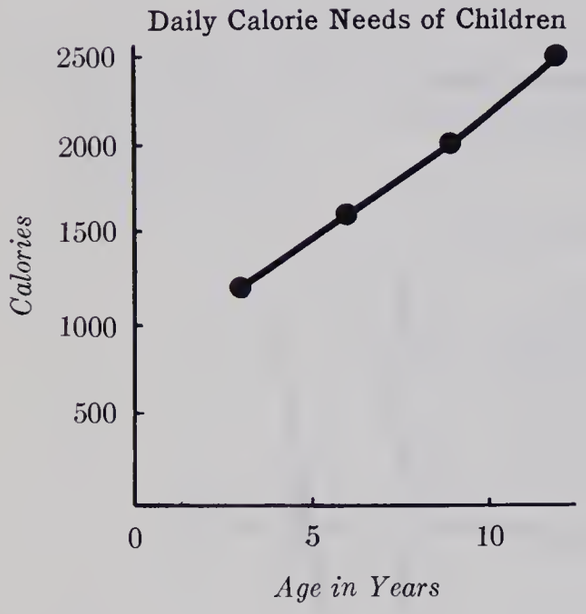




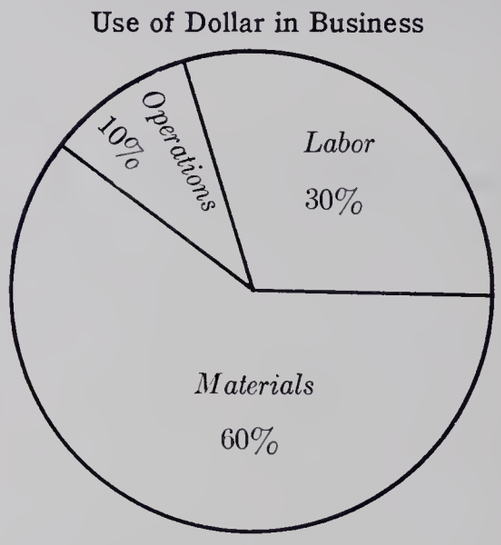
18.



19.



20. Labor: 30%; $\frac{30}{100} \times 360° = 108°$
Operations: 10%; $\frac{10}{100} \times 360° = 36°$
Materials: 60%; $\frac{60}{100} \times 360° = 216°$



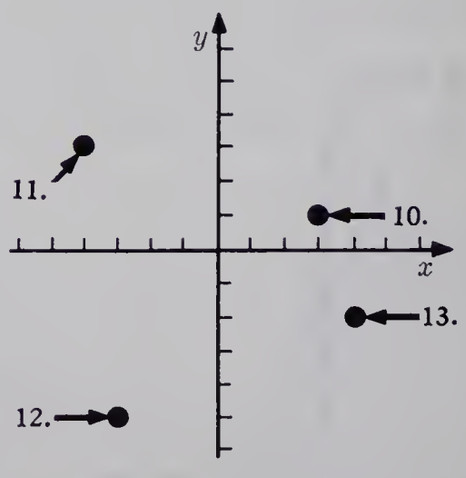
Pages 361–364 · **CHAPTER REVIEW**

1. ordered pair **2.** $a = c, b = d$ **3.** $10(3) + y = 7; y = -23; -23$, *Ans.*

4. $x = -1$, $5(-1) - 2y = 11$; $-5 - 2y = 11$; $-2y = 16$; $y = -8$. $x = 0$; $5(0) - 2y = 11$; $-2y = 11$; $y = -\frac{11}{2}$; $-\frac{11}{2} \notin$ {integers}. $x = 1$; $5(1) - 2y = 11$; $-2y = 6$; $y = -3$; $\{(-1, -8), (1, -3)\}$

5. number lines, origin **6.** abscissa; ordinate **7.** 3, x

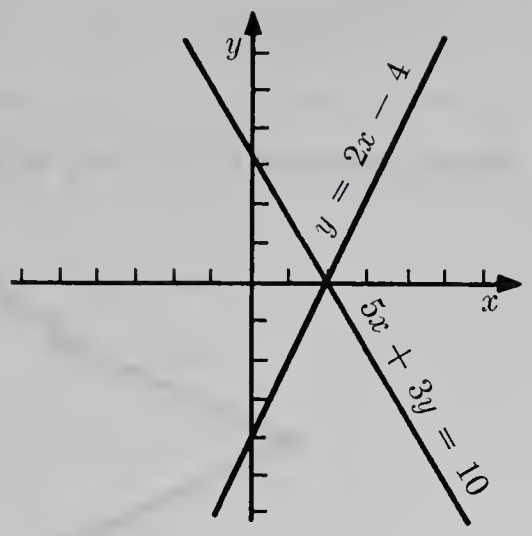
8. first, fourth **9.** third **10–13.**



14. satisfy **15.** $5 + 2y = 7; 2y = 2; y = 1; 1$, *Ans.*

16. constant, one **17.** straight line

18.



19. steepness

20. ordinates, abscissas

21. Slope $= \frac{-6-4}{7-5} = \frac{-10}{2} = -5$

22. zero **23.** no **24.** positive

25. negative **26.** m, b **27.** ordinate, y

28. $y = 4x - 5$; -5, 4, *Ans.* **29.** $y = -7x + 1$

30. $1 = 2(-5) + b$; $1 = -10 + b$; $b = 11$; 11, *Ans.*

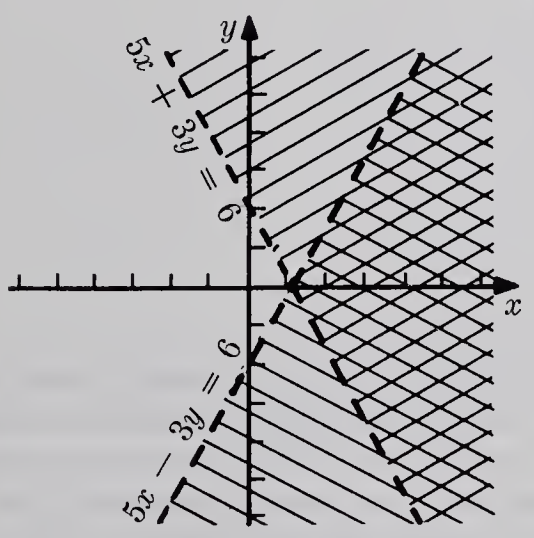
31. $y = -4x + b$; $-6 = -4(2) + b$; $b = 2$; $y = -4x + 2$, *Ans.*

32. Slope $= \frac{1-(-2)}{2-(-4)} = \frac{3}{6} = \frac{1}{2}$; $y = \frac{1}{2}x + b$; $1 = \frac{1}{2} \cdot 2 + b$; $b = 0$; $y = \frac{1}{2}x$ *or* $2y = x$, *Ans.*

33. Find slope of: $4x - 2y = 1$; $-2y = -4x + 1$; $y = 2x - \frac{1}{2}$; $m = 2$. Required line: $y = 2x + b$; $-4 = 2(-1) + b$; $b = -2$; $y = 2x - 2$, *Ans.*

34. half-planes. $y > 3x - 1$; $y < 3x - 1$ **35.** $=$. below, solid **36.** y

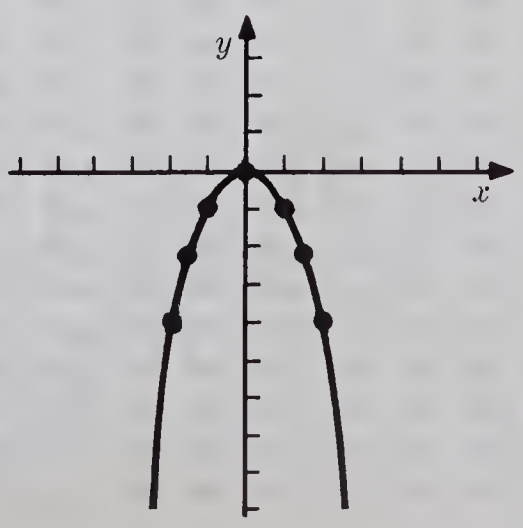
37. Transform each inequality into one having y as one member. $5x + 3y > 6$; $3y > -5x + 6$; $y > -\frac{5}{3}x + 2$ and $5x - 3y > 6$; $-3y > -5x + 6$; $y < \frac{5}{3}x - 2$



38.

x	-3	-2	-1	0	1	2	3
y	-9	-4	-1	0	-1	-4	-9

39.

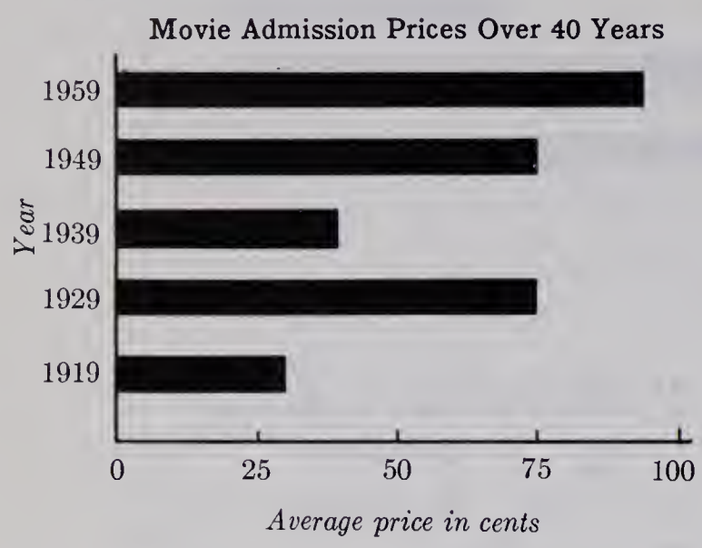


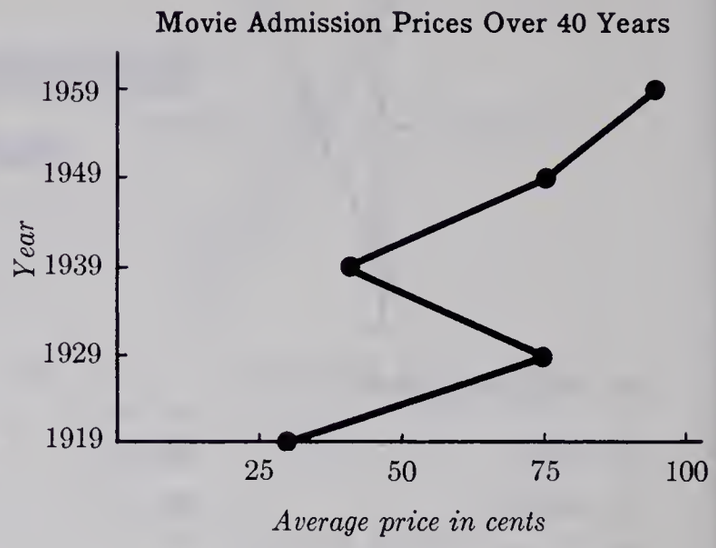
40. Ordinate is least at point where $x = 0$: $y = (0)^2 + 2$; $y = 2$; (0, 2), *Ans.*

41. Ordinate is greatest at point where $x = 0$: $y = 3 - (0)^2$; $y = 3$; (0, 3), *Ans.*

42. plotted

43.

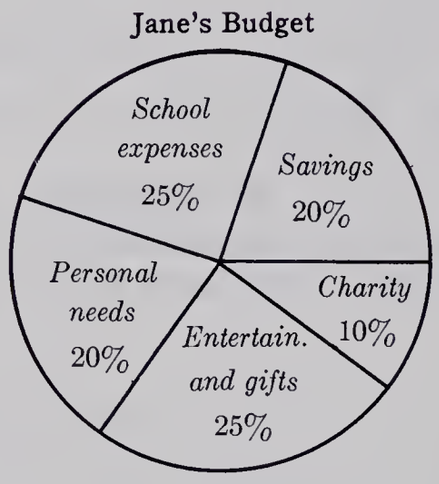




44. parts, total

45. central

46. $\frac{20}{100} \times 360° = 72°$
$\frac{25}{100} \times 360° = 90°$
$\frac{10}{100} \times 360° = 36°$



Page 364 · EXTRA FOR EXPERTS

1.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~
101	~~102~~	103	~~104~~	~~105~~	~~106~~	107	~~108~~	109	~~110~~
~~111~~	~~112~~	113	~~114~~	~~115~~	~~116~~	~~117~~	~~118~~	~~119~~	~~120~~
~~121~~	~~122~~	~~123~~	~~124~~	~~125~~	~~126~~	127	~~128~~	~~129~~	~~130~~
131	~~132~~	~~133~~	~~134~~	~~135~~	~~136~~	137	~~138~~	139	~~140~~
~~141~~	~~142~~	~~143~~	~~144~~	~~145~~	~~146~~	~~147~~	~~148~~	149	~~150~~
151	~~152~~	~~153~~	~~154~~	~~155~~	~~156~~	157	~~158~~	~~159~~	~~160~~
~~161~~	~~162~~	163	~~164~~	~~165~~	~~166~~	167	~~168~~	~~169~~	~~170~~
~~171~~	~~172~~	173	~~174~~	~~175~~	~~176~~	~~177~~	~~178~~	179	~~180~~
181	~~182~~	~~183~~	~~184~~	~~185~~	~~186~~	~~187~~	~~188~~	~~189~~	~~190~~
191	~~192~~	193	~~194~~	~~195~~	~~196~~	197	~~198~~	199	~~200~~

2. (a) 7, because 7 is the largest prime less than $\sqrt{100}$. Stop as soon as you reach a prime p such that $p > \sqrt{N}$, because the cancellation of every pth number from p will be a repetition of cancellations already effected.
(b) 13, since $17 > \sqrt{200}$

3. (a) 15 (b) 10 (c) 10 (d) 11

4. None has been found. $p = n^2 - n + 41$ yields primes for all n such that $n < 41$, but when $n = 41$, $p = (41)^2$, a composite number. $p = n^2 - 79n + 1601$ yields primes for all $n < 80$.

5. Yes, 2. Since 2 is the first even integer (positive) and is divisible only by 1 and itself; all other even numbers are divisible by 2 and are therefore not prime.

CHAPTER 10. Sentences in Two Variables

Page 370 · WRITTEN EXERCISES

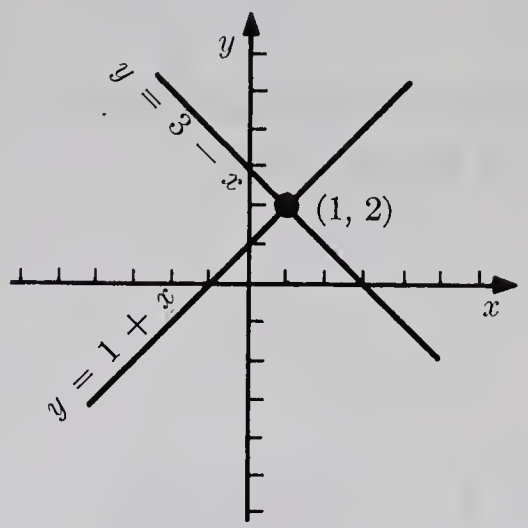
Unless otherwise indicated the scale is one division to one unit.

A **1.**

$y = 3 - x$	
x	y
0	3
3	0
1	2

$y = 1 + x$	
x	y
0	1
−1	0
1	2

$\{(1, 2)\}$

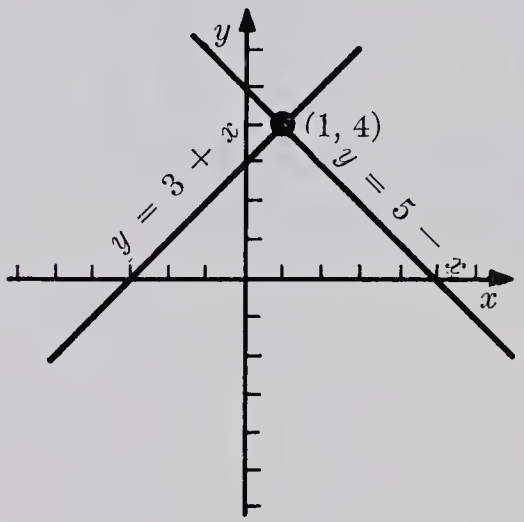


2.

$y = 5 - x$	
x	y
0	5
5	0
1	4

$y = 3 + x$	
x	y
0	3
−3	0
1	4

$\{(1, 4)\}$

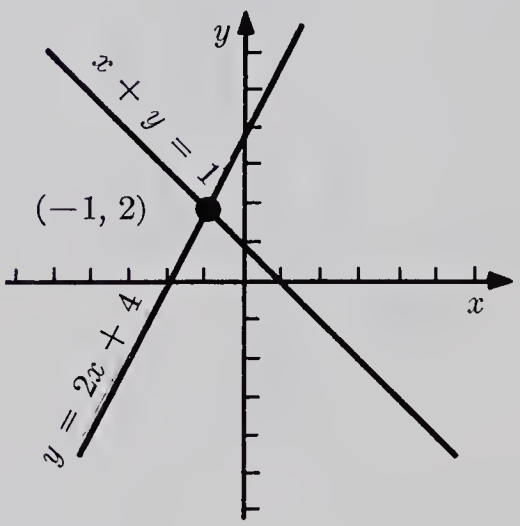


3.

$y = 2x + 4$	
x	y
0	4
−2	0
1	6

$y = 1 - x$	
x	y
0	1
1	0
−1	2

$\{(-1, 2)\}$

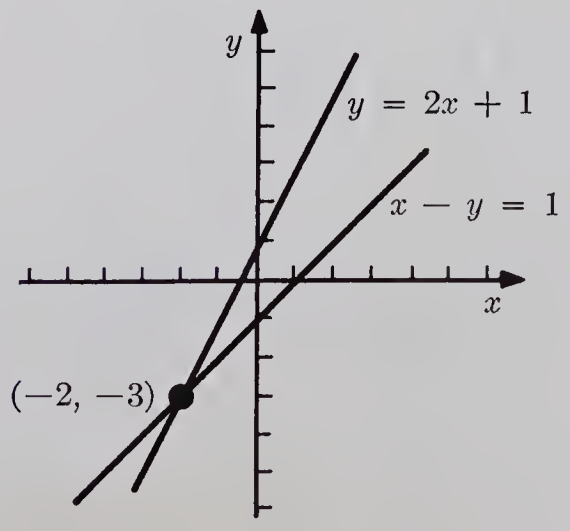


4.

$y = x - 1$	
x	y
0	−1
1	0
2	1

$y = 2x + 1$	
x	y
0	1
$-\frac{1}{2}$	0
1	3

$\{(-2, -3)\}$



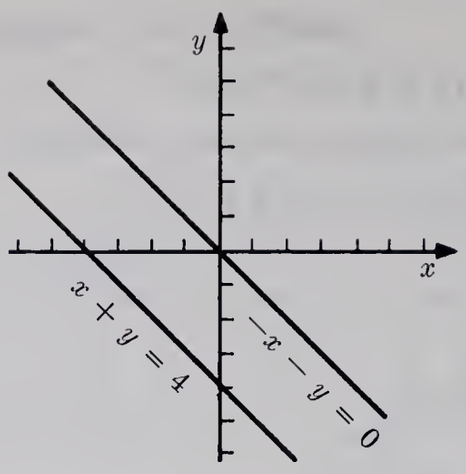
Key to Chapter 10, page 370

5.

$y = -x$	
x	y
0	0
1	−1
3	−3

$y = -x - 4$	
x	y
0	−4
−4	0
1	−5

∅ (the lines are parallel)

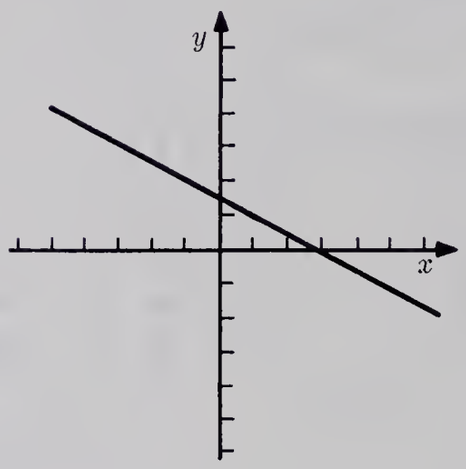


6.

$2y = 3 - x$	
x	y
0	$\frac{3}{2}$
3	0
1	1

$4y = 6 - 2x$	
x	y
0	$\frac{3}{2}$
3	0
1	1

{all real nos.} (the lines coincide)

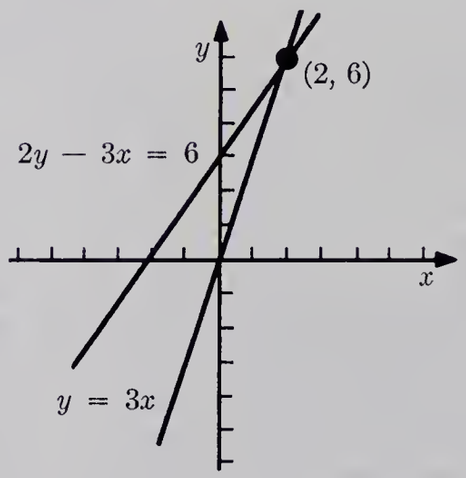


7.

$2y = 3x + 6$	
x	y
0	3
−2	0
2	6

$y = 3x$	
x	y
0	0
1	3
−1	−3

{(2, 6)}

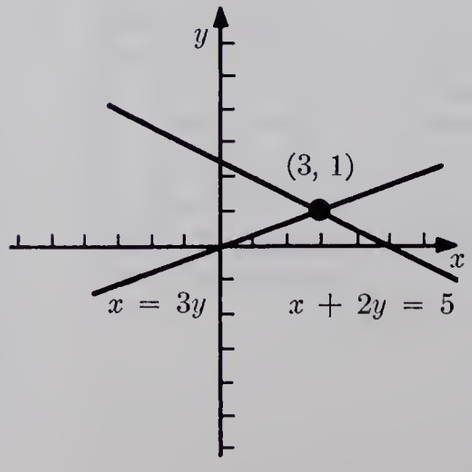


8.

$2y = 5 - x$	
x	y
0	$\frac{5}{2}$
5	0
1	2

$x = 3y$	
x	y
0	0
3	1
6	2

{(3, 1)}

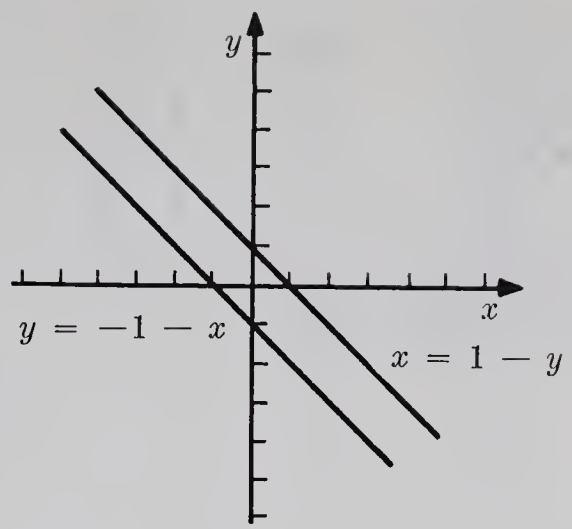


9.

$x = 1 - y$	
x	y
0	1
1	0
2	−1

$y = -1 - x$	
x	y
0	−1
−1	0
2	−3

$\emptyset$ (the lines are parallel)

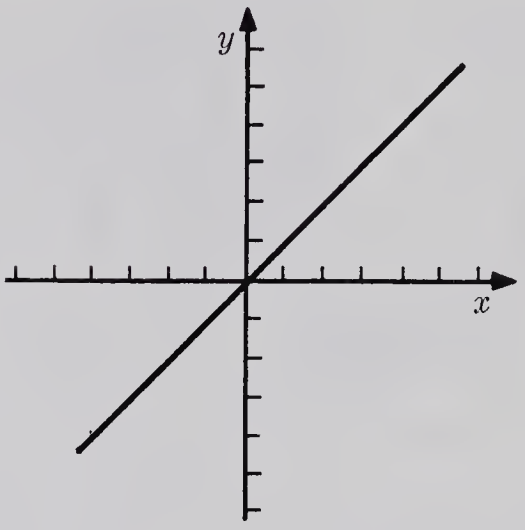


10.

$x = y$	
x	y
0	0
2	2
4	4

$3y = 0 + 3x$	
x	y
0	0
2	2
4	4

{all real nos.} (lines coincide)

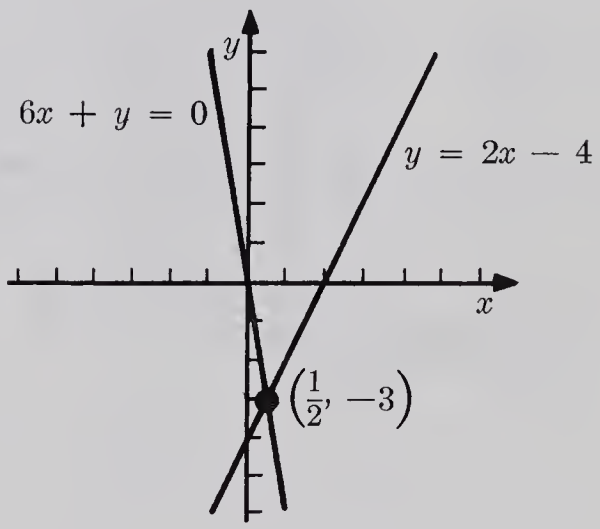


11.

$y = 2x - 4$	
x	y
0	−4
2	0
1	−2

$y = -6x$	
x	y
0	0
1	−6
−1	6

$\{(\frac{1}{2}, -3)\}$

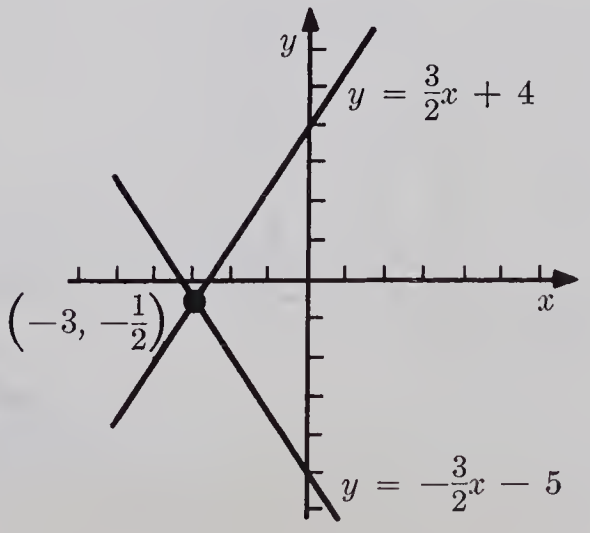


12.

$y = \frac{3}{2}x + 4$	
x	y
0	4
$-\frac{8}{3}$	0
2	7

$y = -\frac{3}{2}x - 5$	
x	y
0	−5
$-\frac{10}{3}$	0
−2	−2

$\{(-3, -\frac{1}{2})\}$

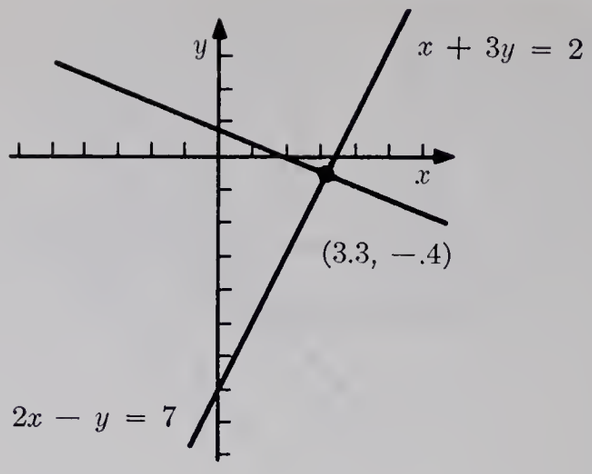


B **13.**

$y = 2x - 7$	
x	y
0	-7
$\frac{7}{2}$	0
1	-5

$3y = 2 - x$	
x	y
0	$\frac{2}{3}$
2	0
-1	1

$\{(3.3, -.4)\}$

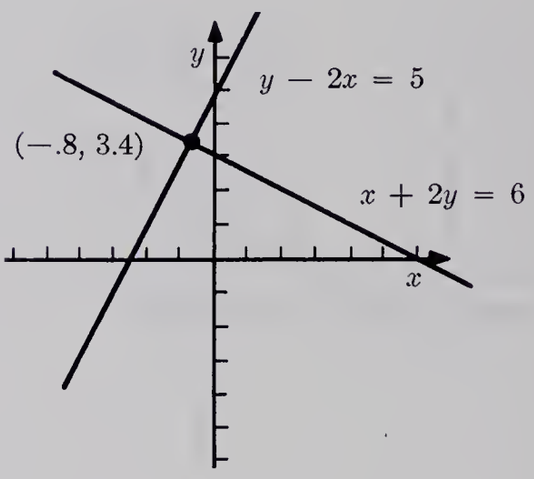


14.

$2y = 6 - x$	
x	y
0	3
6	0
2	2

$y = 5 + 2x$	
x	y
0	5
$-\frac{5}{2}$	0
1	7

$\{(-.8, 3.4)\}$

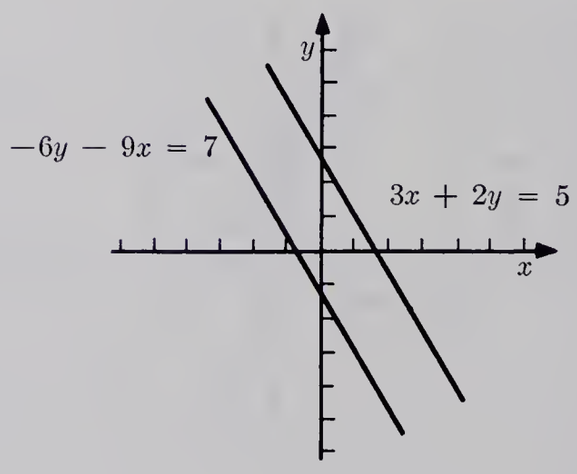


15.

$2y = 5 - 3x$	
x	y
0	$\frac{5}{2}$
$\frac{5}{3}$	0
1	1

$6y = -7 - 9x$	
x	y
0	$-\frac{7}{6}$
$-\frac{7}{9}$	0
1	$-\frac{8}{3}$

$\emptyset$ (the lines are parallel)

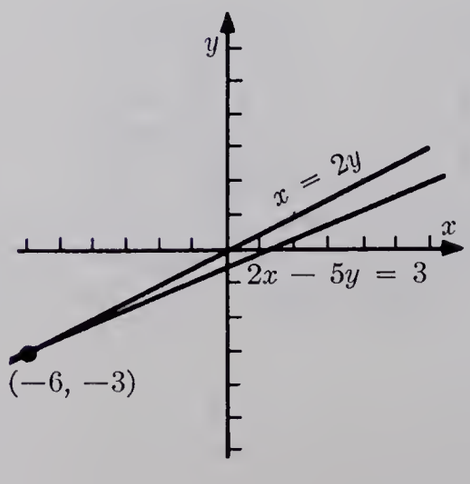


16.

$5y = 2x - 3$	
x	y
0	$-\frac{3}{5}$
$\frac{3}{2}$	0
4	1

$x = 2y$	
x	y
0	0
2	1
4	2

$\{(-6, -3)\}$

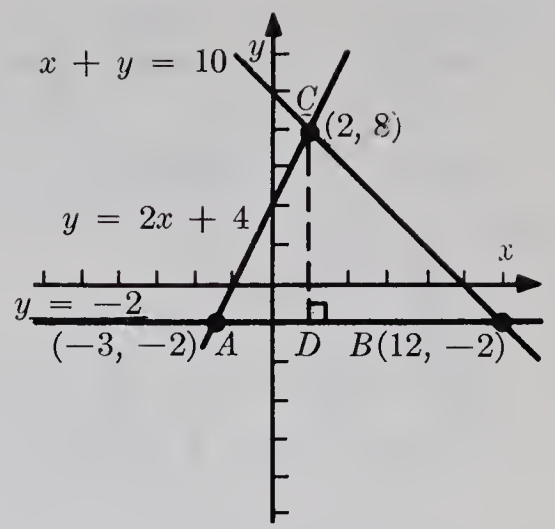


17.

$y = 2x + 4$	
x	y
0	4
−2	0
1	6

$y = 10 - x$	
x	y
0	10
10	0
5	5

$y = -2$	
x	y
0	−2
2	−2
−2	−2



Altitude $= \overline{CD} = 10$
Base $= \overline{AB} = 15$
Area $\triangle ABC = \frac{1}{2}(10)(15) = 75$ sq units

Page 372 · WRITTEN EXERCISES

A

1. $2x = 90; x = 45; (45) + y = 74; y = 29.$ $\{(45, 29)\}$

2. $2w = 150; w = 75; (75) + z = 122; z = 47.$ $\{(75, 47)\}$

3. $4r = 68; r = 17; 2(17) + s = 48; 34 + s = 48; s = 14.$ $\{(17, 14)\}$

4. $6m = 78; m = 13; 3(13) + n = 60; 39 + n = 60; n = 21.$ $\{(13, 21)\}$

5. $p = 3; 2(3) - q = 7; 6 - q = 7; -q = 1; q = -1.$ $\{(3, -1)\}$

6. $2A = 2; A = 1; 5(1) - B = 7; 5 - B = 7; -B = 2; B = -2.$ $\{(1, -2)\}$

7. $2s = 12; s = 6; 3(6) + 2t = 17; 18 + 2t = 17; 2t = -1; t = -\frac{1}{2}.$ $\{(6, -\frac{1}{2})\}$

8. $6w = 0; w = 0; 2(0) + 5z = -10; 0 + 5z = -10; z = -2.$ $\{(0, -2)\}$

9. $7m = 14; m = 2; 2(2) + 3n = -5; 4 + 3n = -5; 3n = -9; n = -3.$ $\{(2, -3)\}$

10. $-2t = 0; t = 0; 2p - 5(0) = 12; 2p = 12; p = 6.$ $\{(6, 0)\}$

11. $-2s = 2; s = -1; 3r - 4(-1) = 1; 3r + 4 = 1; 3r = -3; r = -1.$ $\{(-1, -1)\}$

12. $-2x = 2; x = -1; 2(-1) - 3y = -4; -2 - 3y = -4; -3y = -2; y = \frac{2}{3}.$ $\{(-1, \frac{2}{3})\}$

13. $-10 = -2n; 5 = n; 2 = p - 3(5); 2 = p - 15; 17 = p.$ $\{(5, 17)\}$

14. $-14 = -2t; 7 = t; 6 = s - 3(7); 6 = s - 21; 27 = s.$ $\{(27, 7)\}$

15. $6s = -36; s = -6; 3(-6) + 5t = 2; -18 + 5t = 2; 5t = 20; t = 4.$ $\{(-6, 4)\}$

16. $14x = 42; x = 3; 3(3) + y = 0; 9 + y = 0; y = -9.$ $\{(3, -9)\}$

17. $-4w = -72; w = 18; 3(18) + 8z = 158; 54 + 8z = 158; 8z = 104; z = 13.$ $\{(18, 13)\}$

18. $-3m = -51; m = 17; 2(17) + 11n = 265; 34 + 11n = 265; 11n = 231; n = 21.$ $\{(17, 21)\}$

B

19. $y + x = 8$, $x - y = 4$

$$\begin{array}{r} y + x = 8 \\ -y + x = 4 \\ \hline 2x = 12 \end{array}; x = 6;$$

$y + (6) = 8; y = 2.$ $\{(6, 2)\}$

20. $2x - 3y = 16$, $3y + 2x = -8$

$$\begin{array}{r} 2x - 3y = 16 \\ 2x + 3y = -8 \\ \hline 4x = 8 \end{array}; x = 2;$$

$2(2) - 3y = 16; \quad 4 - 3y = 16; \quad -3y = 12;$
$y = -4.$ $\{(2, -4)\}$

21. $3r - 5s = 0$, $5s + 3r = -60$

$$\begin{array}{r} 3r - 5s = 0 \\ 3r + 5s = -60 \\ \hline 6r = -60 \end{array}; r = -10;$$

$\frac{-10}{5} - \frac{s}{3} = 0; -30 - 5s = 0; -5s = 30; s = -6.$ $\{(-10, -6)\}$

22. $3m - 2n = 18$, $4n - 3m = -24$

$$\begin{array}{r} 3m - 2n = 18 \\ -3m + 4n = -24 \\ \hline 2n = -6 \end{array}; n = -3;$$

$\frac{m}{2} - \frac{(-3)}{3} = 3; 3m + 6 = 18, 3m = 12, m = 4.$ $\{(4, -3)\}$

23. $50[.7(r - c)] = 50[7]$, $70[.5(r + c)] = 70[7]$

$$\begin{array}{r} 35r - 35c = 350 \\ 35r + 35c = 490 \\ \hline 70r = 840 \end{array}; r = 12;$$

$.7(12 - c) = 7; 10[.7(12 - c)] = 10[7]; 84 - 7c = 70; -7c = -14; c = 2.$ $\{(2, 12)\}$

24. $200[.75(r - c)] = 200[3]$. $150r - 150c = 600$
$300[.50(r + c)] = 300[3]$' $150r + 150c = 900$
$300r = 1500$; $r = 5$;
$.75(5 - c) = 3$; $100[.75(5 - c)] = 100[3]$; $375 - 75c = 300$; $-75c = -75$; $c = 1$. $\{(1, 5)\}$

25. $4p - 3q = 1$
$6p - 3q = 2$
$-2p = -1$; $p = \frac{1}{2}$;
$\frac{2(\frac{1}{2})}{5} - \frac{3q}{10} = .1$; $\frac{1}{5} - \frac{3q}{10} = .1$; $2 - 3q = 1$;
$-3q = -1$; $q = \frac{1}{3}$. $\{(\frac{1}{2}, \frac{1}{3})\}$

26. $5u - 3v = 0$
$5u + 6v = -3$
$-9v = 3$; $v = -\frac{1}{3}$;
$\frac{5u}{3} - \left(-\frac{1}{3}\right) = 0$; $\frac{5u}{3} + \frac{1}{3} = 0$; $5u + 1 = 0$;
$5u = -1$; $u = -\frac{1}{5}$. $\{(-\frac{1}{5}, -\frac{1}{3})\}$

Page 374 · **PROBLEMS**

A **1.** $l + w = 72$
$l - w = 12$
$2l = 84$; $l = 42$ ft; $42 + w = 72$; $w = 72 - 42 = 30$ ft. 30 ft, 42 ft, *Ans.*

2. Let x = Mr. Alden's acreage; let y = Mr. Bradford's acreage;
$x - y = 82$
$x + y = 276$
$2x = 358$; $x = 179$ A; $179 + y = 276$; $y = 276 - 179 = 97$ A. 179 A, 97 A, *Ans.*

3. Let x = smaller no.; let y = larger no.;
$2y - x = 21$
$2y + x = 27$
$4y = 48$; $y = 12$; $2(12) - x = 21$; $24 - x = 21$; $-x = -3$; $x = 3$. 3, 12, *Ans.*

4. Let x = Mary's age; let y = Ann's age;
$2x - y = 19$
$2x + y = 41$
$4x = 60$; $x = 15$ yr; $2 \cdot 15 + y = 41$; $y = 41 - 30 = 11$ yr. 11 yr, 15 yr, *Ans.*

5. Let x = lb of white flour; let y = lb of whole-wheat flour;
$3y = x + 11$
$2y = x - 2$
$y = 13$ lb; $2 \cdot 13 = x - 2$; $x = 26 + 2 = 28$ lb. 13 lb, 28 lb, *Ans.*

6. Let x = cost of each cheaper magazine; let y = cost of each more expensive magazine;
$5x = y + 25$
$3x = y - 5$
$2x = 30$; $x = 15¢$; $3 \cdot 15 = y - 5$; $y = 45 + 5 = 50¢$. 15¢, 50¢, *Ans.*

7. Let x = doz at 53¢; let y = doz at 59¢;
$53x + 59y = 1550$
$53x - 59y = 252$
$106x = 1802$; $x = 17$ doz; $53 \cdot 17 + 59y = 1550$; $59y = 1550 - 901 = 649$; $y = 11$ doz.
11 doz, 17 doz, *Ans.*

8. Let x = doz at \$52; y = doz at \$40;
$52x + 40y = 224$. $52x + 40y = 224$
$40y - 52x = 16$' $-52x + 40y = 16$
$80y = 240$; $y = 3$;
$52x + 40(3) = 224$; $52x + 120 = 224$; $52x = 104$; $x = 2$. 2 doz, 3 doz, *Ans.*

9. Let t = cost of each mitt; s = cost of each glove;
$t + 8s = 41.60$
$t + 12s = 59.00$
$-4s = -17.40$; $4s = 17.40$; $s = \$4.35$
$t + 8(4.35) = 41.60$; $t + 34.80 = 41.60$; $t = \$6.80$. \$4.35, \$6.80, *Ans.*

10. Let x = dresses at \$5.95; let y = dresses at \$3.95;

$$\begin{array}{r} 5.95x + 3.95y = 287.00 \\ 5.95x - 3.95y = \ 10.50 \\ \hline 11.90x \qquad\quad = 297.50 \end{array}$$

$11.90x = 297.50$; $119x = 2975$; $x = 25$; $5.95\cdot(25) + 3.95y = 287.00$;

$3.95y = 287.00 - 148.75 = 138.25$; $395y = 13{,}825$; $y = 35$. 25, 35, *Ans.*

Pages 376–377 · WRITTEN EXERCISES

A **1.** $\begin{array}{r} 2x - 2y = 146 \\ 2x - 7y = \ 21 \\ \hline 5y = 125 \end{array}$; $y = 25$;

$x - (25) = 73$; $x = 98$. $\{(98, 25)\}$

2. $\begin{array}{r} 3m - 3n = 156 \\ 3m - 8n = \ 6 \\ \hline 5n = 150 \end{array}$; $n = 30$;

$m - (30) = 52$; $m = 82$. $\{(82, 30)\}$

3. $\begin{array}{r} 2m + 6n = 3 \\ 4m - 3n = 1 \end{array}$ $\begin{array}{r} 2m + 6n = 3 \\ 8m - 6n = 2 \\ \hline 10m \qquad = 5 \end{array}$; $m = \frac{1}{2}$;

$\frac{2(\frac{1}{2})}{3} + 2n = 1$; $\frac{1}{3} + 2n = 1$; $1 + 6n = 3$; $6n = 2$; $n = \frac{1}{3}$. $\{(\frac{1}{2}, \frac{1}{3})\}$

4. $\begin{array}{r} 2c + 5d = -10 \\ 3d - 4c = \ -6 \end{array}$ $\begin{array}{r} 4c + 10d = -20 \\ -4c + \ 3d = \ -6 \\ \hline 13d = -26 \end{array}$; $d = -2$;

$\frac{c}{5} + \frac{(-2)}{2} = -1$; $\frac{c}{5} - 1 = -1$; $c - 5 = -5$; $c = 0$. $\{(0, -2)\}$

5. $\begin{array}{r} 6s - 9t = \ 3 \\ 6s - 8t = \ 14 \\ \hline -t = -11 \end{array}$; $t = 11$; $2s - 3(11) = 1$; $2s - 33 = 1$; $2s = 34$; $s = 17$. $\{(17, 11)\}$

6. $\begin{array}{r} 6a + 15b = \ 54 \\ -6a - \ 8b = -54 \\ \hline 7b = \ 0 \end{array}$; $b = 0$; $2a + 5(0) = 18$; $2a + 0 = 18$; $2a = 18$; $a = 9$. $\{(9, 0)\}$

7. $\begin{array}{r} 5c - 10d = \ 25 \\ -5c + \ 4d = -22 \\ \hline -6d = \ 3 \end{array}$; $d = -\frac{1}{2}$; $c - 2(-\frac{1}{2}) = 5$; $c + 1 = 5$; $c = 4$. $\{(4, -\frac{1}{2})\}$

8. $\begin{array}{r} 3x - \ 2y = \ 0 \\ 9x + 10y = -96 \end{array}$ $\begin{array}{r} 15x - 10y = \ 0 \\ 9x + 10y = -96 \\ \hline 24x \qquad = -96 \end{array}$; $x = -4$;

$\frac{(-4)}{4} - \frac{y}{6} = 0$; $-1 - \frac{y}{6} = 0$; $-6 - y = 0$; $-6 = y$. $\{(-4, -6)\}$

9. $\begin{array}{r} 27q - 24r = \ 3 \\ 12q + 24r = 10 \\ \hline 39q \qquad = 13 \end{array}$; $q = \frac{1}{3}$; $9(\frac{1}{3}) - 8r = 1$; $3 - 8r = 1$; $2 = 8r$; $\frac{1}{4} = r$. $\{(\frac{1}{3}, \frac{1}{4})\}$

10. $\begin{array}{r} 20c - 5d = -50 \\ 3c + 5d = \ 4 \\ \hline 23c \qquad = -46 \end{array}$; $c = -2$; $4(-2) - d = -10$; $-8 - d = -10$; $2 = d$. $\{(-2, 2)\}$

11. $\begin{array}{r} 5p - \ 3w = \ 6 \\ -5p - 35w = 260 \\ \hline -38w = 266 \end{array}$; $w = -7$; $7(-7) + p = -52$; $-49 + p = -52$; $p = -3$. $\{(-3, -7)\}$

12. $\begin{array}{r} 3x + 2y = \ 12 \\ -3x - \ y = -9 \\ \hline y = \ 3 \end{array}$

$\frac{x}{2} + \frac{(3)}{3} = 2$; $\frac{x}{2} + 1 = 2$; $\frac{x}{2} = 1$; $x = 2$. $\{(2, 3)\}$

B **13.** $2(a) + (b) = 3$; $4a + 2b = 6$
$3(a) - 2(b) = 8$; $3a - 2b = 8$
$7a = 14$; $a = 2$;
$2(2) + b = 3$; $4 + b = 3$; $b = -1$; then $x = \frac{1}{a} = \frac{1}{2}$ and $y = \frac{1}{b} = \frac{1}{(-1)} = -1$. $\{(\frac{1}{2}, -1)\}$

14. $\frac{3}{2}\left(\frac{1}{x}\right) - 2\left(\frac{1}{y}\right) = -7$; Let $a = \frac{1}{x}$, $x = \frac{1}{a}$; $\frac{3}{2}(a) - 2(b) = -7$; $-\frac{3}{2}a + 6b = 21$
$\frac{9}{2}\left(\frac{1}{x}\right) + 5\left(\frac{1}{y}\right) = 12$ and $b = \frac{1}{y}$, $y = \frac{1}{b}$. $\frac{9}{2}(a) + 5(b) = 12$ $\frac{9}{2}a + 5b = 12$
$11b = 33$;
$b = 3$;
$\frac{3}{2}a - 2(3) = -7$; $\frac{3}{2}a - 6 = -7$; $\frac{3}{2}a = -1$; $3a = -2$; $a = -\frac{2}{3}$;
then $x = \frac{1}{a} = \frac{1}{(-\frac{2}{3})} = -\frac{3}{2}$ and $y = \frac{1}{b} = \frac{1}{3}$. $\left\{\left(-\frac{3}{2}, \frac{1}{3}\right)\right\}$

15. $4\left(\frac{1}{m}\right) + \frac{3}{2}\left(\frac{1}{n}\right) = 0$; Let $a = \frac{1}{m}$, $m = \frac{1}{a}$; $4a + \frac{3}{2}b = 0$; $8a + 3b = 0$
$\frac{2}{3}\left(\frac{1}{m}\right) + \frac{13}{4}\left(\frac{1}{n}\right) = 1$ and $b = \frac{1}{n}$, $n = \frac{1}{b}$. $\frac{2}{3}a + \frac{13}{4}b = 1$ $8a + 39b = 12$
$-36b = -12$; $b = \frac{1}{3}$;
$4a + \frac{3}{2}(\frac{1}{3}) = 0$; $4a + \frac{1}{2} = 0$; $4a = -\frac{1}{2}$; $a = -\frac{1}{8}$;
then $m = \frac{1}{a} = \frac{1}{(-\frac{1}{8})} = -8$ and $n = \frac{1}{b} = \frac{1}{\frac{1}{3}} = 3$. $\{(-8, 3)\}$

16. $\frac{3}{2}\left(\frac{1}{m}\right) - \frac{3}{4}\left(\frac{1}{n}\right) = 1$; Let $a = \frac{1}{m}$, $m = \frac{1}{a}$
$\frac{4}{3}\left(\frac{1}{m}\right) - \frac{11}{9}\left(\frac{1}{n}\right) = 2$ and $b = \frac{1}{n}$, $n = \frac{1}{b}$.
$\frac{3}{2}(a) - \frac{3}{4}(b) = 1$; $6a - 3b = 4$; $-12a + 6b = -8$
$\frac{4}{3}(a) - \frac{11}{9}(b) = 2$ $12a - 11b = 18$ $12a - 11b = 18$
$-5b = 10$; $b = -2$;
$\frac{3}{2}a - \frac{3}{4}(-2) = 1$; $\frac{3}{2}a + \frac{3}{2} = 1$; $3a + 3 = 2$; $3a = -1$; $a = -\frac{1}{3}$;
then $m = \frac{1}{a} = \frac{1}{(-\frac{1}{3})} = -3$ and $n = \frac{1}{b} = \frac{1}{-2} = -\frac{1}{2}$. $\{(-3, -\frac{1}{2})\}$

17. $2x + 4y - 6x + 3y = -18$; $-4x + 7y = -18$; $-60x + 105y = -270$
$6x - 4y - 21x - 14y = 21$; $-15x - 18y = 21$; $-60x - 72y = 84$
$177y = -354$; $y = -2$;
$\left[\frac{x + 2(-2)}{3}\right] - \left[\frac{2x - (-2)}{2}\right] = -3$; $\frac{x - 4}{3} - \frac{2x + 2}{2} = -3$; $2x - 8 - 6x - 6 = -18$;
$-4x - 14 = -18$; $-4x = -4$; $x = 1$. $\{(1, -2)\}$

18. $3x - 2y + 27x + 3y = -10$; $30x + y = -10$; $120x + 4y = -40$
$18x + 27y - 42x + 24y = 8$; $-24x + 51y = 8$; $-120x + 255y = 40$
$259y = 0$; $y = 0$;
$\frac{2x + 3(0)}{2} - \frac{7x - 4(0)}{3} = \frac{4}{9}$; $\frac{2x}{2} - \frac{7x}{3} = \frac{4}{9}$; $18x - 42x = 8$; $-24x = 8$; $x = -\frac{1}{3}$. $\{(-\frac{1}{3}, 0)\}$

19. $4ax - 6by = 26ab$
$15ax + 6by = 12ab$
$19ax = 38ab$; $x = 2b$;
$2a(2b) - 3by = 13ab$; $4ab - 3by = 13ab$; $-3by = 9ab$; $y = -3a$. $\{(2b, -3a)\}$

20. $12rx + 8sy = -76rs$
$12rx - 21sy = 69rs$
$29sy = -145rs$; $y = -5r$;
$3rx + 2s(-5r) = -19rs$; $3rx - 10rs = -19rs$; $3rx = -9rs$; $x = -3s$. $\{(-3s, -5r)\}$

Page 377 · PROBLEMS

1. Let y = savings; x = rent;
$x + y = 100$; $y + 5 = \frac{x}{2}$; $x + y = 100$; $2y + 10 = x$; $x + y = 100$, $-x + 2y = -10$
$3y = 90$; $y = 30$
$x + (30) = 100$; $x = \$70$, *Ans.*

2. Let x = no. of cars; y = no. of trucks;
$x + y = 30$; $x + 2 = 3y$; $x + y = 30$, $x - 3y = -2$
$4y = 32$; $y = 8$;
$x + (8) = 30$; $x = 22$. 8 trucks, 22 cars, *Ans.*

3. Let x = aunt's age; y = uncle's age;
$x + y = 68$; $2y - x = 40$; $x + y = 68$, $-x + 2y = 40$
$3y = 108$; $y = 36$;
$x + (36) = 68$; $x = 32$ yr, *Ans.*

4. Let x = town's share; y = charity's share;
$x + y = 10{,}000$; $3y - 4000 = x$; $x + y = 10{,}000$, $-x + 3y = 4{,}000$
$4y = 14{,}000$; $y = 3500$; $x + (3500) = 10{,}000$; $x = \$6500$, *Ans.*

5. Let x = width of park; y = length of park; $4(45) = 180$ = perimeter of garden;
$y - x = 150$; $2y + 2x = 4(180) - 20$; $y - x = 150$; $2y + 2x = 700$; $y - x = 150$, $y + x = 350$
$2y = 500$; $y = 250$ M;
$(250) - x = 150$; $-x = -100$; $x = 100$ M. 100 M, 250 M, *Ans.*

6. Let x = width of each print; y = length of each print; $2x + 2y$ = perimeter of each print; $2(2x + 2y)$ = total perimeter of prints;
$2x + 4 = y$; $4x + 4y - 16 = 4(30)$; $2x - y = -4$, $x + y = 34$
$3x = 30$; $x = 10$; $2(10) + 4 = y$; $24 = y$. 10 in, 24 in, *Ans.*

7. Let x = price of 1 pad in cents; y = price of 1 pencil in cents;
$3x + 2y = 34$; $4x + 5y = 57$; $-12x - 8y = -136$, $12x + 15y = 171$
$7y = 35$; $y = 5$; $3x + 2(5) = 34$; $3x = 24$; $x = 8¢$, *Ans.*

8. Let x = price per lb of butter; y = price per doz of eggs;
$2x + 3y = 290$; $x + 2y = 171$; $2x + 3y = 290$, $-2x - 4y = -342$
$-y = -52$; $y = 52¢$
$x + 2(52) = 171$; $x + 104 = 171$; $x = 67¢$. eggs, 52¢/doz; butter, 67¢/lb, *Ans.*

9. Let x = cost of each ball; y = cost of each drink;
$3x + y = 210$; $x + 4y = 125$; $3x + y = 210$, $-3x - 12y = -375$
$-11y = -165$; $y = 15¢$;
$x + 4(15) = 125$; $x + 60 = 125$; $x = 65¢$. 65¢/ball, 15¢/drink, *Ans.*

10. Let x = cost per bat; y = cost per ball;
$7x + 5y = 16.95$; $3x + 6y = 13.05$; $42x + 30y = 101.70$, $-15x - 30y = -65.25$
$27x = 36.45$; $x = \$1.35$;
$3(1.35) + 6y = 13.05$; $4.05 + 6y = 13.05$; $6y = 9.00$; $y = \$1.50$. \$1.35/bat, \$1.50/ball, *Ans.*

11. Let x = amt. at 4%; y = amt. at 6%;

$$\begin{array}{rlrlrl} x + & y = 600 & x + & y = 600 & -4x - 4y &= -2400 \\ .04x + .06y &= 33 & 4x + 6y &= 3300 & 4x + 6y &= 3300 \\ & & & & 2y &= 900;\ y = \$450; \end{array}$$

$x + 450 = 600$; $x = \$150$. \$150 at 4%, \$450 at 6%, *Ans.*

12. Let x = amt. at 6%; y = amt. at 5%;

$$\begin{array}{rlrlrl} y - & x = 500 & -x + & y = 500 & -6x + 6y &= 3{,}000 \\ .05y + .06x &= 113 & 6x + 5y &= 11{,}300 & 6x + 5y &= 11{,}300 \\ & & & & 11y &= 14{,}300;\ y = \$1300; \end{array}$$

$(1300) - x = 500$; $-x = -800$; $x = \$800$. \$800 at 6%, \$1300 at 5%, *Ans.*

Page 378 WRITTEN EXERCISES

A

1. $(3y) + 2y = 5$; $5y = 5$; $y = 1$; $x = 3(1) = 3$. $\{(3, 1)\}$

2. $2x + (5x) = 7$; $7x = 7$; $x = 1$; $y = 5(1) = 5$. $\{(1, 5)\}$

3. $r = 1 + s$; $2(1 + s) - s = 7$; $2 + 2s - s = 7$; $s = 5$; $r - (5) = 1$; $r = 6$. $\{(6, 5)\}$

4. $r = 2 + s$; $3(2 + s) - s = 20$; $6 + 3s - s = 20$; $s = 7$; $r - (7) = 2$; $r = 9$. $\{(9, 7)\}$

5. $r = 12 + s$; $2(12 + s) + 3s = 19$; $24 + 2s + 3s = 19$; $5s = -5$; $s = -1$; $r - (-1) = 12$; $r + 1 = 12$; $r = 11$. $\{(11, -1)\}$

6. $r = 5 + s$; $3(5 + s) + 2s = 5$; $15 + 3s + 2s = 5$; $5s = -10$; $s = -2$; $r - (-2) = 5$; $r + 2 = 5$; $r = 3$. $\{(3, -2)\}$

7. $-n = 18 - 3m$; $n = 3m - 18$; $2m - 3(3m - 18) = 5$; $2m - 9m + 54 = 5$; $-7m = -49$; $m = 7$; $3(7) - n = 18$; $-n = -3$; $n = 3$. $\{(7, 3)\}$

8. $-n = 8 - 2m$; $n = 2m - 8$; $3m - 2(2m - 8) = 11$; $3m - 4m + 16 = 11$; $-m = -5$; $m = 5$; $2(5) - n = 8$; $-n = -2$; $n = 2$. $\{(5, 2)\}$

9. $y = 2x - 8$; $3x + 2(2x - 8) = 33$; $3x + 4x - 16 = 33$; $7x = 49$; $x = 7$; $y - 2(7) = -8$; $y = 6$. $\{(7, 6)\}$

10. $y = 2x - 6$; $2x + 3(2x - 6) = 22$; $2x + 6x - 18 = 22$; $8x = 40$; $x = 5$; $y - 2(5) = -6$; $y = 4$. $\{(5, 4)\}$

11. $2r = 3s$; $r = \dfrac{3s}{2}$; $3\left(\dfrac{3s}{2}\right) + 3s = 33$; $9s + 6s = 66$; $15s = 66$; $s = \frac{22}{5}$; $2r - 3[\frac{22}{5}] = 0$; $10r - 66 = 0$; $10r = 66$; $r = \frac{33}{5}$. $\{(\frac{33}{5}, \frac{22}{5})\}$

12. $s = 33 - 2r$; $3r - 4(33 - 2r) = 0$; $3r - 132 + 8r = 0$; $11r = 132$; $r = 12$; $2(12) + s = 33$; $24 + s = 33$; $s = 9$. $\{(12, 9)\}$

B

13. $w + z = 9$, $3w - z = 15$; $w = 9 - z$; $3(9 - z) - z = 15$; $27 - 3z - z = 15$; $-4z = -12$; $z = 3$; $w + (3) = 9$; $w = 6$. $\{(6, 3)\}$

14. $r - c = 8$, $r + c = 12$; $r = 8 + c$; $(8 + c) + c = 12$; $2c = 4$; $c = 2$; $r - (2) = 8$; $r = 10$. $\{(2, 10)\}$

15. $s - t = 8$, $2s - 3t = 6$; $s = 8 + t$; $2(8 + t) - 3t = 6$; $16 + 2t - 3t = 6$; $-t = -10$; $t = 10$; $s - (10) = 8$; $s = 18$. $\{(18, 10)\}$

16. $x - y = 2$, $x + y = 6$; $x = 2 + y$; $(2 + y) + y = 6$; $2 + 2y = 6$; $2y = 4$; $y = 2$; $x - (2) = 2$; $x = 4$. $\{(4, 2)\}$

17. $w + z = 9$, $4w - z = 16$; $w = 9 - z$; $4(9 - z) - z = 16$; $36 - 4z - z = 16$; $-5z = -20$; $z = 4$; $w + (4) = 9$; $w = 5$. $\{(5, 4)\}$

18. $r - c = 6$, $r + c = 8$; $r = 6 + c$; $(6 + c) + c = 8$; $6 + 2c = 8$; $2c = 2$; $c = 1$; $r - (1) = 6$; $r = 7$. $\{(1, 7)\}$

Page 379 · PROBLEMS

A

1. Let x = larger no.; y = smaller no.; $\frac{1}{2}(x + y) = -\frac{1}{2}$, $\frac{1}{2}(x - y) = \frac{3}{2}$; $x + y = -1$, $x - y = 3$; $x = 3 + y$; $(3 + y) + y = -1$; $3 + 2y = -1$; $2y = -4$; $y = -2$; $x - (-2) = 3$; $x + 2 = 3$; $x = 1$. 1, −2, *Ans.*

2. Let x = distance traveled by Mr. Wright; y = distance traveled by Mr. Black; $x + y = 300$, $x + 12 = 2y$; $y = 300 - x$; $x + 12 = 2(300 - x)$; $x + 12 = 600 - 2x$; $3x = 588$; $x = 196$; $(196) + y = 300$; $y = 104$. 196 mi, 104 mi, *Ans.*

3. Let x = distance traveled by Frank; y = distance traveled by James; $x + y = 9$, $x + 1.5 = 2y$; $x + y = 9$, $2x + 3 = 4y$; $x = 9 - y$; $2(9 - y) + 3 = 4y$; $18 - 2y + 3 = 4y$; $21 = 6y$; $y = 3.5$; $x + (3.5) = 9$; $x = 5.5$. Frank, 5.5 blocks; James, 3.5 blocks, *Ans.*

4. Let n = no. of adults; t = no. of students; $n + t = 500$, $60n + 35t = 25{,}450$; $n = 500 - t$; $60(500 - t) + 35t = 25{,}450$; $30{,}000 - 60t + 35t = 25{,}450$; $-25t = -4550$; $t = 182$; $n + (182) = 500$; $n = 318$ adults, *Ans.*

5. Let x = amt. invested at $3\frac{1}{2}\%$; y = amt. invested at 6%; $x + y = 3000$, $.035x - .06y = 10$; $x + y = 3000$, $35x - 60y = 10{,}000$; $x = 3000 - y$; $35(3000 - y) - 60y = 10{,}000$; $105{,}000 - 35y - 60y = 10{,}000$; $-95y = -95{,}000$; $y = \$1000$; $x + (1000) = 3000$; $x = \$2000$. \$2000 at $3\frac{1}{2}\%$, \$1000 at 6%, *Ans.*

6. Let x = cards/min by 1st sorter; y = cards/min by 2nd sorter; $x + y = 1200$, $2x + \frac{y}{2} = 1200$; $x = 1200 - y$; $2(1200 - y) + \frac{y}{2} = 1200$; $2400 - 2y + \frac{y}{2} = 1200$; $4800 - 4y + y = 2400$; $-3y = -2400$; $y = 800$; $x + (800) = 1200$; $x = 400$. 800 cards/min, 400 cards/min, *Ans.*

7. Let x = 1st no.; y = 2nd no.; $\frac{x + y}{2} = \frac{5}{48}$, $\frac{x - y}{4} = \frac{1}{96}$; $24x + 24y = 5$, $24x - 24y = 1$; $24x = 1 + 24y$; $x = \frac{1 + 24y}{24}$; $24\left[\frac{1 + 24y}{24}\right] + 24y = 5$; $1 + 24y + 24y = 5$; $48y = 4$; $y = \frac{1}{12}$; $24x - 24(\frac{1}{12}) = 1$; $24x - 2 = 1$; $24x = 3$; $x = \frac{1}{8}$. $\frac{1}{8}$, $\frac{1}{12}$, *Ans.*

8. Let x = sum at 6%; y = sum at 4%; $.06x + .04y = 57$, $.04x + .06y = 57 + 6$; $6x + 4y = 5700$, $4x + 6y = 6300$; $6x = 5700 - 4y$; $x = \frac{5700 - 4y}{6}$; $4\left[\frac{5700 - 4y}{6}\right] + 6y = 6300$; $22{,}800 - 16y + 36y = 37{,}800$; $20y = 15{,}000$; $y = \$750$; $4x + 6(750) = 6300$; $4x + 4500 = 6300$; $4x = 1800$; $x = \$450$. \$450, \$750, *Ans.*

9. Let x = no. of dollars invested; n = no. yr at 6%; $\frac{n}{2}$ = no. yr at 5%; $(.06x)n = x + 150$, $(.05x)\frac{n}{2} = x - 375$; (1) $6nx = 100x + 15{,}000$, (2) $5nx = 200x - 75{,}000$; from (2): $n = \frac{200x}{5x} - \frac{75{,}000}{5x} = 40 - \frac{15{,}000}{x}$; subst. in (1): $6x\left[40 - \frac{15{,}000}{x}\right] = 100x + 15{,}000$; $240x - 90{,}000 = 100x + 15{,}000$; $140x = 105{,}000$; $x = \$750$, *Ans.*

10. Let x = lb 66¢ tea in original blend; y = lb 48¢ tea in original blend; $x + y = 100$, $(66x + 48y) - (48x + 66y) = 600$; $x + y = 100$, $18x - 18y = 600$; $x + y = 100$, $3x - 3y = 100$; $x = 100 - y$; $3(100 - y) - 3y = 100$; $300 - 3y - 3y = 100$; $-6y = -200$; $y = 33\frac{1}{3}$; $x + (33\frac{1}{3}) = 100$; $x = 66\frac{2}{3}$; $\frac{66\frac{2}{3}}{33\frac{1}{3}} = 2:1$, *Ans.*

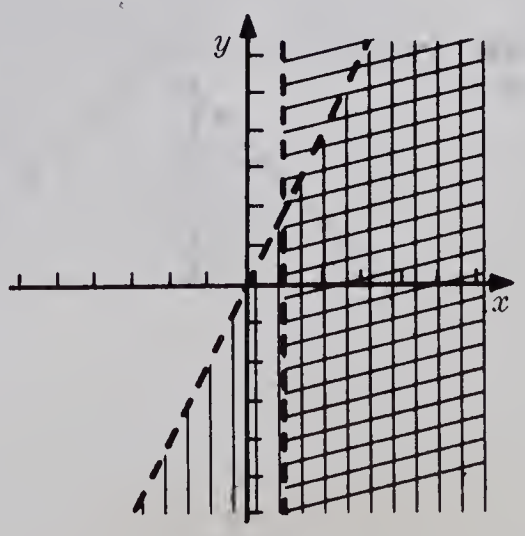
Pages 380–381 · WRITTEN EXERCISES

A **1.** (for boundary)

$y = 2x$	
x	y
0	0
2	4
4	8

$x = 1$	
x	y
1	0
1	2
1	−2

$x > 1$; y may have any real value

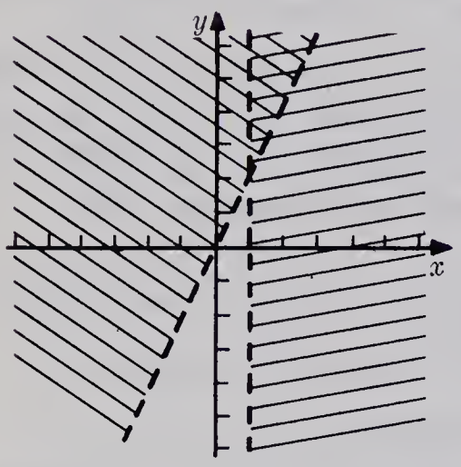


2. (for boundary)

$y = 2x$	
x	y
0	0
2	4
4	8

$x = 1$	
x	y
1	0
1	2
1	−2

$x > 1; y > 2$

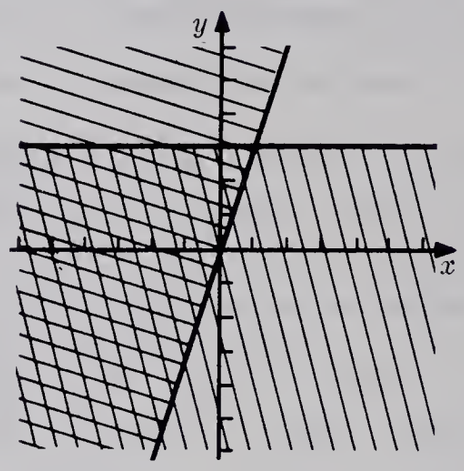


3. (for boundary)

$y = 3x$	
x	y
0	0
1	3
2	6

$y = 3$	
x	y
0	3
−2	3
3	3

$x \leq 1; y \leq 3$

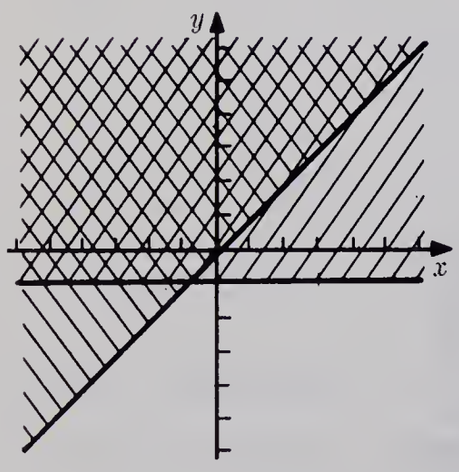


4. (for boundary)

$y = x$	
x	y
0	0
3	3
−2	−2

$y = -1$	
x	y
0	−1
−3	−1
+4	−1

x may have any real value; $y \geq -1$

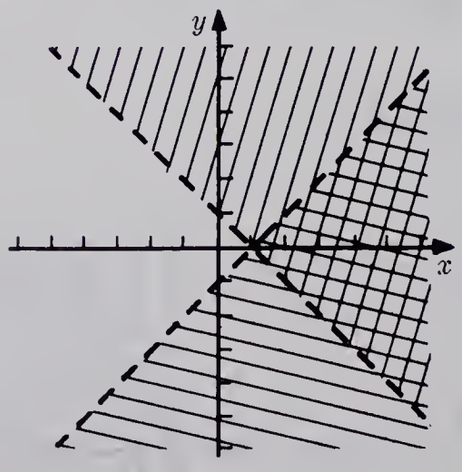


5. (for boundary)

$y = x - 1$	
x	y
0	−1
1	0
3	2

$y = 1 - x$	
x	y
0	1
1	0
3	−2

$x > 1$; y may have any real value

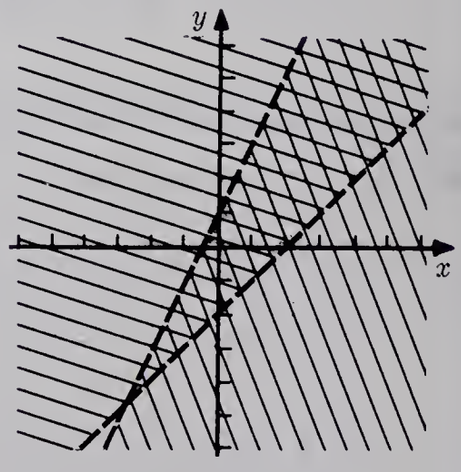


6. (for boundary)

$y = 2x + 1$	
x	y
0	1
1	3
−2	−3

$y = x - 2$	
x	y
0	−2
2	0
4	2

$x > -3; y > -5$

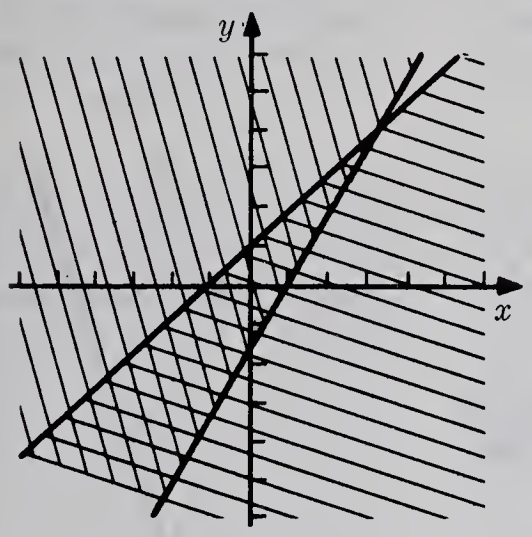


7. (for boundary)

$y = 3x - 6$	
x	y
0	−6
1	−3
−1	−9

$y = 2x + 4$	
x	y
0	4
1	6
−1	2

$x \leq 10; y \leq 24$



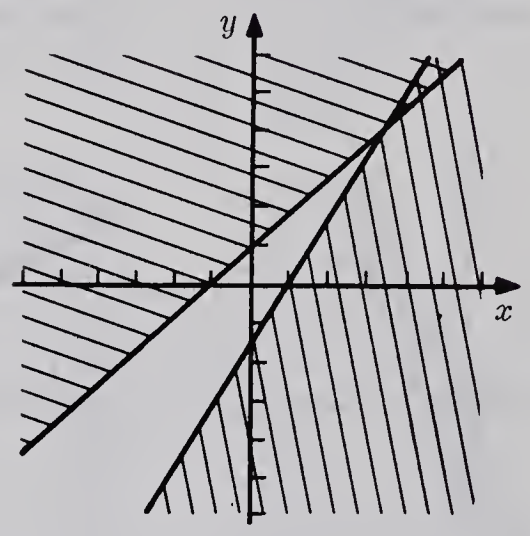
Scale: 1 x-unit = 2
1 y-unit = 4

8. (for boundary)

$y = 3x - 3$	
x	y
0	−3
1	0
−1	−6

$y = 2x + 2$	
x	y
0	2
1	4
−1	0

$x \geq 5; y \geq 12$



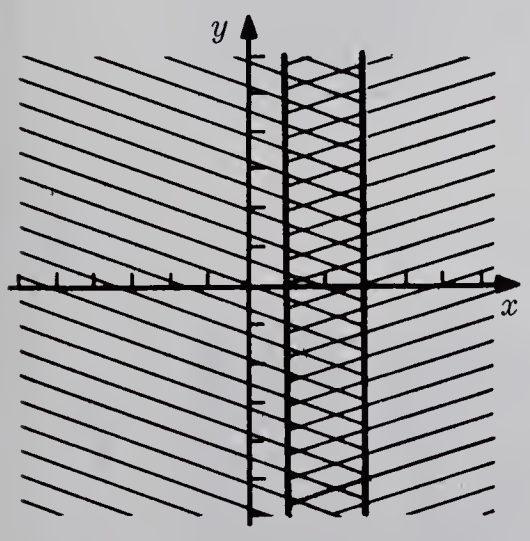
Scale: 1 x-unit = 1
1 y-unit = 2

B **9.** (for boundary)

$x = 1$	
x	y
1	0
1	−2
1	2

$x = 3$	
x	y
3	0
3	−3
3	5

$1 \leq x \leq 3$; y may have any real value

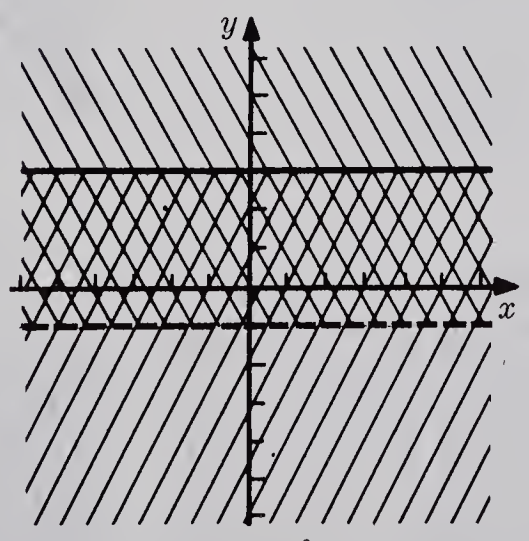


10. (for boundary)

$y - 3 = 0$	
x	y
0	3
−2	3
3	3

$y + 1 = 0$	
x	y
0	−1
2	−1
−2	−1

$-1 < y \leq 3$; x may have any real value

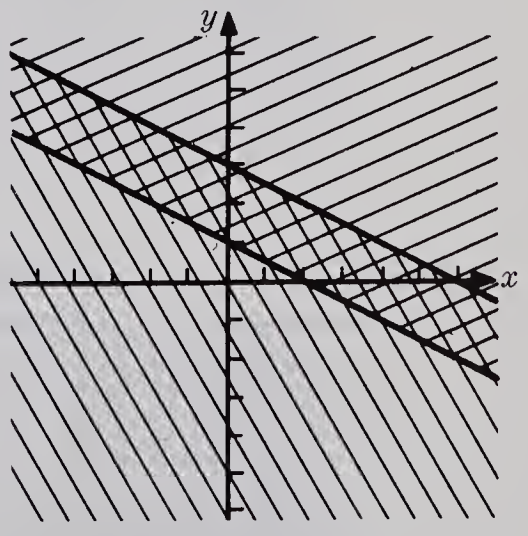


11. (for boundary)

$x + 2y = 2$	
x	y
0	1
2	0
4	−1

$x + 2y = 6$	
x	y
0	3
6	0
4	1

x and y may have any real value

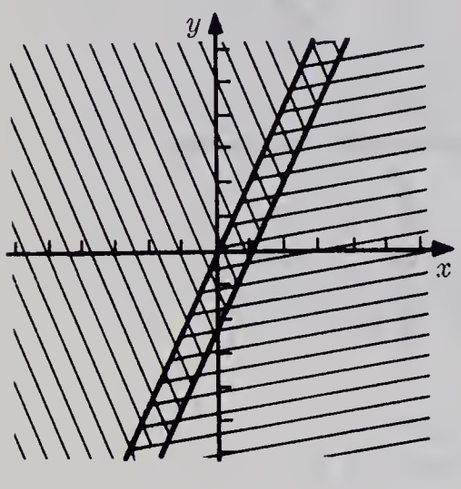


12. (for boundary)

$2x - y = 2$	
x	y
0	-2
2	2
-1	-4

$2x - y = 0$	
x	y
0	0
1	2
2	4

x and y may have any real value

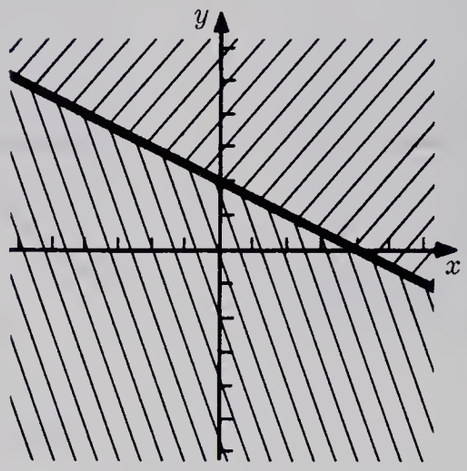


13. (for boundary)

$x + 2y = 4$	
x	y
0	2
4	0
2	1

$2x + 4y = 8$	
x	y
0	2
4	0
2	1

(boundary lines coincide, so the line is the graph of soln. set) x and y may have any real value

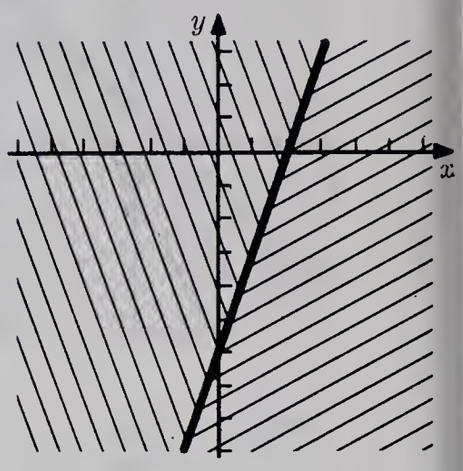


14. (for boundary)

$y = 3x - 6$	
x	y
0	-6
2	0
1	-3

$2y = 6x - 12$	
x	y
0	-6
2	0
1	-3

(boundary lines coincide, so the line is the graph of soln. set) x and y may have any real value



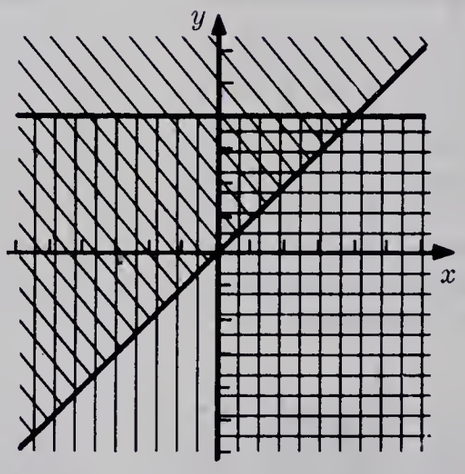
C **15.** (for boundary)

$y = x$	
x	y
0	0
2	2
-3	-3

$x = 0$	
x	y
0	0
0	5
0	-5

$y = 4$	
x	y
0	4
3	4
-3	4

$0 \leq x \leq 4; 0 \leq y \leq 4$



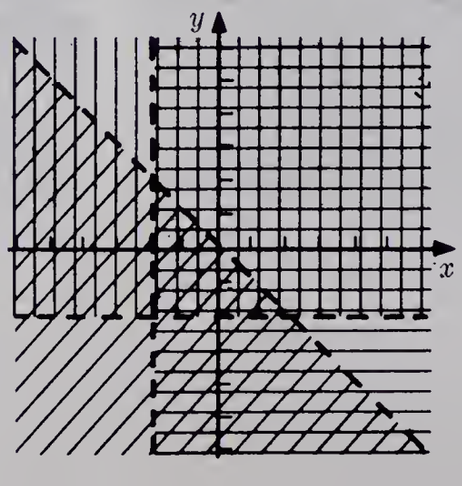
16. (for boundary)

$x + y = 0$	
x	y
0	0
-2	2
3	-3

$x = -2$	
x	y
-2	0
-2	2
-2	-1

$y = -2$	
x	y
0	-2
4	-2
-3	-2

$-2 < x < 2; -2 < y < 2$



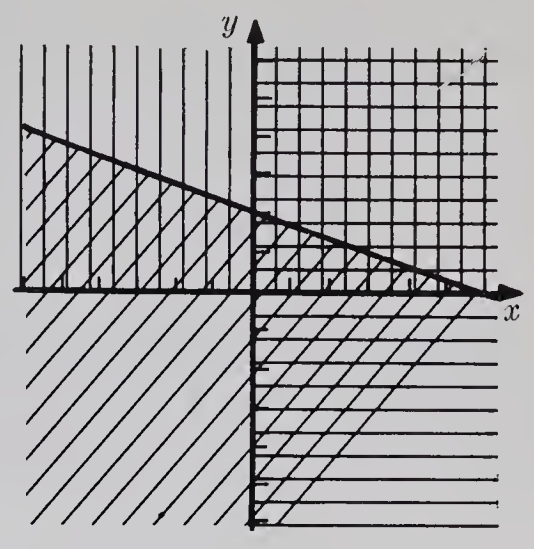
17. (for boundary)

$x + 3y = 6$	
x	y
0	2
6	0
3	1

$x = 0$	
x	y
0	−2
0	0
0	2

$y = 0$	
x	y
−3	0
0	0
3	0

$0 \leq x \leq 6; 0 \leq y \leq 2$



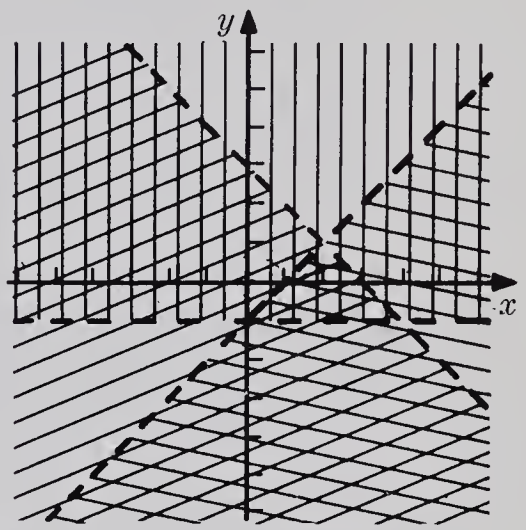
18. (for boundary)

$y = x - 1$	
x	y
0	−1
1	0
2	1

$y = 3 - x$	
x	y
0	3
3	0
1	2

$y = -1$	
x	y
0	−1
3	−1
−3	−1

$0 < x < 4; -1 < y < 1$



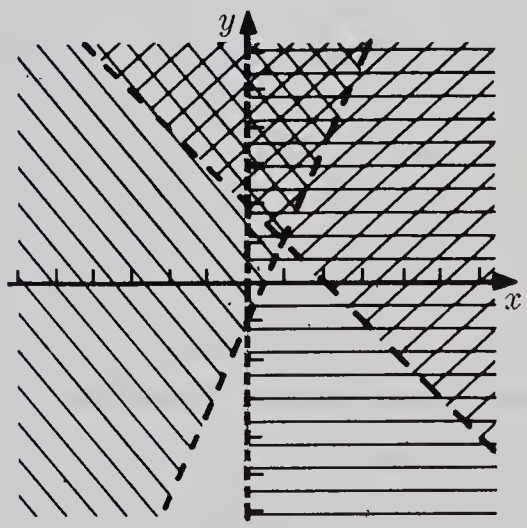
19. (for boundary)

$y = 2x - 1$	
x	y
0	−1
2	3
−2	−5

$x + y = 2$	
x	y
0	2
2	0
1	1

$x = 0$	
x	y
0	−2
0	0
0	2

$x > 0; y > 1$



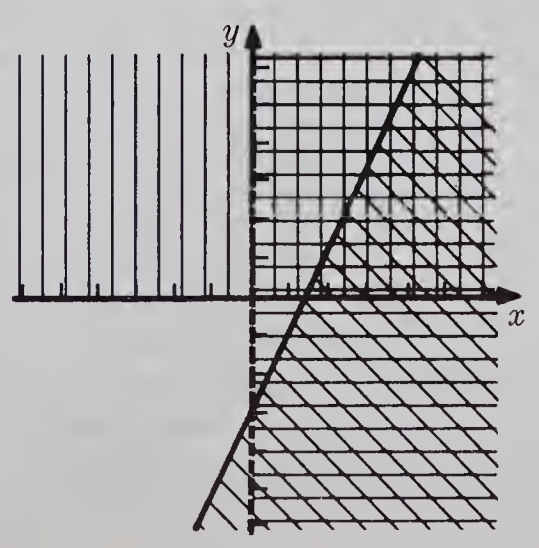
20. (for boundary)

$y = 2x - 3$	
x	y
0	−3
1	−1
−1	−5

$x = 0$	
x	y
0	0
0	3
0	−3

$y = 0$	
x	y
0	0
2	0
4	0

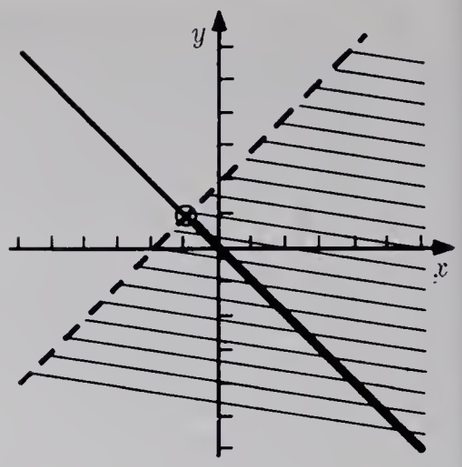
$x \geq \frac{3}{2}; y \geq 0$



21. (for boundary)

$x + y = 0$	
x	y
0	0
3	−3
−2	2

$y = x + 2$	
x	y
0	2
−2	0
1	3

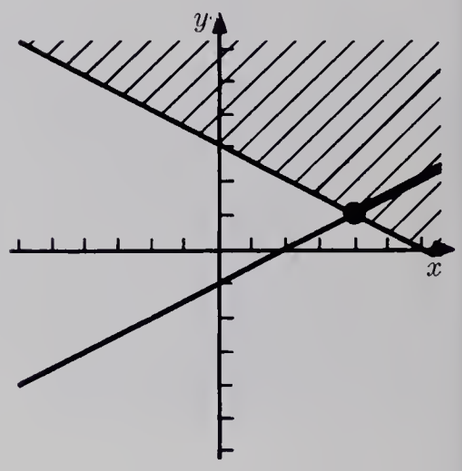


Graph is the heavily-shaded half-line [excludes point at $(-1,1)$].
$x + y = 0$; $x = -y$; $y < (-y) + 2$; $2y < 2$; $y < 1$; since $x = -y$, $y = -x$ and $-x < 1$; then $(-1)(-x) > (-1)(1)$, and $x > -1$. $\therefore x > -1$ and $y < 1$

22. (for boundary)

$x - 2y = 2$	
x	y
0	−1
2	0
4	1

$x + 2y = 6$	
x	y
0	3
6	0
2	2



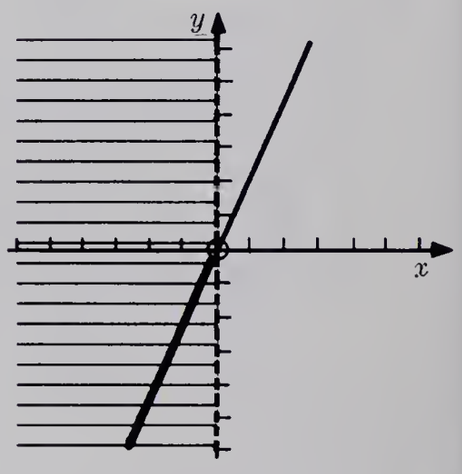
Graph is the heavily-shaded ray [includes point at (4, 1)].

$x - 2y = 2$; $x = 2y + 2$; $(2y + 2) + 2y \geq 6$; $4y + 2 \geq 6$; $4y \geq 4$, $y \geq 1$; but since $x - 2y = 2$, then $-2y = -x + 2$; $2y = x - 2$; and $y = \frac{x}{2} - 1$; $\therefore \frac{x}{2} - 1 \geq 1$; $2\left[\frac{x}{2} - 1\right] \geq 2[1]$; $x - 2 \geq 2$; and $x \geq 4$. $\therefore x \geq 4$ and $y \geq 1$

23. (for boundary)

$y = 2x$	
x	y
0	0
2	4
−2	−4

$x = 0$	
x	y
0	−2
0	0
0	2

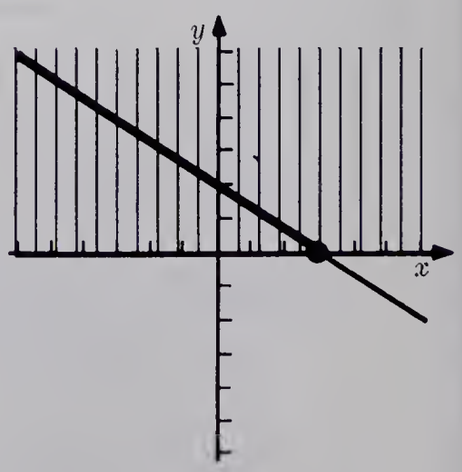


Graph is the heavily-shaded half-line [excludes point at (0, 0)].
$x < 0$ and $x = \frac{y}{2}$ $(y = 2x)$; $\therefore \frac{y}{2} < 0$; $2\left[\frac{y}{2}\right] < 2[0]$; and $y < 0$. $\therefore x < 0$ and $y < 0$

24. (for boundary)

$2x + 3y + 6 = 0$	
x	y
0	2
3	0
6	−2

$y = 0$	
x	y
−2	0
0	0
2	0



Graph is the heavily-shaded ray [includes point at (3, 0)].
$y \geq 0$; $2x + 3y + 6 = 0$; $3y = 6 - 2x$; $y = 2 - \frac{2}{3}x$; $2 - \frac{2}{3}x \geq 0$; $3[2 - \frac{2}{3}x] \geq 3[0]$; $6 - 2x \geq 0$; $-2x \geq -6$; $-1[-2x] \leq -1[-6]$; $2x \leq 6$; $x \leq 3$. $\therefore x \leq 3$ and $y \geq 0$

Pages 382–383 · **PROBLEMS**

A **1.** Let n = tens digit in original no.; y = units digit in original no.; $10n + y$ = original no.; $10y + n$ = new no.;

$$\begin{array}{l} n + y = 13 \\ 10n + y + 27 = 10y + n \end{array}; \quad \begin{array}{l} n + y = 13 \\ 9n - 9y = -27 \end{array}; \quad \begin{array}{l} n + y = 13 \\ n - y = -3 \\ \hline 2n = 10 \end{array}; n = 5;$$

$(5) + y = 13; y = 8$. $10n + y = 58$, *Ans.*

2. Let u = units digit in original no.; t = tens digit in original no.; $10t + u$ = original no.; $10u + t$ = new no.;

$$\begin{array}{l} u + t = 12 \\ (10u + t) - (10t + u) = 36 \end{array}; \quad \begin{array}{l} u + t = 12 \\ 9u - 9t = 36 \end{array}; \quad \begin{array}{l} u + t = 12 \\ u - t = 4 \\ \hline 2u = 16 \end{array}; u = 8;$$

$(8) + t = 12; t = 4$. $10t + u = 48$, *Ans.*

3. Let u = units digit in no.; t = tens digit in no.; $10t + u$ = the no.;

$$\begin{array}{l} u + t = 9 \\ 10t + u = 12t \end{array}; \quad \begin{array}{l} u + t = 9 \\ u - 2t = 0 \\ \hline 3t = 9 \end{array}; t = 3; u + (3) = 9; u = 6.\ 10t + u = 36, \textit{Ans.}$$

4. Let x = tens digit in original no.; y = units digit in original no.; $10x + y$ = original no.; $10y + x$ = new no.;

$$\begin{array}{l} x + y = 12 \\ 10y + x = 13y \end{array}; \quad \begin{array}{l} x + y = 12 \\ x - 3y = 0 \\ \hline 4y = 12 \end{array}; y = 3; x + (3) = 12; x = 9.\ 10x + y = 93, \textit{Ans.}$$

5. Let u = units digit in no.; t = tens digit in no.; $10t + u$ = no.;

$$\begin{array}{l} u = 2t + 1 \\ u + t = 7 \end{array}; \quad \begin{array}{l} u - 2t = 1 \\ u + t = 7 \\ \hline -3t = -6 \end{array}; t = 2; u + (2) = 7; u = 5.\ 10t + u = 25, \textit{Ans.}$$

6. Let r = tens digit; p = units digit; $10r + p$ = the no.;

$$\begin{array}{l} p = 2r \\ r + p = 12 \end{array}; \quad r + (2r) = 12; 3r = 12; r = 4; p = 2(4); p = 8.\ 10r + p = 48, \textit{Ans.}$$

7. Let u = units digit in original no.; t = tens digit in original no.; $10t + u$ = original no.; $10u + t$ = new no.;

$$\begin{array}{l} u + t = 8 \\ 10u + t = 7t \end{array}; \quad \begin{array}{l} u + t = 8 \\ 10u - 6t = 0 \end{array}; \quad t = 8 - u; 10u - 6(8 - u) = 0; 10u - 48 + 6u = 0;$$

$16u = 48; u = 3; (3) + t = 8; t = 5.\ 10t + u = 53$, *Ans.*

8. Let w = units digit in original no.; t = tens digit in original no.; $10t + w$ = original no.; $10w + t$ = new no.;

$$\begin{array}{l} w + t = 6 \\ 10w + t = 12w \end{array}; \quad \begin{array}{l} w + t = 6 \\ -2w + t = 0 \\ \hline 3w = 6 \end{array}; w = 2; (2) + t = 6; t = 4.\ 10t + w = 42, \textit{Ans.}$$

9. Let u = units digit on original check; t = tens digit on original check; $10t + u$ = original no. on check; $10u + t$ = new no.;

$$\begin{array}{l} u + t = 9 \\ (10u + t) - (10t + u) = 9 \end{array}; \quad \begin{array}{l} u + t = 9 \\ 9u - 9t = 9 \end{array}; \quad \begin{array}{l} u + t = 9 \\ u - t = 1 \\ \hline 2u = 10 \end{array}; u = 5;$$

$(5) + t = 9; t = 4$. $10t + u = \$45$, *Ans.*

10. Let t = tens digit in original no.; u = units digit in original no.; $10t + u$ = original no.; $10u + t$ = new no.;

$$\begin{array}{l} t - 2u = 1 \\ (10u + t) - 4 = 3(u + t) \end{array}; \quad \begin{array}{l} t - 2u = 1 \\ 10u + t - 4 = 3u + 3t \end{array}; \quad \begin{array}{l} t - 2u = 1 \\ 7u - 2t = 4 \end{array};$$

$t = 1 + 2u; 7u - 2(1 + 2u) = 4; 7u - 2 - 4u = 4; 3u = 6; u = 2; t - 2(2) = 1; t - 4 = 1;$ $t = 5.\ 10t + u = 52$, *Ans.*

B **11.** Let h = hundreds digit; u = units digit; $(2u)$ = tens digit; $100h + 10(2u) + u$ = the no.;

$$\begin{matrix} 2u = 3h \\ h + (2u) + u = 11 \end{matrix}; \quad \begin{matrix} 2u - 3h = 0 \\ 3u + h = 11 \end{matrix}; \quad h = 11 - 3u; 2u - 3(11 - 3u) = 0; 2u - 33 + 9u = 0;$$

$11u = 33; u = 3; 3(3) + h = 11; 9 + h = 11; h = 2.\ 100h + 10(2u) + u = 263$, *Ans.*

12. Let u = units digit; t = tens digit; $3t$ = hundreds digit; $100(3t) + 10t + u$ = original no.;

$$u + t + (3t) = 19$$
$$[100(3t) + 10t + u] - 198 = [100u + 10t + (3t)]$$

$$\begin{matrix} u + 4t = 19 \\ u + 310t - 198 = 100u + 13t \end{matrix}; \quad \begin{matrix} u + 4t = 19 \\ -99u + 297t = 198 \end{matrix};$$

$u = 19 - 4t$; $-99(19 - 4t) + 297t = 198$; $-1881 + 396t + 297t = 198$; $693t = 2079$;

$\begin{matrix} t = 3 \\ 3t = 9 \end{matrix}$; $u + 4(3) = 19$; $u + 12 = 19$; $u = 7$; $100(3t) + 10t + u = 937$, *Ans.*

13. Let u = units digit in original no.; t = tens digit in original no.; $10t + u$ = original no.;
$10u + t$ = new no.;

$$\frac{10t + u}{t} = 11 + \frac{4}{t}; \quad t\left[\frac{10t + u}{t}\right] = t\left[11 + \frac{4}{t}\right]$$

$$\frac{10u + t}{u} = 10 + \frac{5}{u} \quad u\left[\frac{10u + t}{u}\right] = u\left[10 + \frac{5}{u}\right]$$

$$\begin{matrix} 10t + u = 11t + 4 \\ t + 10u = 10u + 5 \end{matrix}; \quad \begin{matrix} u - t = 4 \\ t = 5 \\ \hline u = 9 \end{matrix}; \quad 10(5) + 9 = 10t + u = 59, \textit{Ans.}$$

14. Let h = hundreds digit in original no.; t = tens digit in original no.; u = units digit in original no.;
$100h + 10t + u$ = original no.; $100u + 10t + h$ = new no.;

$$\frac{(100h + 10t + u) - (100u + 10t + h)}{99} = \frac{100h + 10t + u - 100u - 10t - h}{99}$$
$$= \frac{99h - 99u}{99} = \frac{99(h - u)}{99} = h - u$$

C **15.** Let t = tenths digit in original fraction; h = hundredths digit in original fraction;
$.1t + .01h$ = original fraction; $.1h + .01t$ = new fraction;

$$\begin{matrix} t + h = 9 \\ (.1h + .01t) - (.1t + .01h) = .09 \end{matrix}; \quad \begin{matrix} t + h = 9 \\ 100[(.1h + .01t) - (.1t + .01h)] = 100[.09] \end{matrix};$$

$$\begin{matrix} t + h = 9 \\ 10h + t - 10t - h = 9 \end{matrix}; \quad \begin{matrix} t + h = 9 \\ 9h - 9t = 9 \end{matrix}; \quad \begin{matrix} t + h = 9 \\ -t + h = 1 \\ \hline 2h = 10; h = 5; \end{matrix}$$

$t + (5) = 9$; $t = 4$. $.1t + .01h = .45$, *Ans.*

16. Let x = tenths digit in original fraction; y = hundredths digit in original fraction;
$.1x + .01y$ = original fraction; $.1y + .01x$ = new fraction;

$$\begin{matrix} x + y = 9 \\ .1y + .01x = \frac{4}{7}(.1x + .01y) \end{matrix}; \quad \begin{matrix} x + y = 9 \\ 700[.1y + .01x] = 700[\frac{4}{7}(.1x + .01y)] \end{matrix};$$

$$\begin{matrix} x + y = 9 \\ 70y + 7x = 40x + 4y \end{matrix}; \quad \begin{matrix} x + y = 9 \\ -33x + 66y = 0 \end{matrix}; \quad \begin{matrix} x + y = 9 \\ -x + 2y = 0 \\ \hline 3y = 9; y = 3; \end{matrix}$$

$x + (3) = 9$; $x = 6$; $.1x + .01y = .63$, *Ans.*

17. Let n = tenths digit in original fraction; p = hundredths digit in original fraction;
$.1n + .01p$ = original fraction; $.1p + .01n$ = new fraction;

$$\begin{matrix} n + p = 13 \\ .1p + .01n + .04 = 2(.1n + .01p) \end{matrix}; \quad \begin{matrix} n + p = 13 \\ .1p + .01n + .04 = .2n + .02p \end{matrix};$$

$$\begin{matrix} n + p = 13 \\ 100[.1p + .01n + .04] = 100[.2n + .02p] \end{matrix}; \quad \begin{matrix} n + p = 13 \\ 10p + n + 4 = 20n + 2p \end{matrix}; \quad \begin{matrix} n + p = 13 \\ -19n + 8p = -4 \end{matrix};$$

$n = 13 - p$; $-19(13 - p) + 8p = -4$; $-247 + 19p + 8p = -4$; $27p = 243$; $p = 9$;
$n + (9) = 13$; $n = 4$. $.1n + .01p = .49$, *Ans.*

18. Let t = tenths digit in original fraction; h = hundredths digit in original fraction;
$.1t + .01h$ = original fraction; $.1h + .01t$ = new fraction;

$$\begin{array}{l} t - 2h = 1 \\ .1t + .01h + .01 = 2(.1h + .01t) \end{array}; \quad \begin{array}{l} t - 2h = 1 \\ .1t + .01h + .01 = .2h + .02t \end{array};$$

$$\begin{array}{l} t - 2h = 1 \\ 100[.1t + .01h + .01] = 100[.2h + .02t] \end{array}; \quad \begin{array}{l} t - 2h = 1 \\ 10t + h + 1 = 20h + 2t \end{array};$$

$$\begin{array}{l} t - 2h = 1 \\ 8t - 19h = 1 \end{array}; \quad \begin{array}{l} -8[t - 2h] = -8[1] \\ 8t - 19h = -1 \end{array}; \quad \begin{array}{l} -8t + 16h = -8 \\ \underline{8t - 19h = -1} \\ -3h = -9; h = 3; \end{array}$$

$t - 2(3) = 1, t - 6 = 1, t = 7. .1t + .01h = .73$, *Ans.*

Pages 384–385 · **PROBLEMS**

A **1.** Let x = rate of boat in still water; y = speed of water;

$$\begin{array}{l} \frac{3}{4}(x + y) = 6 \\ \frac{3}{2}(x - y) = 6 \end{array}; \quad \begin{array}{l} 4[\frac{3}{4}(x + y)] = 4[6] \\ 2[\frac{3}{2}(x - y)] = 2[6] \end{array}; \quad \begin{array}{l} 3x + 3y = 24 \\ 3x - 3y = 12 \end{array}; \quad \begin{array}{l} x + y = 8 \\ \underline{x - y = 4} \\ 2x = 12; x = 6 \text{ mph, } \textit{Ans.} \end{array}$$

2. Let x = speed without wind; y = speed of wind;

$$\begin{array}{l} \frac{1}{20}(x + y) = 1 \\ \frac{1}{15}(x - y) = 1 \end{array}; \quad \begin{array}{l} 20[\frac{1}{20}(x + y)] = 20[1] \\ 15[\frac{1}{15}(x - y)] = 15[1] \end{array}; \quad \begin{array}{l} x + y = 20 \\ \underline{x - y = 15} \\ 2x = 35; x = 17\frac{1}{2} \text{ mph, } \textit{Ans.} \end{array}$$

3. Let x = speed of wind; y = speed of plane without wind;

$$\begin{array}{l} \frac{3}{2}(y - x) = 450 \\ \frac{5}{4}(y + x) = 450 \end{array}; \quad \begin{array}{l} 2[\frac{3}{2}(y - x)] = 2[450] \\ 4[\frac{5}{4}(y + x)] = 4[450] \end{array}; \quad \begin{array}{l} 3(y - x) = 900 \\ 5(y + x) = 1800 \end{array}; \quad \begin{array}{l} y - x = 300 \\ \underline{y + x = 360} \\ 2y = 660; y = 330; \end{array}$$

$\frac{3}{2}[(330) - x] = 450; 2\{\frac{3}{2}[330 - x]\} = 2\{450\}; 3(330 - x) = 900; 990 - 3x = 900; -3x = -90;$
$x = 30$ mph, *Ans.*

4. Let t = speed of plane without wind; v = speed of wind;

$$\begin{array}{l} \frac{3}{2}(t + v) = 540 \\ \frac{9}{5}(t - v) = 540 \end{array}; \quad \begin{array}{l} 2[\frac{3}{2}(t + v)] = 2[540] \\ 5[\frac{9}{5}(t - v)] = 5[540] \end{array}; \quad \begin{array}{l} 3(t + v) = 1080 \\ 9(t - v) = 2700 \end{array}; \quad \begin{array}{l} t + v = 360 \\ \underline{t - v = 300} \\ 2t = 660; t = 330 \text{ mph, } \textit{Ans.} \end{array}$$

5. Let n = speed without current; y = speed of current;

$$\begin{array}{l} \frac{3}{5}(n + y) = 3 \\ \frac{3}{2}(n - y) = 3 \end{array}; \quad \begin{array}{l} 5[\frac{3}{5}(n + y)] = 5[3] \\ 2[\frac{3}{2}(n - y)] = 2[3] \end{array}; \quad \begin{array}{l} 3(n + y) = 15 \\ 3(n - y) = 6 \end{array}; \quad \begin{array}{l} n + y = 5 \\ \underline{n - y = 2} \\ 2n = 7; n = \frac{7}{2}; \end{array}$$

$\frac{3}{2}[(\frac{7}{2}) - y] = 3; \frac{21}{4} - \frac{3}{2}y = 3; 4[\frac{21}{4} - \frac{3}{2}y] = 4[3]; 21 - 6y = 12; -6y = -9; y = 1\frac{1}{2}$ mph, *Ans.*

6. Let x = speed in still water; y = speed of current;

$$\begin{array}{l} \frac{2}{3}[x + y] = 6 \\ 3[x - y] = 6 \end{array}; \quad \begin{array}{l} 3\{\frac{2}{3}[x + y]\} = 3\{6\} \\ x - y = 2 \end{array}; \quad \begin{array}{l} 2(x + y) = 18 \\ x - y = 2 \end{array}; \quad \begin{array}{l} x + y = 9 \\ \underline{x - y = 2} \\ 2x = 11; x = 5\frac{1}{2} \text{ mph, } \textit{Ans.} \end{array}$$

B **7.** Let x = speed in still water; y = speed of current. Then $\frac{4}{x + y}$ = time to go downstream; $\frac{4}{x - y}$ = time to go upstream; $\frac{1}{x - y}$ = time to go one mile against current; $\frac{3}{x + y}$ = time to go three miles with current;

$$\frac{4}{x + y} + \frac{4}{x - y} = \frac{8}{3}; \quad \frac{1}{x - y} = \frac{3}{x + y}; \quad x + y = 3(x - y); x + y = 3x - 3y; 4y = 2x; 2y = x.$$

$$\frac{4}{2y + y} + \frac{4}{2y - y} = \frac{8}{3}; \frac{4}{3y} + \frac{4}{y} = \frac{8}{3}; 4 + 12 = 8y; y = 2; x = 2(2) = 4 \text{ mph, } \textit{Ans.}$$

Average rate $R = \frac{\text{distance}}{\text{time}}; R = \frac{8}{\frac{8}{3}} = 3$ mph, *Ans.*

8. Let x = rate in still air; y = speed of wind. Then $x + y$ = rate with wind; $x - y$ = rate against wind;

$\frac{15}{x+y} + \frac{15}{x-y} = 3\frac{1}{2}$; $3\left(\frac{5}{x+y}\right) + \frac{15}{x-y} = \frac{7}{2}$, or, $3\left(\frac{2}{x-y}\right) + \frac{15}{x-y} = \frac{7}{2}$; $\frac{21}{x-y} = \frac{7}{2}$;

$\frac{2}{x-y} = \frac{5}{x+y}$

$42 = 7x - 7y$; $6 = x - y$; $y + 6 = x$; $\frac{2}{(y+6)-y} = \frac{5}{(y+6)+y}$; $\frac{1}{3} = \frac{5}{2y+6}$; $2y + 6 = 15$; $y = 4\frac{1}{2}$ mph, *Ans.* Average rate $R = \frac{d}{t} = \frac{30}{\frac{7}{2}} = \frac{60}{7} = 8\frac{4}{7}$ mph, *Ans.*

9. Let x = speed of plane in still air; y = speed of wind. Then $(x + y)$ = speed with wind; $(x - y)$ = speed against wind. t = time with wind; w = time against wind;

$t + w = \frac{15}{2}$; $t + w = \frac{15}{2}$

$w - t = 1$; $-t + w = 1$

$2w = \frac{17}{2}$; $w = \frac{17}{4}$; $t + (\frac{17}{4}) = \frac{15}{2}$; $t = \frac{15}{2} - \frac{17}{4} = \frac{13}{4}$.

Then, $t(x+y) = \frac{13}{4}(x+y)$ = distance with wind; $w(x-y) = \frac{17}{4}(x-y)$ = distance against wind;

$\frac{13}{4}(x+y) = \frac{1105}{2}$; $4[\frac{13}{4}(x+y)] = 4[\frac{1105}{2}]$; $13(x+y) = 2(1105)$; $x + y = 170$

$\frac{17}{4}(x-y) = \frac{1105}{2}$; $4[\frac{17}{4}(x-y)] = 4[\frac{1105}{2}]$; $17(x-y) = 2(1105)$; $x - y = 130$

$2x = 300$; $x = 150$;

$\frac{13}{4}[(150) + y] = \frac{1105}{2}$; $4\{\frac{13}{4}[150 + y]\} = 4\{\frac{1105}{2}\}$; $13(150 + y) = 2(1105)$; $150 + y = 170$; $y = 20$. 150 mph, 20 mph, *Ans.*

10. Let d = distance upstream in one unit of time; then $2d$ = distance downstream in one unit of time;

$s - c = d$; $s + c = 2(s - c)$; $s + c = 2s - 2c$; $3c = s$; $c = \frac{s}{3}$ *or* $s = 3c$, *Ans.*

$s + c = 2d$

Pages 385–386 · **PROBLEMS**

A **1.** Let x = man's age now; y = son's age now. Then $x + 5$ = man's age in 5 yr; $y + 5$ = son's age in 5 yr;

$x = 5y$; $(5y) + 5 = 3(y + 5)$; $5y + 5 = 3y + 15$; $2y = 10$; $y = 5$ yr, *Ans.*

$x + 5 = 3(y + 5)$

2. Let x = Ruth's age now; y = father's age now. Then $x - 1$ = Ruth's age 1 yr ago; $y - 1$ = father's age 1 yr ago;

$y = 7x$; $(7x) - 1 = 9(x - 1)$; $7x - 1 = 9x - 9$; $-2x = -8$; $x = 4$ yr, *Ans.*

$(y - 1) = 9(x - 1)$

3. Let x = Joe's present age; y = Jill's present age. Then $x - 3$ = Joe's age 3 yr ago; $y - 3$ = Jill's age 3 yr ago; $x + 6$ = Joe's age 6 yr hence; $y + 6$ = Jill's age 6 yr hence;

$(x - 3) = 2(y - 3) + 1$; $x - 3 = 2y - 6 + 1$; $x - 2y = -2$

$(x + 6) = \frac{1}{2}(y + 6) + 10$; $2(x + 6) = 2(\frac{1}{2}[y + 6] + 10)$; $2x + 12 = y + 6 + 20$;

$x - 2y = -2$; $x - 2y = -2$; $x - 2y = -2$

$2x - y = 14$; $-2[2x - y] = -2[14]$; $-4x + 2y = -28$

$-3x = -30$; $x = 10$;

$(10 - 3) = 2(y - 3) + 1$; $7 = 2y - 6 + 1$; $12 = 2y$; $y = 6$. Joe, 10 yr, *Ans.*

4. Let x = Jerry's present age; y = Jeff's present age. Then $x - 5$ = Jerry's age 5 yr ago; $y - 5$ = Jeff's age 5 yr ago; $x + 10$ = Jerry's age 10 yr hence; $y + 10$ = Jeff's age 10 yr hence;

$(x - 5) = \frac{2}{3}(y - 5)$; $3[x - 5] = 3[\frac{2}{3}(y - 5)]$; $3x - 15 = 2(y - 5)$;

$(x + 10) = \frac{5}{6}(y + 10)$; $6[x + 10] = 6[\frac{5}{6}(y + 10)]$; $6x + 60 = 5(y + 10)$;

$3x - 15 = 2y - 10$; $3x - 2y = 5$; $-2[3x - 2y] = -2[5]$; $-6x + 4y = -10$

$6x + 60 = 5y + 50$; $6x - 5y = -10$; $6x - 5y = -10$; $6x - 5y = -10$

$-y = -20$; $y = 20$;

$(x - 5) = \frac{2}{3}[(20) - 5]$; $x - 5 = \frac{2}{3}(15)$; $x - 5 = 10$; $x = 15$. Jerry, 15 yr; Jeff, 20 yr, *Ans.*

5. Let p = Janet's present age; q = Phil's present age. Then $p - 4$ = Janet's age 4 yr ago; $q - 4$ = Phil's age 4 yr ago;

$$\begin{matrix} p = \frac{4}{5}q \\ (p-4) = \frac{3}{4}(q-4) \end{matrix}; \frac{4}{5}q - 4 = \frac{3}{4}q - 3; 20[\tfrac{4}{5}q - 4] = 20[\tfrac{3}{4}q - 3]; 16q - 80 = 15q - 60;$$

$q = 20; p = \frac{4}{5}(20) = 16$. Janet, 16 yr; Phil, 20 yr, *Ans.*

6. Let n = daughter's age; x = son's age. Then $3(n + x)$ = wife's age; $3(n + x) + x$ = father's age;

$$n + x + [3(n+x)] + [3(n+x) + x] = 69; \begin{matrix} x = 2n \end{matrix}; n + x + [3n + 3x] + [3n + 3x + x] = 69;$$

$n + x + 3n + 3x + 3n + 4x = 69; 7n + 8x = 69; 7n + 8(2n) = 69; 7n + 16n = 69; 23n = 69;$
$n = 3; x = 2(3) = 6$ yr, *Ans.*

B **7.** Let R = Ray's age; J = Joyce's age;

$$\begin{matrix} \frac{3}{5}R + \frac{3}{4}J = R + 3 \\ \frac{2}{5}R + \frac{1}{2}J = R - 3 \end{matrix}; \begin{matrix} 20[\frac{3}{5}R + \frac{3}{4}J] = 20[R+3] \\ 10[\frac{2}{5}R + \frac{1}{2}J] = 10[R-3] \end{matrix}; \begin{matrix} 12R + 15J = 20R + 60 \\ 4R + 5J = 10R - 30 \end{matrix};$$

$$\begin{matrix} -8R + 15J = 60 \\ -6R + 5J = -30 \end{matrix}; \begin{matrix} -8R + 15J = 60 \\ -3[-6R + 5J] = -3[-30] \end{matrix}; \begin{matrix} -8R + 15J = 60 \\ 18R - 15J = 90 \\ \hline 10R = 150 \end{matrix}; R = 15 \text{ yr, } Ans.$$

8. Let M = Mary's age now; J = Jane's age now; d = no. yr ago when Mary was as old as Jane is now;

$$\begin{matrix} M = 2(J-d) \\ M - d = J \end{matrix}; -d = J - M; d = M - J; M = 2[J - (M - J)]; M = 2(J - M + J);$$

$M = 2(2J - M); M = 4J - 2M; 3M = 4J; J = \frac{3}{4}M$ *or* $M = \frac{4}{3}J$, *Ans.*

9. Let J = Jane's age now; M = Mary's age now. Then $M + 3$ = Mary's age in 3 yr; $J + 3$ = Jane's age in 3 yr; $J - 4$ = Jane's age 4 yr ago; from preceding problem, $M = \frac{4}{3}J$; then since $M + 3 = 3(J - 4)$, $\frac{4}{3}J + 3 = 3(J - 4); 3[\frac{4}{3}J + 3] = 3[3J - 12];$
$4J + 9 = 9J - 36; 45 = 5J; 9 = J; M = \frac{4}{3}(9) = 12$. J, 9 yr; M, 12 yr, *Ans.*

10. Let B = Bob's present age; b = brother's present age; d = no. yr hence when twice brother's age will equal Bob's present age;

$$\begin{matrix} 2(b+d) = B \\ 8(b) = B + d \end{matrix}; d = 8b - B; 2[(b + [8b - B])] = B; 2[b + 8b - B] = B; 2[9b - B] = B;$$

$18b - 2B = B; 18b = 3B; 6b = B$ *or* $b = \frac{1}{6}B$, *Ans.*

Page 387 · PROBLEMS

A **1.** Let $\frac{x}{y}$ = original fraction;

$$\begin{matrix} y = x + 3 \\ \frac{x+1}{y+1} = \frac{2}{3} \end{matrix}; (3)(y+1)\left[\frac{x+1}{y+1}\right] = (3)(y+1)[\tfrac{2}{3}]; 3(x+1) = 2(y+1); 3x + 3 = 2y + 2;$$

$-2y = -3x - 1; -1(-2y) = -1(-3x - 1); 2y = 3x + 1; 2(x + 3) = 3x + 1;$
$2x + 6 = 3x + 1; -x = -5; x = 5; y = (5) + 3 = 8$ $\frac{x}{y} = \frac{5}{8}$, *Ans.*

2. Let $\frac{p}{n}$ = original fraction;

$$\begin{matrix} p + 5 = n \\ \frac{p+1}{n+1} = \frac{1}{2} \end{matrix}; 2(n+1)\left[\frac{p+1}{n+1}\right] = 2(n+1)[\tfrac{1}{2}]; 2(p+1) = (n+1)(1); 2p + 2 = n + 1;$$

$2p - n = -1; 2p - (p + 5) = -1; 2p - p - 5 = -1; p = 4; (4) + 5 = n; 9 = n.$

$\frac{p}{n} = \frac{4}{9}$, *Ans.*

3. Let $\frac{x}{y}$ = original fraction;

$y - 3 = x$
$\frac{x-1}{y} = \frac{1}{2}$; $2y\left(\frac{x-1}{y}\right) = 2y(\frac{1}{2})$; $2(x-1) = y(1)$; $2x - 2 = y$; $(2x-2) - 3 = x$;

$2x - 5 = x$; $-5 = -x$; $5 = x$; $y - 3 = (5)$; $y = 8$. $\frac{x}{y} = \frac{5}{8}$, *Ans.*

4. Let $\frac{w}{r}$ = original fraction;

$r - 5 = w$
$\frac{w-1}{r} = \frac{1}{3}$; $3r\left(\frac{w-1}{r}\right) = 3r(\frac{1}{3})$; $3(w-1) = r(1)$; $3w - 3 = r$;

$(3w - 3) - 5 = w$; $3w - 8 = w$; $-8 = -2w$; $w = 4$; $r - 5 = (4)$; $r = 9$.

$\frac{w}{r} = \frac{4}{9}$, *Ans.*

5. Let $\frac{p}{q}$ = original fraction;

$\frac{p}{q} = \frac{3}{4}$; $4q\left(\frac{p}{q}\right) = 4q(\frac{3}{4})$; $4p = 3q$; $p = \frac{3q}{4}$; $3\left(\frac{3q}{4}\right) + 21 = 4q$;

$\frac{p+7}{q} = \frac{4}{3}$; $3q\left(\frac{p+7}{q}\right) = 3q(\frac{4}{3})$; $3(p+7) = q(4)$; $3p + 21 = 4q$

$\frac{9q}{4} + 21 = 4q$; $4\left[\frac{9q}{4} + 21\right] = 4[4q]$; $9q + 84 = 16q$; $84 = 7q$; $12 = q$; $\frac{p}{12} = \frac{3}{4}$; $12\left(\frac{p}{12}\right) = 12(\frac{3}{4})$;

$p = 3(3)$; $p = 9$. $\frac{p}{q} = \frac{9}{12}$, *Ans.*

6. Let $\frac{x}{y}$ = original fraction;

$\frac{x}{y} = \frac{2}{3}$; $3y\left(\frac{x}{y}\right) = 3y(\frac{2}{3})$; $3(x) = y(2)$; $3x = 2y$;

$\frac{x+10}{y} = \frac{3}{2}$; $2y\left(\frac{x+10}{y}\right) = 2y(\frac{3}{2})$; $2(x+10) = y(3)$; $2x + 20 = 3y$

$x = \frac{2}{3}y$; $2(\frac{2}{3}y) + 20 = 3y$; $\frac{4}{3}y + 20 = 3y$; $3(\frac{4}{3}y + 20) = 3(3y)$; $4y + 60 = 9y$; $60 = 5y$; $12 = y$;

$3x = 2(12)$; $3x = 24$; $x = 8$. $\frac{x}{y} = \frac{8}{12}$, *Ans.*

B **7.** Let p = tens digit in original fraction's num.; q = units digit in original fraction's num.; then $\frac{10p+q}{10q+p}$ = original fraction;

$\frac{10p+q}{10q+p} = \frac{4}{7}$; $7(10q+p)\left[\frac{10p+q}{10q+p}\right] = 7(10q+p)[\frac{4}{7}]$

$\frac{10p+q+11}{10q+p-22} = \frac{7}{4}$; $4(10q+p-22)\left[\frac{10p+q+11}{10q+p-22}\right] = 4(10q+p-22)[\frac{7}{4}]$;

$7(10p+q) = (10q+p)(4)$; $70p + 7q = 40q + 4p$; $66p = 33q$;

$4(10p+q+11) = (10q+p-22)(7)$; $40p + 4q + 44 = 70q + 7p - 154$; $33p - 66q = -198$;

$2p = q$; $p - 2(2p) = -6$; $-3p = -6$; $p = 2$;

$p - 2q = 6$;

$\frac{10(2)+q}{10q+(2)} = \frac{4}{7}$; $\frac{20+q}{10q+2} = \frac{4}{7}$; $7(10q+2)\left[\frac{20+q}{10q+2}\right] = 7(10q+2)[\frac{4}{7}]$; $7(20+q) = (10q+2)[4]$;

$140 + 7q = 40q + 8$; $132 = 33q$; $4 = q$. $\frac{10p+q}{10q+p} = \frac{10(2)+4}{10(4)+2} = \frac{24}{42}$, *Ans.*

8. Let x = tens digit in denom. of original fraction; y = units digit in denom. of original fraction then $10x + y$ = denom. of original fraction; $\frac{x+y}{10x+y}$ = original fraction;

$\frac{x+y}{10x+y} = \frac{1}{7}$; $7(10x+y)\left[\frac{x+y}{10x+y}\right] = 7(10x+y)[\frac{1}{7}]$

$\frac{x+y+3}{10x+y+3} = \frac{1}{4}$; $4(10x+y+3)\left[\frac{x+y+3}{10x+y+3}\right] = 4(10x+y+3)[\frac{1}{4}]$;

$$\begin{matrix} 7(x+y) = (10x+y)(1) \\ 4(x+y+3) = (10x+y+3)(1) \end{matrix}; \quad 7x+7y = 10x+y \quad 4x+4y+12 = 10x+y+3; \quad \begin{matrix} 6y = 3x \\ 3y-6x+9 = 0 \end{matrix};$$

$$\begin{matrix} 2y = x \\ 3y-6x+9 = 0 \end{matrix}; \; 3y-6(2y)+9 = 0; \; 3y-12y = -9; \; -9y = -9; \; y = 1; \; \frac{x+(1)}{10x+(1)} = \frac{1}{7};$$

$$7(10x+1)\left[\frac{x+1}{10x+1}\right] = 7(10x+1)[\tfrac{1}{7}]; \; 7(x+1) = (10x+1)1; \; 7x+7 = 10x+1;$$

$-3x = -6$, $x = 2$. $\dfrac{x+y}{10x+y} = \dfrac{(2)+(1)}{10(2)+(1)} = \dfrac{3}{21}$, *Ans.*

9. Let x = tens digit in num. of original fraction; y = units digit in num. of original fraction; then $\dfrac{10x+y}{10y+x}$ = original fraction;

$$\frac{10x+y+11}{10y+x+7} = \frac{1}{2} \quad 2(10y+x+7)\left[\frac{10x+y+11}{10y+x+7}\right] = 2(10y+x+7)[\tfrac{1}{2}]$$
$$\frac{x+y}{y-x} = 2 \quad (y-x)\left[\frac{x+y}{y-x}\right] = (y-x)[2]$$

$$2(10x+y+11) = (10y+x+7)(1); \quad 20x+2y+22 = 10y+x+7; \quad 19x-8y = -15;$$
$$1(x+y) = 2y-2x \qquad\qquad x+y = 2y-2x \qquad\qquad 3x-y = 0$$

$$\begin{matrix} 19x-8y = -15 \\ -8(3x-y) = -8(0) \end{matrix}; \quad \begin{matrix} 19x-8y = -15 \\ -24x+8y = 0 \\ \hline -5x = -15 \end{matrix}; \; x = 3; \; \frac{10(3)+y+11}{10y+(3)+7} = \frac{1}{2}; \; \frac{30+y+11}{10y+10} = \frac{1}{2};$$

$$\frac{41+y}{10y+10} = \frac{1}{2}; \; 2[10y+10]\left[\frac{41+y}{10y+10}\right] = 2[10y+10][\tfrac{1}{2}]; \; 2(41+y) = 1(10y+10);$$

$82+2y = 10y+10$; $-8y = -72$; $y = 9$. $\dfrac{10x+y}{10y+x} = \dfrac{10(3)+(9)}{10(9)+(3)} = \dfrac{39}{93}$, *Ans.*

10. Let x = tens digit in num. of original fraction; y = units digit in num. of original fraction; then $\dfrac{(100)2+10x+y}{100(y)+10x+2}$ = original fraction;

$$\frac{200+10x+y}{100y+10x+2+16} = \frac{1}{2} \quad 2(100y+10x+18)\left[\frac{200+10x+y}{100y+10x+18}\right] = 2(100y+10x+18)[\tfrac{1}{2}]$$
$$\frac{10x+y+89}{100y+10x+2} = \frac{1}{4} \quad 4(100y+10x+2)\left[\frac{10x+y+89}{100y+10x+2}\right] = 4(100y+10x+2)[\tfrac{1}{4}]$$

$$\begin{matrix} 2(200+10x+y) = (100y+10x+18)(1) \\ 4(10x+y+89) = (100y+10x+2)(1) \end{matrix}; \quad \begin{matrix} 400+20x+2y = 100y+10x+18 \\ 40x+4y+356 = 100y+10x+2 \end{matrix};$$

$$\begin{matrix} 10x-98y = -382 \\ 30x-96y = -354 \end{matrix}; \quad \begin{matrix} -3(10x-98y) = -3(-382) \\ 30x-96y = -354 \end{matrix}; \quad \begin{matrix} -30x+294y = 1146 \\ 30x-96y = -354 \\ \hline 198y = 792 \end{matrix}; \; y = 4;$$

$$\frac{200+10x+(4)}{100(4)+10x+2+16} = \frac{1}{2}; \; \frac{204+10x}{418+10x} = \frac{1}{2}; \; 2(418+10x)\left[\frac{204+10x}{418+10x}\right] = 2(418+10x)[\tfrac{1}{2}];$$

$2(204+10x) = (418+10x)1$; $408+20x = 418+10x$; $10x = 10$; $x = 1$.

$\dfrac{(100)2+10(x)+(y)}{100(y)+10(x)+2} = \dfrac{200+10(1)+(4)}{100(4)+10(1)+2} = \dfrac{214}{412}$, *Ans.*

C **11.** Let x = tens digit in num. of fraction; y = units digit in num. of fraction; then $\dfrac{10x+y}{10y+x}$ = fraction;

$$\frac{10x+y}{10y+x} = \frac{4}{7}; \; 7(10y+x)\left[\frac{10x+y}{10y+x}\right] = 7(10y+x)[\tfrac{4}{7}]; \; 7(10x+y) = (10y+x)(4);$$

$70x+7y = 40y+4x$; $66x = 33y$; $2x = y$, $x \in \{1, 2, \ldots, 9\}$, $y \in \{1, 2, \ldots, 9\}$.

$\dfrac{10x+y}{10y+x} = \dfrac{12}{21}, \dfrac{24}{42}, \dfrac{36}{63}, \dfrac{48}{84}$, *Ans.*

$y = 2x$	
x	y
1	2
2	4
3	6
4	8

12. Let x = hundreds digit in num. of fraction; y = units digit in num. of fraction;

then $\dfrac{100x + (10)0 + y}{100y + (10)0 + x}$ = fraction;

$\dfrac{100x + y}{100y + x} = \dfrac{34}{67}$; $(67)(100y + x)\left[\dfrac{100x + y}{100y + x}\right] = (67)(100y + x)[\frac{34}{67}]$;

$67(100x + y) = (100y + x)(34)$; $6700x + 67y = 3400y + 34x$; $6666x = 3333y$; $2x = y$;

$x \in \{1, 2, \ldots, 9\}$
$y \in \{1, 2, \ldots, 9\}$.

$y = 2x$	
x	y
1	2
2	4
3	6
4	8

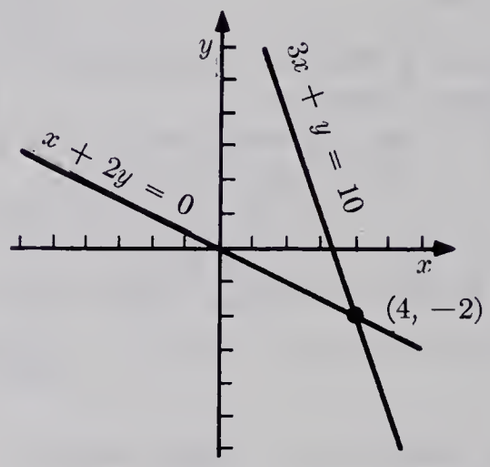
$\dfrac{100x + (10)0 + y}{100y + (10)0 + x} = \dfrac{102}{201}, \dfrac{204}{402}, \dfrac{306}{603}, \dfrac{408}{804}$, *Ans.*

Pages 389–390 · **CHAPTER TEST**

1.

$y = 10 - 3x$	
x	y
0	10
$\frac{10}{3}$	0
2	4

$2y = -x$	
x	y
0	0
2	−1
−2	1



$\{(4, -2)\}$

2. $2x = 45$; $x = \frac{45}{2}$; $(\frac{45}{2}) + y = 37$; $2[\frac{45}{2} + y] = 2[37]$; $45 + 2y = 74$; $2y = 29$; $y = \frac{29}{2}$. $\{(\frac{45}{2}, \frac{29}{2})\}$

3. $-6w = 6$; $w = -1$; $5r - 4(-1) = 14$; $5r + 4 = 14$; $5r = 10$; $r = 2$. $\{(2, -1)\}$

4. $2t = 44$; $t = 22$; $2s + (22) = 78$; $2s = 56$; $s = 28$. $\{(28, 22)\}$

5. Let x = no. correct questions; y = no. incorrect questions; then $x + y$ = no. questions on test;

$3x - 1y = 40$, $-2[3x - y] = -2[40]$; $-6x + 2y = -80$
$4x - 2y = 50$ $\quad 4x - 2y = 50$ $\quad 4x - 2y = 50$
$-2x \qquad = -30$; $x = 15$;

$3(15) - y = 40$, $45 - y = 40$; $-y = -5$; $y = 5$. $x + y = (15) + (5) = 20$, *Ans.*

6. $-3[x - y] = -3[26]$; $-3x + 3y = -78$
$3x - 8y = 3$ $\quad 3x - 8y = 3$
$-5y = -75$; $y = 15$; $x - (15) = 26$; $x = 41$. $\{(41, 15)\}$

7. $2[8w + 5z] = 2[31]$; $16w + 10z = 62$
$5[3w - 2z] = 5[0]$ $\quad 15w - 10z = 0$
$31w \qquad = 62$; $w = 2$;

$3(2) - 2z = 0$; $6 - 2z = 0$; $-2z = -6$; $z = 3$. $\{(2, 3)\}$

8. $m = 2 + n$; $3(2 + n) - 2n = 1$; $6 + 3n - 2n = 1$; $n = -5$; $m - (-5) = 2$; $m + 5 = 2$; $m = -3$. $\{(-3, -5)\}$

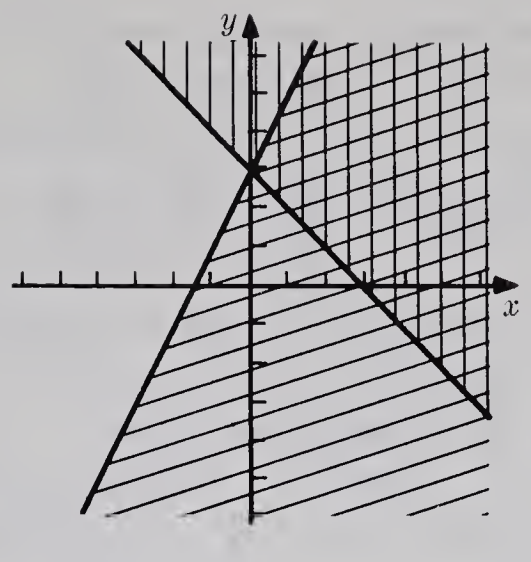
9. $s = (3 - 2r)$; $3r - 2(3 - 2r) = 57$; $3r - 6 + 4r = 57$; $7r = 63$, $r = 9$; $2(9) + s = 3$; $18 + s = 3$; $s = -15$. $\{(9, -15)\}$

10. $r = 30 - 2s$; $2(30 - 2s) - s = -45$; $60 - 4s - s = -45$; $-5s = -105$; $s = 21$; $r + 2(21) = 30$; $r + 42 = 30$; $r = -12$. $\{(-12, 21)\}$

11. (for boundary)

$y = 2x + 3$	
x	y
0	3
2	7
−2	−1

$y = 3 - x$	
x	y
0	3
3	0
2	1



$x > 0$; y may be any real no.

12. Let x = tens digit; y = units digit; then $10x + y$ = number

$x + y = 15$; $y = 15 - x$; $y = 15 - x$;

$(10y + x) = (10x + y) + 9$' $9y - 9x = 9$' $y - x = 1$ '

$(15 - x) - x = 1$; $15 - 2x = 1$; $-2x = -14$; $x = 7$; $(7) + y = 15$; $y = 8$.

$10x + y = 10(7) + (8) = 78$, *Ans.*

13. Let x = speed of boat in still water; y = speed of current;

$4(x - y) = 48$; $x - y = 12$

$3(x + y) = 48$' $x + y = 16$

$2x \quad = 28$; $x = 14$;

$4(14 - y) = 48$; $56 - 4y = 48$; $-4y = -8$; $y = 2$ mph, *Ans.*

14. Let x = George's present age; y = Henry's present age. Then $x + 5$ = George's age in 5 yr; $y + 5$ = Henry's age in 5 yr;

$y - x = 20$; $y - x = 20$; $y - x = 20$

$2(x + 5) = y + 5$' $2x + 10 = y + 5$' $-y + 2x = -5$

$x = 15$; $y - (15) = 20$; $y = 35$.

George, 15 yr; Henry, 35 yr, *Ans.*

15. Let x = num. of original fraction; y = denom. of original fraction; then $\frac{x}{y}$ = original fraction;

$\frac{x}{y} = \frac{3}{4}$; $4y\left[\frac{x}{y}\right] = 4y[\frac{3}{4}]$; $4x = 3y$; $4x = 3y$;

$\frac{x+3}{y+3} = \frac{4}{5}$; $5(y+3)\left[\frac{x+3}{y+3}\right] = 5(y+3)[\frac{4}{5}]$; $5(x+3) = (y+3)4$; $5x + 15 = 4y + 12$;

$4x = 3y$; $4x - 3y = 0$; $-4[4x - 3y] = -4[0]$; $-16x + 12y = 0$

$5x - 4y = -3$' $5x - 4y = -3$' $3[5x - 4y] = 3[-3]$' $15x - 12y = -9$

$-x \quad = -9$; $x = 9$;

$4(9) = 3y$; $36 = 3y$; $12 = y$. $\frac{x}{y} = \frac{9}{12}$, *Ans.*

Pages 390–393 · CHAPTER REVIEW

1. same axes **2.** ordered pair *or* solution set **3.** root

4. (6, 2) **5.** (−6, −4) **6.** (3, 5)

7. $4x = 20$; $x = 5$; $2(5) - y = 9$; $10 - y = 9$; $-y = -1$; $y = 1$. $\{(5, 1)\}$

8. $r = -30$; $2(-30) - t = 40$; $-60 - t = 40$; $-t = 100$; $t = -100$. $\{(-30, -100)\}$

9. $10m = 40$; $m = 4$; $5(4) - 3n = 11$; $20 - 3n = 11$; $-3n = -9$; $n = 3$. $\{(4, 3)\}$

10. equals

11. Let x = 1st no. (smaller); y = 2nd no. (larger); $y + 5 = 2x$, $x + y = 100$, *Ans.*

12. Let l = length, in ft; w = width, in ft; $l = 5w$, $2l + 2w = 72$, *Ans.*

13. Let x = length of longer part; y = length of shorter part;

$x + y = 7$; $x + y = 7$; $y = 7 - x$; $3x - 5(7 - x) = 9$; $3x - 35 + 5x = 9$; $8x = 44$;

$3x - 9 = 5y$' $3x - 5y = 9$'

$x = 5\frac{1}{2}$; $(\frac{11}{2}) + y = 7$, $y = \frac{3}{2} = 1\frac{1}{2}$. $5\frac{1}{2}$ ft, $1\frac{1}{2}$ ft, *Ans.*

14. $-3(x + 3y) = -3(26)$; $-3x - 9y = -78$
$3x + 2y = 29$; $3x + 2y = 29$
$-7y = -49$; $y = 7$; $x + 3(7) = 26$; $x + 21 = 26$; $x = 5$. $\{(5, 7)\}$

15. $-4(7r - 10s) = -4(1)$; $-28r + 40s = -4$
$56r - 40s = 28$; $56r - 40s = 28$
$28r = 24$; $r = \frac{6}{7}$;
$7(\frac{6}{7}) - 10s = 1$; $6 - 10s = 1$; $-10s = -5$; $s = \frac{1}{2}$. $\{(\frac{6}{7}, \frac{1}{2})\}$

16. $7(9w + 5z) = 7(33)$; $63w + 35z = 231$
$5(6w - 7z) = 5(-9)$; $30w - 35z = -45$
$93w = 186$; $w = 2$;
$9(2) + 5z = 33$; $18 + 5z = 33$; $5z = 15$; $z = 3$. $\{(2, 3)\}$

17. $u = 11 + v$; $2(11 + v) + 3v = 7$; $22 + 2v + 3v = 7$; $5v = -15$; $v = -3$; $2u + 3(-3) = 7$; $2u - 9 = 7$; $2u = 16$; $u = 8$. $\{(8, -3)\}$

18. $-z = 18 - 2w$, $z = 2w - 18$; $3w - 2(2w - 18) = 38$; $3w - 4w + 36 = 38$; $-w = 2$; $w = -2$; $2(-2) - z = 18$; $-4 - z = 18$; $-z = 22$; $z = -22$. $\{(-2, -22)\}$

19. $p = 10 - 2q$; $6(10 - 2q) + 3q = -3$; $60 - 12q + 3q = -3$; $-9q = -63$; $q = 7$; $p + 2(7) = 10$; $p + 14 = 10$; $p = -4$. $\{(-4, 7)\}$

20. half-plane **21.** (0, 10) **22.** (0, −5) **23.** position (place) **24.** 3; 1

25. Let x = tens digit; y = units digit; $10x + y$ = the no.;
$x - y = 2$
$x + y = 16$
$2x = 18$; $x = 9$; $9 - y = 2$; $-y = -7$; $y = 7$. $10x + y = 10(9) + (7) = 97$, *Ans.*

26. Let x = tens digit in original no.; y = units digit in original no.; then $10x + y$ = original no.;
$x + y = 12$; $x + y = 12$; $x + y = 12$
$10y + x + 36 = 10x + y$; $-9x + 9y = -36$; $-x + y = -4$
$2y = 8$; $y = 4$;
$x + (4) = 12$; $x = 8$. $10x + y = 10(8) + (4) = 84$, *Ans.*

27. greater **28.** 205

29. Let x = Norman's speed without wind; y = speed of wind;
$\frac{5}{4}(x - y) = 15$; $4[\frac{5}{4}(x - y)] = 4[15]$; $5(x - y) = 60$; $x - y = 12$
$\frac{3}{4}(x + y) = 15$; $4[\frac{3}{4}(x + y)] = 4[15]$; $3(x + y) = 60$; $x + y = 20$
$2x = 32$; $x = 16$ mph, *Ans.*

30. Let y = rate in still water; x = rate of current;
$\frac{13}{2}(y - x) = 91$; $2[\frac{13}{2}(y - x)] = 2[91]$; $13(y - x) = 182$; $y - x = 14$
$\frac{7}{2}(y + x) = 91$; $2[\frac{7}{2}(y + x)] = 2[91]$; $7(y + x) = 182$; $y + x = 26$
$2y = 40$; $y = 20$ mph, *Ans.*

31. Let n = Bertrand's present age; p = Allen's present age; then $n + 4$ = Bertrand's age in 4 yr; $p + 4$ = Allen's age in 4 yr;
$p = \frac{4}{5}n$; $6[p + 4] = 6[\frac{5}{6}(n + 4)]$; $6p + 24 = 5(n + 4)$; $6p + 24 = 5n + 20$;
$(p + 4) = \frac{5}{6}(n + 4)$
$6(\frac{4}{5}n) + 24 = 5n + 20$; $\frac{24}{5}n + 24 = 5n + 20$; $5[\frac{24}{5}n + 24] = 5[5n + 20]$;
$24n + 120 = 25n + 100$; $20 = n$; $p = \frac{4}{5}(20)$; $p = 16$. Bertrand, 20 yr; Allen, 16 yr, *Ans.*

32. Let x = Jack's present age; y = June's present age; then $x - 3$ = Jack's age 3 yr ago; $y - 3$ = June's age 3 yr ago; $x + 2$ = Jack's age in 2 yr; $y + 2$ = June's age in 2 yr;
$(y - 3) = \frac{2}{3}(x - 3)$; $3(y - 3) = 3[\frac{2}{3}(x - 3)]$; $3y - 9 = 2x - 6$; $3y - 2x = 3$;
$(y + 2) = \frac{3}{4}(x + 2)$; $4(y + 2) = 4[\frac{3}{4}(x + 2)]$; $4y + 8 = 3x + 6$; $4y - 3x = -2$;
$-3(3y - 2x) = -3(3)$; $-9y + 6x = -9$
$2(4y - 3x) = 2(-2)$; $8y - 6x = -4$
$-y = -13$; $y = 13$; $(13) - 3 = \frac{2}{3}(x - 3)$; $10 = \frac{2}{3}x - 2$;
$\frac{2}{3}x = 12$; $\frac{3}{2}[\frac{2}{3}x] = \frac{3}{2}[12]$; $x = 18$. Jack, 18 yr; June, 13 yr, *Ans.*

33. Let x = Mr. Granger's present age; y = Tim's present age; then $x - 1$ = Mr. Granger's age last yr; $y - 1$ = Tim's age last yr;

$$\begin{matrix} x = 2y + 10 \\ (x - 1) + 4 = 3(y - 1) \end{matrix}; x + 3 = 3y - 3; x - 3y = -6; (2y + 10) - 3y = -6; -y = -16;$$

$y = 16$; $x = 2(16) + 10 = 32 + 10 = 42$. Mr. G., 42 yr; Tim, 16 yr, *Ans.*

34. Let $\frac{x}{y}$ = fraction;

$$\begin{matrix} \frac{x}{y+1} = \frac{5}{8} \\ \frac{x-4}{y} = \frac{2}{5} \end{matrix}; \quad \begin{matrix} 8(y+1)\left[\frac{x}{y+1}\right] = 8(y+1)[\frac{5}{8}] \\ 5y\left[\frac{x-4}{y}\right] = 5y[\frac{2}{5}] \end{matrix}; \quad \begin{matrix} 8x = 5y + 5 \\ 5x - 20 = 2y \end{matrix}; \quad \begin{matrix} 8x - 5y = 5 \\ 5x - 2y = 20 \end{matrix};$$

$$\begin{matrix} -2(8x - 5y) = -2(5) \\ 5(5x - 2y) = 5(20) \end{matrix}; \quad \begin{matrix} -16x + 10y = -10 \\ 25x - 10y = 100 \end{matrix}; \quad 9x = 90; x = 10;$$

$\frac{10}{y+1} = \frac{5}{8}$; $8(y+1)\left(\frac{10}{y+1}\right) = 8(y+1)[\frac{5}{8}]$; $80 = 5y + 5$; $75 = 5y$; $y = 15$. $\frac{x}{y} = \frac{10}{15}$, *Ans.*

35. Let $\frac{x}{y}$ = fraction;

$$\begin{matrix} \frac{x}{y} = \frac{2}{5} \\ \frac{y+5}{x-1} = 5 \end{matrix}; \quad \begin{matrix} 5y\left(\frac{x}{y}\right) = 5y(\frac{2}{5}) \\ (x-1)\left(\frac{y+5}{x-1}\right) = (x-1)(5) \end{matrix}; \quad \begin{matrix} 5x = 2y \\ y + 5 = 5x - 5 \end{matrix}; \quad \begin{matrix} 5x - 2y = 0 \\ -5x + y = -10 \end{matrix}$$

$-y = -10$; $y = 10$;

$\frac{2}{5} = \frac{x}{10}$; $10[\frac{2}{5}] = 10\left[\frac{x}{10}\right]$; $4 = x$. $\frac{x}{y} = \frac{4}{10}$, *Ans.*

Pages 393–395 · EXTRA FOR EXPERTS

1. Let x = no. of $3\frac{1}{2}$ yd pieces and y = no. of $4\frac{1}{2}$ yd pieces where x and $y \in$ {nonnegative integers}. $3\frac{1}{2}x + 4\frac{1}{2}y = 55$; $7x + 9y = 110$; $x = \frac{110 - 9y}{7}$. Since $110 + 9 = 119$ is $7 \cdot 17$, let $y = 7s - 1$, $s \in$ {positive integers}. Then $x = \frac{110 - 9(7s - 1)}{7} = \frac{119 - 7 \cdot 9 \cdot s}{7}$ $= 17 - 9s$. 8 $3\frac{1}{2}$ yd pieces, 6 $4\frac{1}{2}$ yd pieces, *Ans.*

s	1
x	8
y	6

2. Let x = no. of loaves of bread. If k and $n \in$ {positive integers}, $x = 4k + 1$ and $x = 5n + 4$. Then $4k + 1 = 5n + 4$; $4k = 5n + 3$. Since $5 + 3 = 8 = 4 \cdot 2$, let $n = 4s + 1$ where $s \in$ {nonnegative integers}. Then $4k = 5(4s + 1) + 3 = 4(5s + 2)$; $k = 5s + 2$, and $x = 20s + 9$. Since x is near 72, take $s = 3$; then $x = 69$ loaves, *Ans.*

3. Let x = the integer. If l, m, $n \in$ {positive integers}, $x = 2l + 1$; $x = 3m + 2$; $x = 7n + 6$. Then, $2l + 1 = 3m + 2$; $2l = 3m + 1$. Since $3 + 1 = 2 \cdot 2$, let $m = 2r + 1$, $r \in$ {nonnegative integers}. Then $2l = 3(2r + 1) + 1 = 2(3r + 2)$; $l = 3r + 2$, and $x = 6r + 5$. But $x = 7n + 6$. Hence, $6r + 5 = 7n + 6$, $7n = 6r - 1$. Because $6 \cdot 6 - 1 = 35 = 7 \cdot 5$, let $r = 7t + 6$, $t \in$ {nonnegative integers}. Then, $7n = 6(7t + 6) - 1 = 7(6t + 5)$; $n = 6t + 5$, and $x = 42t + 41$. For $t = 0$, $x = 41$, *Ans.*

4. Let x = no. of roosters, y = no. of hens, and z = no. of chicks.

Then $\begin{matrix} x + y + z = 100 \\ 5x + 3y + \frac{1}{3}z = 100 \end{matrix}$; $\begin{matrix} 3x + 3y + 3z = 300 \\ 5x + 3y + \frac{1}{3}z = 100 \end{matrix}$

$-2x + \frac{8}{3}z = 200$; $\frac{8}{3}z = 200 + 2x = 2(100 + x)$; $z = \frac{3}{4}(100 + x)$.

Let $100 + x = 4t$, $t \in$ {positive integers}; $x = 4t - 100$, $z = 3t$, and $y = 100 - (x + z) = 200 - 7t$. Since $x \geq 5$, take $t = 27$ and 28.

t	27	28
x	8	12
y	11	4
z	81	84

8 roosters, 11 hens, 81 chicks or 12 roosters, 4 hens, 84 chicks, *Ans.*

5. Let x = no. of dimes, y = no. of nickels, z = no. of pennies; $x, y, z \in$ {integers ≥ 1}.
Then $x + y + z = 15$
$10x + 5y + z = 100$
$-9x - 4y = -85$; $9x = 85 - 4y$; $x = \dfrac{85 - 4y}{9}$.

Since $85 - 4 = 9 \cdot 9$, let $y = 9t + 1$, $t \in$ {nonnegative integers}. Then, $x = \dfrac{85 - 4(9t + 1)}{9} = 9 - 4t$. Also $z = 15 - (x + y) = 15 - (5t + 10) = 5 - 5t$.

t	0
x	9
y	1
z	5

9 dimes, 1 nickel, 5 pennies, *Ans.*

6. $4a - 11b + 12c = 22$
$4a + 20b - 16c = 68$
$-31b + 28c = -46$; $-31b = -28c - 46$; $b = \dfrac{2(14c + 23)}{31}$.

Let $14c + 23 = 31t$, where $t \in$ {nonnegative integers}. Then $b = \dfrac{2(31t)}{31} = 2t$; $c = \dfrac{31t - 23}{14}$. Since $31 \cdot 3 - 23 = 70 = 5 \cdot 14$, let $t = 14s + 3$ where $s \in$ {nonnegative integers}. Then $c = \dfrac{31(14s + 3) - 23}{14} = \dfrac{31 \cdot 14s + 70}{14} = 31s + 5$ and $b = 28s + 6$.

$a = \dfrac{11b - 12c + 22}{4} = \dfrac{11(28s + 6) - 12(31s + 5) + 22}{4} = -16s + 7$.

Thus s can only be 0. For $s = 0$, $t = 3$; and $a = 7$, $b = 6$, $c = 5$, *Ans.*

7. Let x = no. of 3¢ stamps and y = no. of 5¢ stamps, x and $y \in$ {integers ≥ 1}. Then $3x + 5y = 50$; $x = \dfrac{50 - 5y}{3}$; $x = \dfrac{5(10 - y)}{3}$. Let $10 - y = 3t$, $t \in$ {nonnegative integers}. Then $x = \dfrac{5(3t)}{3} = 5t$ and $y = 10 - 3t$ with $0 < t \leq 3$.

t	3	2	1
x	15	10	5
y	1	4	7

15 3¢ stamps, 1 5¢ stamp;
10 3¢ stamps, 4 5¢ stamps;
or 5 3¢ stamps, 7 5¢ stamps, *Ans.*

8. Let x = no. of atoms of carbon, y = no. of atoms of hydrogen, z = no. of atoms of oxygen; $x, y, z \in$ {positive integers}. Then $12x + y + 16z = 46$; $y = 46 - 12x - 16z$.

C	x	1	1	2	2	3
H	y	18	2	6	No	No
O	z	1	2	1	2	1

$\therefore C_1H_{18}O_1$, $C_1H_2O_2$, $C_2H_6O_1$, *Ans.*

CHAPTER 11. The Real Numbers

Pages 399–400 · **WRITTEN EXERCISES**

A **1.** (8)(16) ? (7)(15); 128 > 105; $\frac{8}{15} > \frac{7}{16}$
2. (−22)(25) ? (−12)(45); −550 < −540; $-\frac{22}{45} < -\frac{12}{25}$
3. (19)(35) ? (56)(12); 665 < 672; $\frac{19}{12} < \frac{56}{35}$
4. (−46)(56) ? (−53)(49); −2576 > −2597; $-\frac{46}{49} > -\frac{53}{56}$
5. (200)(126) ? (240)(105); 25,200 = 25,200; $\frac{200}{105} = \frac{240}{126}$

6. $(138)(216)\ ?\ (184)(162);\ 29{,}808 = 29{,}808;\ \frac{138}{162} = \frac{184}{216}$

7. $-\frac{52}{3}\ ?\ -\frac{400}{12};\ (-52)(12)\ ?\ (-400)(3);\ -624 > -1200;\ -17\frac{1}{3} > -\frac{400}{12}$

8. $\frac{321}{15}\ ?\ \frac{106}{5};\ (321)(5)\ ?\ (106)(15);\ 1605 > 1590;\ \frac{321}{15} > 21\frac{1}{5}$

9. $(1)(31)\ ?\ (16)(2);\ 31 < 32;\ \frac{1}{2} < \frac{16}{31}.\ \left\{-\frac{4}{9}, \frac{1}{2}, \frac{16}{31}\right\}$

10. $(2)(14)\ ?\ (9)(3);\ 28 > 27;\ \frac{2}{3} > \frac{9}{14}.\ \left\{-\frac{3}{7}, \frac{9}{14}, \frac{2}{3}\right\}$

11. $\frac{25}{20} = \frac{5}{4}, \frac{24}{21} = \frac{8}{7};\ (5)(7)\ ?\ (8)(4);\ 35 > 32;\ \frac{5}{4} > \frac{8}{7};\ (5)(3)\ ?\ (4)(4);\ 15 < 16,\ \frac{5}{4} < \frac{4}{3}.\ \left\{\frac{24}{21}, \frac{25}{20}, \frac{4}{3}\right\}$

12. $\frac{22}{7} = 3\frac{1}{7}, \frac{10}{3} = 3\frac{1}{3}, \frac{157}{50} = 3\frac{7}{50};\ (1)(3)\ ?\ (1)(7);\ 3 < 7;\ \frac{1}{7} < \frac{1}{3};\ (1)(50)\ ?\ (7)(7);\ 50 > 49;$
$\frac{1}{7} > \frac{7}{50};\ \frac{7}{50} < \frac{1}{7} < \frac{1}{3}.\ \left\{3, \frac{157}{50}, \frac{22}{7}, \frac{10}{3}\right\}$

13. $-\frac{11}{4} = -2\frac{3}{4}, -\frac{87}{32} = -2\frac{23}{32}, -\frac{16}{6} = -2\frac{2}{3};\ (-3)(32)\ ?\ (-23)(4);\ -96 < -92,\ -\frac{3}{4} < -\frac{23}{32};$
$(-3)(3)\ ?\ (-2)(4);\ -9 < -8,\ -\frac{3}{4} < -\frac{2}{3};\ (-23)(3)\ ?\ (-2)(32);\ -69 < -64,\ -\frac{23}{32} < -\frac{2}{3}.$
$\left\{-\frac{11}{4}, -\frac{87}{32}, -\frac{16}{6}\right\}$

14. $-\frac{3}{2} = -1\frac{1}{2}, -\frac{99}{70} = -1\frac{29}{70}, -\frac{7}{5} = -1\frac{2}{5}, -\frac{17}{12} = -1\frac{5}{12};\ (-1)(70)\ ?\ (-29)(2);$
$-70 < -58, -\frac{1}{2} < -\frac{29}{70};\ (-29)(5)\ ?\ (-2)(70);\ -145 < -140,\ -\frac{29}{70} < -\frac{2}{5};\ (-1)(12)\ ?\ (-5)(2);$
$-12 < -10;\ -\frac{1}{2} < -\frac{5}{12};\ (-29)(12)\ ?\ (-5)(70);\ -348 > -350;\ -\frac{29}{70} > -\frac{5}{12};$
$-\frac{1}{2} < -\frac{5}{12} < -\frac{29}{70} < -\frac{2}{5}.\ \left\{-\frac{3}{2}, -\frac{17}{12}, -\frac{99}{70}, -\frac{7}{5}\right\}$

15. $1 - \frac{11}{12} = \frac{12}{12} - \frac{11}{12} = \frac{1}{12};\ \frac{11}{12} + \frac{1}{2}\left(\frac{1}{12}\right) = \frac{22}{24} + \frac{1}{24} = \frac{23}{24}$

16. $\frac{5}{7} - \frac{9}{14} = \frac{10}{14} - \frac{9}{14} = \frac{1}{14};\ \frac{9}{14} + \frac{1}{2}\left(\frac{1}{14}\right) = \frac{18}{28} + \frac{1}{28} = \frac{19}{28}$

17. $\frac{1}{10} - \frac{1}{100} = \frac{10}{100} - \frac{1}{100} = \frac{9}{100};\ \frac{1}{100} + \frac{1}{2}\left(\frac{9}{100}\right) = \frac{2}{200} + \frac{9}{200} = \frac{11}{200}$

18. $-\frac{1}{100} - \left(-\frac{1}{1000}\right) = -\frac{10}{1000} - \left(-\frac{1}{1000}\right) = -\frac{9}{1000};$
$-\frac{1}{1000} + \frac{1}{2}\left(-\frac{9}{1000}\right) = -\frac{2}{2000} + \left(-\frac{9}{2000}\right) = -\frac{11}{2000}$

19. $-6\frac{1}{7} - \left(-7\frac{2}{5}\right) = -6\frac{5}{35} - \left(-7\frac{14}{35}\right) = 1\frac{9}{35};$
$-7\frac{2}{5} + \frac{1}{2}\left(1\frac{9}{35}\right) = -7\frac{14}{35} + \frac{1}{2}\left(\frac{44}{35}\right) = -7\frac{14}{35} + \frac{22}{35} = -6\frac{49}{35} + \frac{22}{35} = -6\frac{27}{35}$

20. $5\frac{1}{8} - 4\frac{2}{3} = 5\frac{3}{24} - 4\frac{16}{24} = 4\frac{27}{24} - 4\frac{16}{24} = \frac{11}{24};\ 4\frac{2}{3} + \frac{1}{2}\left(\frac{11}{24}\right) = 4\frac{32}{48} + \frac{11}{48} = 4\frac{43}{48}$

B **21.** $1\frac{1}{4} - \frac{7}{8} = \frac{10}{8} - \frac{7}{8} = \frac{3}{8};\ \frac{7}{8} + \frac{1}{3}\left(\frac{3}{8}\right) = \frac{7}{8} + \frac{1}{8} = 1$

22. $-\frac{1}{4} - \left(-\frac{3}{2}\right) = -\frac{1}{4} - \left(-\frac{6}{4}\right) = \frac{5}{4};\ -\frac{3}{2} + \frac{1}{5}\left(\frac{5}{4}\right) = -\frac{6}{4} + \frac{1}{4} = -\frac{5}{4}$

23. If $a < b$: $a + \frac{1}{2}(b - a) = \dfrac{2a + b - a}{2} = \dfrac{a + b}{2}.$

If $b < a$: $b + \frac{1}{2}(a - b) = \dfrac{2b + a - b}{2} = \dfrac{a + b}{2}.$

24. (a) $p + \dfrac{q - p}{3} = \dfrac{3p + q - p}{3} = \dfrac{2p + q}{3}$ (b) $p - \dfrac{p - q}{3} = \dfrac{3p - p + q}{3} = \dfrac{2p + q}{3}$

25. For each integer there is a next larger one, obtained by adding 1 to the given integer; $2 + 1 = 3$.

26. The property of density implies that no matter how small you choose the rational number greater than 2, it is possible to find another rational number between 2 and this number, i.e., greater than 2 and less than the number chosen.

27. a. True b. False c. False d. True

28–31. In each case $\dfrac{a}{c}$ and $\dfrac{b}{d}$ represent rational numbers by definition. Since the system of rational numbers is closed under addition, subtraction, multiplication, and division (except by 0), each of the given expressions represents a rational number.

Page 403 · WRITTEN EXERCISES

A **1.** .08 **2.** .06 **3.** .21875 **4.** 1.125 **5.** $-.\overline{90}$ **6.** $-1.\overline{6}$ **7.** $-.\overline{285714}$ **8.** −.2125 **9.** 2.025

10. $.\overline{7428571}$ **11.** $-.2\overline{6}$ **12.** $-.\overline{380952}$ **13.** $\frac{666}{1000} = \frac{333}{500}$ **14.** $\frac{4}{10} = \frac{2}{5}$

15.
$$\begin{aligned} 10N &= 3.37\overline{7} \\ N &= .33\overline{7} \\ \hline 9N &= 3.040 \end{aligned}$$
$$N = \frac{3.04}{9} = \frac{304}{900} = \frac{76}{225}$$

16.
$$\begin{aligned} 100N &= 33.7\overline{37} \\ N &= .3\overline{37} \\ \hline 99N &= 33.400 \end{aligned}$$
$$N = \frac{33.4}{99} = \frac{334}{990} = \frac{167}{495}$$

17.
$$\begin{aligned} 100N &= 27.\overline{27} \\ N &= .\overline{27} \\ \hline 99N &= 27.00 \end{aligned}$$
$$N = \frac{27}{99} = \frac{3}{11}$$

18. $1000N = 303.\overline{303}$
$N = .\overline{303}$
$999N = 303.000$
$N = \frac{303}{999} = \frac{101}{333}$

19. $1000N = -2567.\overline{567}$
$N = -2.\overline{567}$
$999N = -2565.000$
$N = \frac{-2565}{999} = -\frac{95}{37}$

20. $10N = -12.02\overline{2}$
$N = -1.20\overline{2}$
$9N = -10.820$
$N = \frac{-10.82}{9} = -\frac{1082}{900} = -\frac{541}{450}$

B **21.** $.27 - .27\overline{27} = -.00\overline{27}$; any no. N such that $.27 < N < .27\overline{27}$ will be between them, e.g., .2727

22. $.33 - .3\overline{33} = -.00\overline{33}$; any no. N such that $.33 < N < .\overline{33}$ will be between them, e.g., .3325

23. $.9 - .9\overline{9} = -.0\overline{9}$; any no. N such that $.9 < N < .\overline{9}$ will be between them, e.g., .95

24. $.16 - .16\overline{16} = -.00\overline{16}$; any no. N such that $.16 < N < .\overline{16}$ will be between them, e.g., .161

25. $\frac{1}{12} = .08\overline{3}$; $.0833\overline{3} - .0833 = .0000\overline{3}$; any no. N such that $.0833 < N < .08\overline{3}$ will be between them, e.g., .08332

26. $\frac{1}{11} = .\overline{09}$; $.090909\overline{09} - .0909 = .000009\overline{09}$; any no. N such that $.0909 < N < .\overline{09}$ will be between them, e.g., .090905

27. $\{.\overline{1}, .\overline{2}, .\overline{3}, \cdots, .\overline{8}\}$; $2(.\overline{1}) = .\overline{2}$, $3(.\overline{1}) = .\overline{3}$, $\cdots$, $8(.\overline{1}) = .\overline{8}$

28. $\{.\overline{09}, .\overline{18}, .\overline{27}, \cdots, .\overline{90}\}$; $2(.\overline{09}) = .\overline{18}$, $3(.\overline{09}) = .\overline{27}$, $\cdots$, $10(.\overline{09}) = .\overline{90}$

29. $\{.\overline{142857}, .\overline{285714}, .\overline{428571}, \cdots, .\overline{857142}\}$; $2(.\overline{142857}) = .\overline{285714}$, $3(.\overline{142857}) = .\overline{428571}$, $\cdots$, $6(.142857) = .857142$

30. $\{.\overline{076923}, .\overline{153846}, .\overline{230769}, \cdots, .\overline{923076}\}$; $2(.\overline{076923}) = .\overline{153846}$, $3(.\overline{076923}) = .\overline{230769}$, $\cdots$, $12(.\overline{076923}) = .\overline{923076}$

Pages 405–407 · **WRITTEN EXERCISES**

A **1.** $\sqrt{4(121)} = \sqrt{2^2(11^2)} = 2(11) = 22$

2. $\sqrt{3^2(7^2)} = 3(7) = 21$

3. $\sqrt{4^2(9^2)} = 4(9) = 36$

4. $\sqrt{8^2(6^2)} = 8(6) = 48$

5. $-\sqrt{2^2(17^2)} = -[2(17)] = -34$

6. $-\sqrt{2^2(14^2)} = -[2(14)] = -28$

7. $-\sqrt{\frac{3^2(11^2)}{5^2}} = -\left[\frac{3(11)}{5}\right] = -\frac{33}{5}$

8. $-\sqrt{\frac{2^2(11^2)}{3^2}} = -\left[\frac{2(11)}{3}\right] = -\frac{22}{3}$

9. $\pm\sqrt{\frac{1}{2^2(9^2)}} = \pm\frac{1}{2(9)} = \pm\frac{1}{18}$

10. $\pm\sqrt{\frac{1}{3^2(13^2)}} = \pm\frac{1}{3(13)} = \pm\frac{1}{39}$

11. $\pm\sqrt{\frac{4^2}{5^2(7^2)}} = \pm\frac{4}{5(7)} = \pm\frac{4}{35}$

12. $\pm\sqrt{\frac{7^2}{9^2(4^2)}} = \pm\frac{7}{9(4)} = \pm\frac{7}{36}$

13. $x = \pm\sqrt{36} = \pm 6$. $\{6, -6\}$

14. $y = \pm\sqrt{49} = \pm 7$. $\{7, -7\}$

15. $4a^2 = 1$; $a^2 = \frac{1}{4}$; $a = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$. $\{\frac{1}{2}, -\frac{1}{2}\}$

16. $9b^2 = 4$; $b^2 = \frac{4}{9}$; $b = \pm\sqrt{\frac{4}{9}} = \pm\frac{2}{3}$. $\{\frac{2}{3}, -\frac{2}{3}\}$

17. $3t^2 = 27$; $t^2 = 9$; $t = \pm\sqrt{9} = \pm 3$. $\{3, -3\}$

18. $5k^2 = 125$; $k^2 = 25$; $k = \pm\sqrt{25} = \pm 5$. $\{5, -5\}$

B **19.** $(r + s)^2 = r^2 + 2rs + s^2 = (r^2 + s^2) + 2rs = 12 + 2(2) = 16$; $r + s = \sqrt{16} = 4$

20. $(r - s)^2 = r^2 - 2rs + s^2$; $(5)^2 = r^2 + s^2 - 2(3)$; $r^2 + s^2 = 25 + 6 = 31$

21. $(x - y)^2 = 5$; $x^2 - 2xy + y^2 = 5$; $x^2 - 2(1) + y^2 = 5$; $x^2 + y^2 = 7$;
$(x + y)^2 = x^2 + 2xy + y^2 = x^2 + y^2 + 2xy = 7 + 2(1) = 9$; $x + y = -\sqrt{9} = -3$

22. $(x + y)^2 = x^2 + 2xy + y^2$; $(8)^2 = (x^2 + y^2) + 2xy$; $64 = 40 + 2xy$; $2xy = 24$;
$(x - y)^2 = x^2 - 2xy + y^2 = x^2 + y^2 - 2xy = 40 - 24 = 16$; $x - y = \sqrt{16} = 4$

23. $\sqrt{(-a)^2} \neq -a$ since $\sqrt{(-a)^2} = |-a| = a$

24. The fallacy occurs in the reasoning: $(x - 1)^2 = (1 - x)^2$; $\therefore x - 1 = 1 - x$. If $x > 1$ then $x - 1$ is positive and $\sqrt{(x - 1)^2} = x - 1$, whereas $1 - x$ is negative and $\sqrt{(1 - x)^2} = -(1 - x) = x - 1$. This leads to $x - 1 = x - 1$. If $x < 1$ then $x - 1$ is negative and $\sqrt{(x - 1)^2} = -(x - 1) = 1 - x$, whereas $1 - x$ is positive and $\sqrt{(1 - x)^2} = 1 - x$. This leads to $1 - x = 1 - x$.

C **25.** $\sqrt{x^2} = x + 18$; $x^2 = x^2 + 36x + 324$; $36x = -324$; $x = -9$. $\{-9\}$

26. $\sqrt{y^2} = y - 50$; $y^2 = y^2 - 100y + 2500$; $100y = 2500$; $y = 25$ (reject, $625 \neq 25 - 50$). $\emptyset$

27. *Prove:* If $a \geq 0$ and $b > 0$, then $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

To show that $\frac{\sqrt{a}}{\sqrt{b}} \geq 0$:

If $a \geq 0$, then $\sqrt{a} \geq 0$	Defn. of principal root
If $b > 0$, then $\sqrt{b} > 0$	Defn. of principal root
$\therefore \frac{\sqrt{a}}{\sqrt{b}} \geq 0$	Quotient of 2 positive nos. is positive

To show that $\left(\frac{\sqrt{a}}{\sqrt{b}}\right)^2 = \frac{a}{b}$:

$\left(\frac{\sqrt{a}}{\sqrt{b}}\right)^2 = \left(\frac{\sqrt{a}}{\sqrt{b}}\right)\left(\frac{\sqrt{a}}{\sqrt{b}}\right)$	Defn. of a power
$= \frac{(\sqrt{a})(\sqrt{a})}{(\sqrt{b})(\sqrt{b})}$	Defn. of the product of fractions
$= \frac{\sqrt{a^2}}{\sqrt{b^2}}$	Product prop. of square root
$= \frac{a}{b}$	Defn. of square root

28.

$(\sqrt{a})^3 = \sqrt{a} \cdot \sqrt{a} \cdot \sqrt{a}$	Defn. of a power
$= \sqrt{a \cdot a \cdot a}$	Product prop. of square roots
$= \sqrt{a^3}$	Defn. of a power

29. If $a > b$ and $b > 0$, then $a \cdot a > a \cdot b$ Mult. prop. of inequality

and $a \cdot b > b \cdot b$	Mult. prop. of inequality
$\therefore a \cdot a > b \cdot b$	Transitive prop. of inequality
$a^2 > b^2$	Defn. of a power

To show that a positive no. cannot have two different positive square roots: assume the two positive square roots of x^2 are p and q. If $p > q$, then $p^2 > q^2$. Since p^2 and $q^2 = x^2$, this implies that $x^2 > x^2$ which is a contradiction. If $q > p$, the same contradiction develops.

30. If $b < a$ and $a < 0$, then $b \cdot b > a \cdot b$ Mult. prop. of inequality

and $a \cdot b > a \cdot a$	Mult. prop. of inequality
$\therefore b \cdot b > a \cdot a$	Transitive prop. of inequality
$a^2 < b^2$	Defn. of a power

To show that a positive no. cannot have two differentne gative square roots: assume the two negative square roots of x^2 are p and q. If $p > q$, then $p^2 < q^2$. Since p^2 and $q^2 = x^2$, this implies that $x^2 < x^2$ which is a contradiction. If $q > p$, the same contradiction develops.

Pages 410–411 · WRITTEN EXERCISES

A **1.** $\sqrt{3.61} = \sqrt{\frac{361}{100}} = \frac{19}{10} = 1.9$ **2.** $\sqrt{4.41} = \sqrt{\frac{441}{100}} = \frac{21}{10} = 2.1$

3. $\sqrt{10.24} = \sqrt{\frac{1024}{100}} = \frac{32}{10} = 3.2$ **4.** $\sqrt{30.25} = \sqrt{\frac{3025}{100}} = \frac{55}{10} = 5.5$

5. $\sqrt{1764} = 42$ **6.** $\sqrt{5329} = 73$

7. $-\sqrt{316.84} = -\sqrt{2^2(79.21)} = -2\sqrt{79.21} = -2\sqrt{\frac{7921}{100}} = -2(\frac{89}{10}) = -2(8.9) = -17.8$

8. $-\sqrt{408.04} = -\sqrt{2^2(102.01)} = -2\sqrt{102.01} = -2(10.10) = -20.2$

To find $\sqrt{102.01}$: Let $a^2 \doteq 102.01$ $\sqrt{102.01} = 10.10$

$a = 10$

$102.01 \div 10 \doteq 10.20$

$2)20.20$

$a = 10.10$

$102.01 \div 10.10 = 10.10$

9. $\sqrt{24{,}336} = \sqrt{2^2(6084)} = \sqrt{2^2(78^2)} = 2(78) = 156$

10. $\sqrt{110{,}889} = \sqrt{9^2(1369)} = \sqrt{9^2(37^2)} = 9(37) = 333$

11. $\sqrt{2} \doteq 1.41$ **12.** $\sqrt{3} \doteq 1.73$

13. Let $a^2 \doteq 50.3$

$a = 7$

$50.3 \div 7 \doteq 7.1$

$2)14.1$

$a = 7.05$

$50.3 \div 7.05 \doteq 7.13$

$2)14.18$

$a = 7.09$

$50.3 \div 7.09 \doteq 7.094$

$\sqrt{50.3} \doteq 7.09$

14. Let $a^2 \doteq 10.7$

$a = 3$

$10.7 \div 3 \doteq 3.5$

$2)6.5$

$a = 3.25$

$10.7 \div 3.25 \doteq 3.292$

$2)6.542$

$a = 3.271$

$10.7 \div 3.271 \doteq 3.2711$

$\sqrt{10.7} \doteq 3.27$

15. $-\sqrt{356} = -\sqrt{2^2(89)} = -2\sqrt{89} \doteq -2(9.434) \doteq -18.87$

16. $-\sqrt{279} = -\sqrt{3^2(31)} = -3\sqrt{31} \doteq -3(5.568) \doteq -16.70$

17. $\sqrt{27} \doteq 5.196 \doteq 5.20$

18. $\sqrt{35} \doteq 5.916 \doteq 5.92$

B **19.** $x = \pm\sqrt{960} = \pm\sqrt{8^2(15)} = \pm 8\sqrt{15} \doteq \pm 8(3.873) \doteq \pm 31.0$. $\{31.0, -31.0\}$

20. $y = \pm\sqrt{405} = \pm\sqrt{9^2(5)} = \pm 9\sqrt{5} \doteq \pm 9(2.236) \doteq \pm 20.1$. $\{20.1, -20.1\}$

21. $t^2 = 3.2$; $t = \pm\sqrt{3.2} = \pm\sqrt{.16(20)} = \pm\sqrt{.4^2(2^2)(5)} = \pm .4(2)\sqrt{5} = \pm .8\sqrt{5} \doteq \pm .8(2.236) \doteq \pm 1.8$. $\{1.8, -1.8\}$

22. $u^2 = 12.6$; $u = \pm\sqrt{12.6} = \pm\sqrt{.09(140)} = \pm\sqrt{.3^2(2^2)(35)} = \pm .3(2)\sqrt{35} = \pm .6\sqrt{35} \doteq \pm .6(5.916) \doteq \pm 3.5$. $\{3.5, -3.5\}$

23. $z^2 = 160$; $z = \pm\sqrt{160} = \pm\sqrt{4^2(10)} = \pm 4\sqrt{10} \doteq \pm 4(3.162) \doteq \pm 12.6$. $\{12.6, -12.6\}$

24. $w^2 = 125$; $w = \pm\sqrt{125} = \pm\sqrt{5^2(5)} = \pm 5\sqrt{5} \doteq \pm 5(2.236) \doteq \pm 11.2$. $\{11.2, -11.2\}$

25.

Let $a^2 \doteq 21$ — $a = 4$ — $21 \div 4 \doteq \underline{5}$ — $2\overline{)9}$ — $a = 4.5$

Let $a^2 \doteq 21$ — $a = 5$ — $21 \div 5 \doteq \underline{4}$ — $2\overline{)9}$ — $a = 4.5$

$21 \div 4.5 \doteq \underline{4.6}$ — $2\overline{)9.1}$ — $a = 4.55$

$21 \div 4.55 \doteq \underline{4.61}$ — $2\overline{)9.16}$ — $a = 4.58$

$21 \div 4.58 \doteq 4.585$

$\sqrt{21} \doteq 4.58$

26.

Let $a^2 \doteq 90$ — $a = 9$ — $90 \div 9 = \underline{10}$ — $2\overline{)19}$ — $a = 9.5$

Let $a^2 \doteq 90$ — $a = 10$ — $90 \div 10 = \underline{9}$ — $2\overline{)19}$ — $a = 9.5$

$90 \div 9.5 \doteq \underline{9.4}$ — $2\overline{)18.9}$ — $a = 9.45$

$90 \div 9.45 \doteq \underline{9.52}$ — $2\overline{)18.97}$ — $a = 9.485$

$90 \div 9.485 \doteq 9.488$

$\sqrt{90} \doteq 9.49$

C **27.** $p^2 = 1.18$; $p = \pm\sqrt{1.18} = \pm\sqrt{\dfrac{118}{100}} = \dfrac{\pm\sqrt{2(59)}}{10} \doteq \dfrac{\pm(1.414)(7.681)}{10} \doteq \pm 1.1$. $\{1.1, -1.1\}$

28. $x^2 + 2x + 1 + x^2 - 2x + 1 = 55$; $2x^2 + 2 = 55$; $2x^2 = 53$; $x^2 = \frac{53}{2}$;
$x = \pm\sqrt{\dfrac{53}{2}} = \dfrac{\pm\sqrt{53}}{\sqrt{2}} \doteq \dfrac{\pm 7.280}{1.414} \doteq \pm 5.1$. $\{5.1, -5.1\}$

29. $11y^2 = 36$; $y^2 = \dfrac{36}{11}$; $y = \pm\sqrt{\dfrac{36}{11}} \doteq \dfrac{\pm 6}{3.317} \doteq \pm 1.8$. $\{1.8, -1.8\}$

30. $9z^2 = 11$; $z^2 = \dfrac{11}{9}$; $z = \pm\sqrt{\dfrac{11}{9}} \doteq \dfrac{\pm 3.317}{3} \doteq \pm 1.1$. $\{1.1, -1.1\}$

31. Assume $a + \sqrt{b}$ is rational: then $(a + \sqrt{b}) - a$ is rational (set of rational nos. is closed under subtraction). Since $(a + \sqrt{b}) - a = \sqrt{b}$, $\sqrt{b}$ is rational, which is a contradiction. Thus, $a + \sqrt{b}$ must be irrational.

32. Consider some positive integer n. Assume that its cube root is a fraction $\dfrac{a}{b}$ in lowest terms, i.e., $\sqrt[3]{n} = \dfrac{a}{b}$, where a, b, n are positive integers, and a and b have no common factors.

If $\sqrt[3]{n} = \dfrac{a}{b}$, then $n = \dfrac{a^3}{b^3}$, where $\dfrac{a^3}{b^3}$ is in lowest terms. Since n is an integer, b^3 must equal 1, and $b = 1$, which means that $\dfrac{a}{b} = \dfrac{a}{1} = a$. Thus, for the cube root of a positive integer n to be rational, the root must be an integer, $\dfrac{a}{1}$.

As there are many positive integers which have no integral cube roots, the symbol $\sqrt[3]{n}$ may represent an irrational number.

Page 411 · PROBLEMS

A **1.** $x^2 = 75$; $x = \sqrt{75} \doteq 8.7$ in, *Ans.*

2. $x^2 = 46$; $x = \sqrt{46} \doteq 6.8$ ft, *Ans.*

3. $87.92 \doteq 3.14r^2$; $r^2 \doteq 28$; $r \doteq \sqrt{28} \doteq 5.29$ cm, *Ans.*

4. $15.708 \doteq .7854d^2$; $d^2 \doteq 20$; $d \doteq \sqrt{20} \doteq 4.47$ cm, *Ans.*

5. $x^2 = \frac{35}{9}$; $x = \sqrt{\frac{35}{9}} \doteq \frac{5.916}{3} \doteq 2.0$ M, *Ans.*

6. $x^2 = \frac{63}{16}$; $x = \sqrt{\frac{63}{16}} = \frac{\sqrt{9 \cdot 7}}{4} \doteq \frac{3(2.646)}{4} = \frac{7.938}{4} \doteq 2.0$ M, *Ans.*

7. Let x = width; $3x$ = length;
$3x^2 = 225$;
$x^2 = 75$;
$x = \sqrt{75} \doteq 8.7$ M;
$3x \doteq 3(8.7) \doteq 26.1$ M. 26.1 M, 8.7 M, *Ans.*

8. Let x = width; $2x$ = length;
$2x^2 = 1814$;
$x^2 = 907$;
$x = \sqrt{907}$

To find $\sqrt{907}$:
Let $a^2 \doteq 907$
$a = 30$
$907 \div 30 \doteq 30.23$
$2\overline{)60.23}$
$a = 30.115$
$907 \div 30.115 \doteq 30.117$
$\sqrt{907} \doteq 30.1$. 30.1 cm, 60.2 cm, *Ans.*

Page 413 · WRITTEN EXERCISES

1. $10^2 \stackrel{?}{=} 6^2 + 8^2$; $100 = 36 + 64$; yes

2. $15^2 \stackrel{?}{=} 9^2 + 12^2$; $225 = 81 + 144$; yes

3. $52^2 \stackrel{?}{=} 20^2 + 48^2$; $2{,}704 = 400 + 2{,}304$; yes

4. $51^2 \stackrel{?}{=} 24^2 + 45^2$; $2{,}601 = 576 + 2{,}025$; yes

5. $c^2 = 15^2 + 20^2$; $c^2 = 225 + 400 = 625$; $c = 25.00$ in

6. $c^2 = 32^2 + 24^2$; $c^2 = 1024 + 576 = 1600$; $c = 40.00$ M

7. $c^2 = 6^2 + (1\frac{3}{4})^2$; $c^2 = 36 + (\frac{7}{4})^2 = 36 + \frac{49}{16} = \frac{576}{16} + \frac{49}{16} = \frac{625}{16}$; $c = \frac{25}{4} = 6\frac{1}{4} = 6.25$ mi

8. $c^2 = (2\frac{6}{7})^2 + 3^2$; $c^2 = (\frac{20}{7})^2 + 3^2 = \frac{400}{49} + 9 = \frac{400}{49} + \frac{441}{49} = \frac{841}{49}$; $c = \frac{29}{7} = 4\frac{1}{7} \doteq 4.14$ yd

9. $41^2 = a^2 + 9^2$; $1681 = a^2 + 81$; $a^2 = 1600$; $a = \sqrt{1600} = \sqrt{4^2 \cdot 10^2} = 40.00$ M

10. $25^2 = 10^2 + b^2$; $625 = 100 + b^2$; $b^2 = 525$; $b = \sqrt{525} = \sqrt{5^2(21)} = 5\sqrt{21} \doteq 5(4.583) \doteq 22.92$ ft

Pages 413–414 · PROBLEMS

A **1.** $c = 17$, $a = 8$; $17^2 = 8^2 + b^2$; $289 = 64 + b^2$; $b^2 \doteq 225$; $b = 15$ ft, *Ans.*

2. $c = 34$, $a = 16$; $34^2 = 16^2 + b^2$; $1156 = 256 + b^2$; $b^2 = 900$; $b = 30$ ft, *Ans.*

3. $a = 80$, $b = 50$; $c^2 = 50^2 + 80^2 = 2500 + 6400 = 8900$;
$c = \sqrt{8900} = \sqrt{10^2(89)} = 10\sqrt{89} \doteq 10(9.434) = 94.34$ ft, *Ans.*

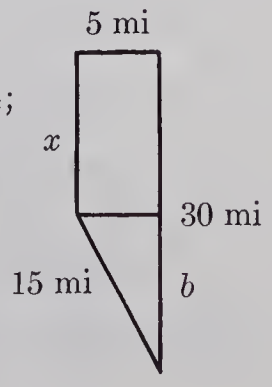
4. $a = 16$, $b = 16$; $c^2 = 16^2 + 16^2 = 256 + 256 = 512$;
$c = \sqrt{512} = \sqrt{16^2(2)} = 16\sqrt{2} \doteq 16(1.414) \doteq 22.62$ in, *Ans.*

B **5.** Let $x = a$, $x + 2 = b$, $x + 4 = c$; $(x + 4)^2 = x^2 + (x + 2)^2$; $x^2 + 8x + 16 = x^2 + x^2 + 4x + 4$;
$x^2 - 4x - 12 = 0$; $(x + 2)(x - 6) = 0$; $x + 2 = 0$ or $x - 6 = 0$; $x = -2$ (reject);
$x = 6$ in, $x + 2 = 8$ in, $x + 4 = 10$ in, *Ans.*

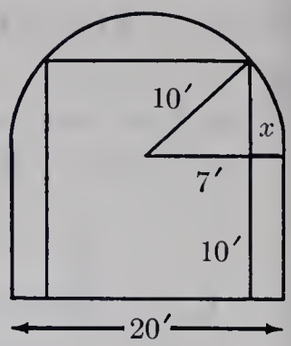
6. Let $x = a$, $x + 1 = b$, $x + 2 = c$; $(x + 2)^2 = x^2 + (x + 1)^2$;
$x^2 + 4x + 4 = x^2 + x^2 + 2x + 1$; $x^2 - 2x - 3 = 0$; $(x + 1)(x - 3) = 0$; $x + 1 = 0$ or
$x - 3 = 0$; $x = -1$ (reject), $x = 3$ ft, $x + 1 = 4$ ft, $x + 2 = 5$ ft, *Ans.*

7. Let $3x = a$, $4x = b$; $35^2 = (3x)^2 + (4x)^2$; $1225 = 9x^2 + 16x^2$; $25x^2 = 1225$; $x^2 = 49$; $x = 7$;
$3x = 21$ ft, $4x = 28$ ft, *Ans.*

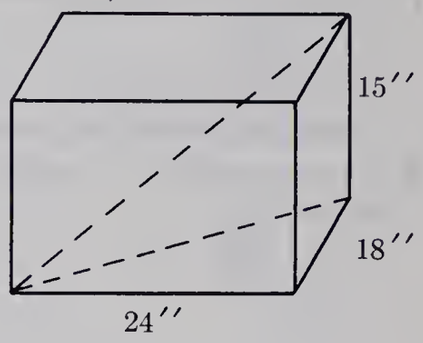
8. Let x = no. of miles he travels south; $x + b = 30$; $15^2 = 5^2 + b^2$;
$225 = 25 + b^2$; $b^2 = 200$; $b = \sqrt{200} = \sqrt{10^2(2)} = 10\sqrt{2} \doteq 10(1.414) = 14.14$;
$x \doteq 30 - 14.14 = 15.86$ mi, *Ans.*



C **9.** Let x = amt. by which height of truck exceeds 10 ft; $10^2 = x^2 + 7^2$;
$100 = x^2 + 49$; $x^2 = 51$; $x = \sqrt{51} \doteq 7.14$ ft;
height of truck $\doteq 10 + 7.14 = 17.14$ ft, *Ans.*



10. Let c = length of diagonal of base of box, d = length of diagonal of box;
$c^2 = 24^2 + 18^2$; $c^2 = 576 + 324 = 900$; $c = \sqrt{900} = 30$.
$d^2 = 15^2 + c^2$; $d^2 = 15^2 + 900$; $d^2 = 225 + 900 = 1125$
$d = \sqrt{1125} = 15\sqrt{5} \doteq 33.54$ in, *Ans.*



Pages 415–416 · **WRITTEN EXERCISES**

A

1. $(3 \cdot 8)\sqrt{17^2} = 24 \cdot 17 = 408$

2. $(5 \cdot 13)\sqrt{26^2} = 65 \cdot 26 = 1690$

3. $\sqrt{3 \cdot 2 \cdot 6} = \sqrt{6 \cdot 6} = 6$

4. $2\sqrt{3 \cdot 7 \cdot 2} = 2\sqrt{42}$

5. $\sqrt{\frac{3}{5} \cdot \frac{5}{3}} = \sqrt{1} = 1$

6. $(2 \cdot 3)\sqrt{\frac{3}{5} \cdot \frac{5}{4}} = 6\sqrt{\frac{3}{4}} = \frac{6}{2}\sqrt{3} = 3\sqrt{3}$

7. $\sqrt{\frac{1}{2} \cdot \frac{3}{5}} = \sqrt{\frac{3}{10}} = \sqrt{\frac{3}{10} \cdot \frac{10}{10}} = \frac{1}{10}\sqrt{30}$

8. $(\frac{2}{3} \cdot \frac{1}{5})\sqrt{\frac{5}{7} \cdot \frac{7}{20}} = \frac{2}{15}\sqrt{\frac{1}{4}} = \frac{2}{15} \cdot \frac{1}{2} = \frac{1}{15}$

9. $\sqrt{\frac{42}{5} \cdot \frac{30}{7}} = \sqrt{36} = 6$

10. $\sqrt{\frac{15}{2} \cdot \frac{10}{3}} = \sqrt{25} = 5$

11. $-10\sqrt{a^2} = -10a$

12. $-12\sqrt{b^2} = -12b$

13. $\frac{15}{5}\sqrt{\frac{10}{2}} = 3\sqrt{5}$

14. $\frac{3}{6}\sqrt{\frac{22}{2}} = \frac{1}{2}\sqrt{11}$

15. $\sqrt{81 \cdot 2} = 9\sqrt{2}$

16. $2\sqrt{9 \cdot 2} = 6\sqrt{2}$

17. $4\sqrt{49 \cdot 2} = 28\sqrt{2}$

18. $\sqrt{\frac{75}{3}} = \sqrt{25} = 5$

19. $\sqrt{\frac{2}{98}} = \sqrt{\frac{1}{49}} = \frac{1}{7}$

20. $\frac{3}{4}\sqrt{4 \cdot 3} = \frac{6}{4}\sqrt{3} = \frac{3}{2}\sqrt{3}$

21. $\frac{6\sqrt{9 \cdot 3}}{2} = 3 \cdot 3\sqrt{3} = 9\sqrt{3}$

22. $\frac{6\sqrt{\ \cdot 5}}{3} = 2 \cdot 2\sqrt{5} = 4\sqrt{5}$

23. $\frac{21}{7}\sqrt{\frac{5}{25}} = \frac{3\sqrt{5}}{5}$

24. $\sqrt{\frac{a^5}{a}} = \sqrt{a^4} = a^2$

25. $12\sqrt{\frac{27}{2} \cdot \frac{2}{2}} = \frac{12\sqrt{54}}{2} = 6\sqrt{9 \cdot 6} = 18\sqrt{6}$

26. $4\sqrt{\frac{5}{8} \cdot \frac{2}{2}} = \frac{4\sqrt{10}}{4} = \sqrt{10}$

27. $6\sqrt{\frac{11}{12} \cdot \frac{3}{3}} = \frac{6\sqrt{33}}{6} = \sqrt{33}$

28. $\sqrt{\frac{3y}{y}} = \sqrt{3}$

29. $\frac{1}{3}\sqrt{\frac{60}{6}} = \frac{\sqrt{10}}{3}$

30. $\sqrt{36c^2 \cdot 2} = 6c\sqrt{2}$

31. $\sqrt{4d^4 \cdot 10d} = 2d^2\sqrt{10d}$

32. $3\sqrt{9k^2 \cdot 10k} = 9k\sqrt{10k}$

B

33. $15\sqrt{a^2b^2} = 15ab$

34. $-14\sqrt{c^2d^2} = -14cd$

35. $\sqrt{x^2} - 3\sqrt{x} = x - 3\sqrt{x}$

36. $2\sqrt{y} - \sqrt{y^2} = 2\sqrt{y} - y$

37. $-2\sqrt{6 \cdot 3 \cdot 8} = -2\sqrt{6^2 \cdot 2^2} = -2 \cdot 6 \cdot 2 = -24$

38. $5\sqrt{14 \cdot 7 \cdot 8} = 5\sqrt{7^2 \cdot 4^2} = 5 \cdot 7 \cdot 4 = 140$

39. $\sqrt{21y \cdot y \cdot 7} = \sqrt{3 \cdot 7^2 \cdot y^2} = 7y\sqrt{3}$

40. $\sqrt{x \cdot 3x \cdot 24} = \sqrt{2^3 \cdot 3^2 \cdot x^2} = 6x\sqrt{2}$

41. $4a^2 \cdot 5a = 20a^3$

42. $9b^2 \cdot 2b = 18b^3$

43. $6\sqrt{6 \cdot 2} + 2\sqrt{6 \cdot 3} = 6\sqrt{2^2 \cdot 3} + 2\sqrt{3^2 \cdot 2} = 12\sqrt{3} + 6\sqrt{2}$

44. $10\sqrt{10 \cdot 5} - 5\sqrt{10 \cdot 15} = 10\sqrt{5^2 \cdot 2} - 5\sqrt{5^2 \cdot 6} = 50\sqrt{2} - 25\sqrt{6}$

45. $-6\sqrt{\frac{7}{6} \cdot \frac{6}{6}} = \frac{-6\sqrt{42}}{6} = -\sqrt{42}$

46. $5\sqrt{\frac{7}{5} \cdot \frac{5}{5}} = \frac{5\sqrt{35}}{5} = \sqrt{35}$

47. $-5\sqrt{(.04)(60)} = -5(.2)\sqrt{60} = -\sqrt{4 \cdot 15} = -2\sqrt{15}$

48. $\frac{647}{100}\sqrt{\frac{35}{100}} = \frac{647\sqrt{35}}{100 \cdot 10} = \frac{647\sqrt{35}}{1000} = .647\sqrt{35}$

49. $\frac{2\sqrt{7}}{\sqrt{7}} + \frac{\sqrt{14}}{\sqrt{7}} = 2\sqrt{\frac{7}{7}} + \sqrt{\frac{14}{7}} = 2\sqrt{1} + \sqrt{2} = 2 + \sqrt{2}$

50. $\dfrac{50\sqrt{34}}{2\sqrt{17}} = 25\sqrt{\dfrac{34}{17}} = 25\sqrt{2}$ (*Note:* $5\sqrt{17} - 3\sqrt{17} = (5-3)\sqrt{17} = 2\sqrt{17}$)

51. $\dfrac{5\sqrt{2}}{\sqrt{2}} + \dfrac{\sqrt{8}}{\sqrt{2}} = 5\sqrt{\dfrac{2}{2}} + \sqrt{\dfrac{8}{2}} = 5\sqrt{1} + \sqrt{4} = 5 + 2 = 7$

52. $\dfrac{\sqrt{3}}{\sqrt{3}} - \dfrac{\sqrt{12}}{\sqrt{3}} = \sqrt{\dfrac{3}{3}} - \sqrt{\dfrac{12}{3}} = \sqrt{1} - \sqrt{4} = 1 - 2 = -1$

53. $\dfrac{7\sqrt{a^3}}{\sqrt{a}} + \dfrac{\sqrt{a}}{\sqrt{a}} = 7\sqrt{\dfrac{a^3}{a}} + \sqrt{\dfrac{a}{a}} = 7\sqrt{a^2} + \sqrt{1} = 7a + 1$

54. $\dfrac{\sqrt{b}}{\sqrt{b}} - \dfrac{4\sqrt{b^3}}{\sqrt{b}} = \sqrt{\dfrac{b}{b}} - 4\sqrt{\dfrac{b^3}{b}} = \sqrt{1} - 4\sqrt{b^2} = 1 - 4b$

Pages 416–417 · **PROBLEMS**

B **1.** $(3x)^2 - x^2 = \frac{3}{4}$; $9x^2 - x^2 = \frac{3}{4}$; $8x^2 = \frac{3}{4}$; $x^2 = \frac{3}{32}$;
$x = \pm\sqrt{\dfrac{3}{32}\cdot\dfrac{2}{2}} = \dfrac{\pm\sqrt{6}}{8} \doteq \dfrac{\pm 2.449}{8} \doteq \pm.3$; $3x \doteq \pm.9$. $\pm.9, \pm.3$ *Ans.*

2. $(4x)^2 - (3x)^2 = 12$; $16x^2 - 9x^2 = 12$; $7x^2 = 12$; $x^2 = \frac{12}{7}$;
$x = \pm\sqrt{\dfrac{12}{7}\cdot\dfrac{7}{7}} = \dfrac{\pm\sqrt{4\cdot 21}}{7} = \dfrac{\pm 2\sqrt{21}}{7} \doteq \dfrac{\pm 2(4.583)}{7} = \dfrac{\pm 9.166}{7} \doteq \pm 1.3$;
$4x \doteq \pm 5.2$; $3x \doteq \pm 3.9$; *Ans.*

3. $3x \cdot 2x = 128$; $6x^2 = 128$; $x^2 = \frac{64}{3}$;
$x = \sqrt{\dfrac{64}{3}\cdot\dfrac{3}{3}} = \dfrac{8\sqrt{3}}{3} \doteq \dfrac{8(1.732)}{3} = \dfrac{13.856}{3} \doteq 4.6$; $3x \doteq 13.8$ in, $2x \doteq 9.2$ in, *Ans.*

4. Let x = length of side of square; c = length of diagonal of square; $x^2 = 70$; $x = \sqrt{70}$;
$c^2 = (\sqrt{70})^2 + (\sqrt{70})^2 = 70 + 70 = 140$;
$c = \sqrt{140} = \sqrt{4(35)} = 2\sqrt{35} \doteq 2(5.916) \doteq 11.8$ in, *Ans.*

5. $s = \dfrac{30 + 60 + 70}{2} = 80$ cm;
$A = \sqrt{80(80-30)(80-60)(80-70)} = \sqrt{80\cdot 50\cdot 20\cdot 10} = \sqrt{16\cdot 5\cdot 25\cdot 2\cdot 4\cdot 5\cdot 5\cdot 2}$
$= \sqrt{4^2\cdot 5^2\cdot 5^2\cdot 4^2\cdot 5} = 4\cdot 5\cdot 5\cdot 4\sqrt{5} = 400\sqrt{5} \doteq 400(2.236) \doteq 894.4$ sq cm, *Ans.*

6. $c^2 = .5^2 + .8^2 = .25 + .64 = .89$; $c = \sqrt{.89} = \sqrt{\dfrac{89}{100}} \doteq \dfrac{9.434}{10} \doteq .9$ M;
$r \doteq \frac{1}{2}(.5 + .8 - .9) = \frac{1}{2}(.4) = .2$ M, *Ans.*

C **7.** $(4.6)^2 = s^2 + s^2$; $2s^2 = 21.16$;
$s^2 = 10.58$; $s = \sqrt{10.58} = \sqrt{\dfrac{1058}{100}} = \dfrac{\sqrt{1058}}{10} \doteq \dfrac{32.5}{10} \doteq 3.3$, *Ans.*
To find $\sqrt{1058}$:

Let $a^2 \doteq 1058$
$a = 33$
$1058 \div 33 \doteq 32$
$2\overline{)65}$
$a = 32.5$
$1058 \div 32.5 \doteq 32.55$
$\sqrt{1058} \doteq 32.5$

8. $s^2 = 25$; $s = 5$ in; $d^2 = 5^2 + 5^2 = 25 + 25 = 50$;
$d = \sqrt{50} = \sqrt{25\cdot 2} = 5\sqrt{2} \doteq 5(1.414) = 7.07$; $r \doteq \frac{1}{2}(7.07) \doteq 3.5$ in, *Ans.*

9. Circle: $A = \pi r^2$; $\dfrac{22}{7}r^2 \doteq 77$; $r^2 \doteq \dfrac{49}{2}$; $r \doteq \sqrt{\dfrac{49}{2}} = \sqrt{\dfrac{49}{2}\cdot\dfrac{2}{2}} \doteq \dfrac{7\sqrt{2}}{2}$; $d = 2r \doteq 7\sqrt{2}$ in
Square: $A = s^2$; $s^2 = 25$; $s = 5$; $d^2 = 5^2 + 5^2 = 25 + 25 = 50$; $d = \sqrt{50} = \sqrt{25\cdot 2} = 5\sqrt{2}$ in
Since diameter of circle is greater than diagonal of square, square will fit in circle.

10. For equilateral triangle:

$s^2 = (\frac{1}{2}s)^2 + h^2$ $\qquad A = \frac{1}{2}bh$

$4^2 = 2^2 + h^2$ $\qquad A = \frac{1}{2}(4)(2\sqrt{3}) = 4\sqrt{3} \doteq 4(1.732) \doteq 6.9$ sq in

$h^2 = 16 - 4 = 12$

$h = \sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}$ in

For square:

$4^2 = s^2 + s^2$ $\qquad A = s^2$

$2s^2 = 16$ $\qquad A = (2\sqrt{2})^2 = 4 \cdot 2 = 8$ sq in

$s^2 = 8$

$s = \sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$ in $\qquad 6.9$ sq in < 8 sq in, *Ans.*

11. $2^2 = 1^2 + h^2$; $h^2 = 4 - 1 = 3$; $h = \sqrt{3} \doteq 1.7$, *Ans.*

12. $s^2 = \left(\frac{s}{2}\right)^2 + 6^2$; $s^2 = \frac{s^2}{4} + 36$; $4s^2 = s^2 + 144$; $3s^2 = 144$; $s^2 = 48$;

$s = \sqrt{48} = \sqrt{16 \cdot 3} = 4\sqrt{3} \doteq 4(1.732) \doteq 6.9$, *Ans.*

Pages 418–419 · **WRITTEN EXERCISES**

A **1.** $6\sqrt{5}$ **2.** $-9\sqrt{11}$ **3.** $6\sqrt{2} + 4\sqrt{3}$ **4.** $3\sqrt{2} - 2\sqrt{5}$ **5.** $\sqrt{2} - \sqrt{4 \cdot 2} = \sqrt{2} - 2\sqrt{2} = -\sqrt{2}$

6. $\sqrt{4 \cdot 3} + \sqrt{3} = 2\sqrt{3} + \sqrt{3} = 3\sqrt{3}$ **7.** $\sqrt{2} - \sqrt{\frac{1}{2} \cdot \frac{2}{2}} = \sqrt{2} - \frac{\sqrt{2}}{2} = \frac{1}{2}\sqrt{2}$

8. $\sqrt{6} + \sqrt{\frac{2}{3} \cdot \frac{3}{3}} = \sqrt{6} + \frac{\sqrt{6}}{3} = \frac{4}{3}\sqrt{6}$

9. $2\sqrt{16 \cdot 2} - 3\sqrt{25 \cdot 2} = 8\sqrt{2} - 15\sqrt{2} = -7\sqrt{2}$

10. $2\sqrt{25 \cdot 2} - 3\sqrt{9 \cdot 2} = 10\sqrt{2} - 9\sqrt{2} = \sqrt{2}$

11. $3\sqrt{9 \cdot 7} + \frac{1}{4}\sqrt{4 \cdot 7} = 9\sqrt{7} + \frac{1}{4} \cdot 2\sqrt{7} = 9\sqrt{7} + \frac{1}{2}\sqrt{7} = \frac{18}{2}\sqrt{7} + \frac{1}{2}\sqrt{7} = \frac{19\sqrt{7}}{2}$

12. $2\sqrt{25 \cdot 6} - \frac{3}{8}\sqrt{16 \cdot 6} = 10\sqrt{6} - \frac{3}{8} \cdot 4\sqrt{6} = 10\sqrt{6} - \frac{3}{2}\sqrt{6} = \frac{20}{2}\sqrt{6} - \frac{3}{2}\sqrt{6} = \frac{17\sqrt{6}}{2}$

13. $4\sqrt{\frac{5}{2} \cdot \frac{2}{2}} - 10\sqrt{\frac{1}{10} \cdot \frac{10}{10}} = \frac{4\sqrt{10}}{2} - \frac{10\sqrt{10}}{10} = 2\sqrt{10} - \sqrt{10} = \sqrt{10}$

14. $10\sqrt{\frac{3}{5} \cdot \frac{5}{5}} + 30\sqrt{\frac{5}{3} \cdot \frac{3}{3}} = \frac{10\sqrt{15}}{5} + \frac{30\sqrt{15}}{3} = 2\sqrt{15} + 10\sqrt{15} = 12\sqrt{15}$

15. $\sqrt{3} - 4\sqrt{16 \cdot 3} + 3\sqrt{25 \cdot 3} = \sqrt{3} - 16\sqrt{3} + 15\sqrt{3} = 0$

16. $\sqrt{7} + 4\sqrt{4 \cdot 7} - 3\sqrt{9 \cdot 7} = \sqrt{7} + 8\sqrt{7} - 9\sqrt{7} = 0$

B **17.** $\sqrt{3} + 2\sqrt{9 \cdot 3} - 6\sqrt{\frac{1}{3} \cdot \frac{3}{3}} = \sqrt{3} + 6\sqrt{3} - \frac{6}{3}\sqrt{3} = \sqrt{3} + 6\sqrt{3} - 2\sqrt{3} = 5\sqrt{3}$

18. $3\sqrt{10} - 4\sqrt{9 \cdot 10} + 5\sqrt{\frac{1}{10} \cdot \frac{10}{10}} = 3\sqrt{10} - 12\sqrt{10} + \frac{5}{10}\sqrt{10}$

$= 3\sqrt{10} - 12\sqrt{10} + \frac{1}{2}\sqrt{10} = \frac{-17\sqrt{10}}{2}$

19. $12\sqrt{\frac{2}{3} \cdot \frac{3}{3}} - 2\sqrt{\frac{3}{2} \cdot \frac{2}{2}} + \sqrt{9 \cdot 6} = \frac{12}{3}\sqrt{6} - \frac{2}{2}\sqrt{6} + 3\sqrt{6} = 4\sqrt{6} - \sqrt{6} + 3\sqrt{6} = 6\sqrt{6}$

20. $10\sqrt{\frac{2}{5} \cdot \frac{5}{5}} + \sqrt{\frac{5}{2} \cdot \frac{2}{2}} - \frac{1}{4}\sqrt{4 \cdot 10} = \frac{10}{5}\sqrt{10} + \frac{1}{2}\sqrt{10} - \frac{2}{4}\sqrt{10} = 2\sqrt{10}$

21. $\frac{1}{2}\sqrt{3} + 2\sqrt{\frac{1}{3} \cdot \frac{3}{3}} - \frac{1}{6}\sqrt{25 \cdot 3} = \frac{1}{2}\sqrt{3} + \frac{2}{3}\sqrt{3} - \frac{5}{6}\sqrt{3} = \frac{3\sqrt{3}}{6} + \frac{4\sqrt{3}}{6} - \frac{5\sqrt{3}}{6} = \frac{2\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$

22. $6\sqrt{\frac{5}{4 \cdot 3} \cdot \frac{3}{3}} + \sqrt{\frac{4 \cdot 3}{5} \cdot \frac{5}{5}} - \frac{1}{5}\sqrt{4 \cdot 15} = \frac{6}{2 \cdot 3}\sqrt{15} + \frac{2}{5}\sqrt{15} - \frac{2}{5}\sqrt{15} = \sqrt{15}$

C **23.** $2\sqrt{4 \cdot 3x} - \frac{2x^2}{3}\sqrt{\frac{36 \cdot 3}{x} \cdot \frac{x}{x}} = 4\sqrt{3x} - \frac{12x^2}{3x}\sqrt{3x} = 4\sqrt{3x} - 4x\sqrt{3x} = (4 - 4x)\sqrt{3x}$ *or* $4(1 - x)\sqrt{3x}$

24. $a^5\sqrt{\frac{c}{a^2 \cdot a} \cdot \frac{a}{a}} + 2\sqrt{a^6 \cdot ca} = \frac{a^5}{a^2}\sqrt{ca} + 2a^3\sqrt{ca} = a^3\sqrt{ca} + 2a^3\sqrt{ca} = 3a^3\sqrt{ac}$

25. $\sqrt{\frac{36n^2 + 64n^2}{36 \cdot 64}} = \sqrt{\frac{100n^2}{36 \cdot 64}} = \frac{10n}{6 \cdot 8} = \frac{5n}{24}$ **26.** $\sqrt{\frac{25t^2 - 9t^2}{9 \cdot 25}} = \sqrt{\frac{16t^2}{9 \cdot 25}} = \frac{4t}{3 \cdot 5} = \frac{4t}{15}$

27. $a\sqrt{2} - \sqrt{6} = \sqrt{4 \cdot 6} - a\sqrt{4 \cdot 2}$; $a\sqrt{2} - \sqrt{6} = 2\sqrt{6} - 2a\sqrt{2}$; $3a\sqrt{2} = 3\sqrt{6}$;

$a = \frac{3\sqrt{6}}{3\sqrt{2}} = \sqrt{\frac{6}{2}} = \sqrt{3}$. $\{\sqrt{3}\}$

28. $\sqrt{9 \cdot 10} - b\sqrt{9 \cdot 5} = b\sqrt{16 \cdot 5} - \sqrt{121 \cdot 10}$; $3\sqrt{10} - 3b\sqrt{5} = 4b\sqrt{5} - 11\sqrt{10}$; $14\sqrt{10} = 7b\sqrt{5}$; $b = \dfrac{14\sqrt{10}}{7\sqrt{5}} = 2\sqrt{\dfrac{10}{5}} = 2\sqrt{2}$. $\{2\sqrt{2}\}$

29. $4\sqrt{x} + 2\sqrt{x} = 6$; $6\sqrt{x} = 6$; $\sqrt{x} = 1$; $x = 1$. $\{1\}$

30. $2 + 2\sqrt{y} = 3\sqrt{y}$; $2 = \sqrt{y}$; $y = 4$. $\{4\}$

Page 420 · **WRITTEN EXERCISES**

A **1.** $2^2 - (\sqrt{3})^2 = 4 - 3 = 1$ **2.** $4^2 - (\sqrt{5})^2 = 16 - 5 = 11$

3. $(\sqrt{2})^2 - (\sqrt{3})^2 = 2 - 3 = -1$ **4.** $(\sqrt{7})^2 - (\sqrt{6})^2 = 7 - 6 = 1$

5. $(2\sqrt{3})^2 + 6\sqrt{3} - 10\sqrt{3} - 15 = 12 - 4\sqrt{3} - 15 = -3 - 4\sqrt{3}$

6. $(5\sqrt{2})^2 + 5\sqrt{2} - 20\sqrt{2} - 4 = 50 - 15\sqrt{2} - 4 = 46 - 15\sqrt{2}$

7. $1 + 2\sqrt{7} + 7 = 8 + 2\sqrt{7}$

8. $25 - 2(5\sqrt{10}) + (\sqrt{10})^2 = 25 - 10\sqrt{10} + 10 = 35 - 10\sqrt{10}$

9. $(5\sqrt{2})^2 - 2(5\sqrt{2}) + 1 = 50 - 10\sqrt{2} + 1 = 51 - 10\sqrt{2}$

10. $(3\sqrt{7})^2 + 2(6\sqrt{7}) + 4 = 63 + 12\sqrt{7} + 4 = 67 + 12\sqrt{7}$

11. $6\sqrt{12} + 2\sqrt{18} = 6\sqrt{4 \cdot 3} + 2\sqrt{9 \cdot 2} = 12\sqrt{3} + 6\sqrt{2}$

12. $10\sqrt{50} - 5\sqrt{150} = 10\sqrt{25 \cdot 2} - 5\sqrt{25 \cdot 6} = 50\sqrt{2} - 25\sqrt{6}$

B **13.** $8(\sqrt{3})^2 - 12\sqrt{3} + 2\sqrt{3} - 3 = 24 - 10\sqrt{3} - 3 = 21 - 10\sqrt{3}$

14. $5(\sqrt{7})^2 + 10\sqrt{7} - 2\sqrt{7} - 4 = 35 + 8\sqrt{7} - 4 = 31 + 8\sqrt{7}$

15. $2(\sqrt{6})^2 + 6\sqrt{18} - \sqrt{18} - 3(\sqrt{3})^2 = 12 + 5\sqrt{18} - 9 = 3 + 5\sqrt{9 \cdot 2} = 3 + 15\sqrt{2}$

16. $12(\sqrt{15})^2 - 18\sqrt{75} + 2\sqrt{75} - 3(\sqrt{5})^2 = 180 - 16\sqrt{75} - 15 = 165 - 16\sqrt{25 \cdot 3} = 165 - 80\sqrt{3}$

17. $\dfrac{1(\sqrt{5}+1)}{(\sqrt{5}-1)(\sqrt{5}+1)} = \dfrac{\sqrt{5}+1}{(\sqrt{5})^2 - 1^2} = \dfrac{\sqrt{5}+1}{5-1} = \dfrac{\sqrt{5}+1}{4}$

18. $\dfrac{1(\sqrt{7}-1)}{(\sqrt{7}+1)(\sqrt{7}-1)} = \dfrac{\sqrt{7}-1}{(\sqrt{7})^2 - 1^2} = \dfrac{\sqrt{7}-1}{7-1} = \dfrac{\sqrt{7}-1}{6}$

19. $\dfrac{\sqrt{2}(\sqrt{2}-3)}{(\sqrt{2}+3)(\sqrt{2}-3)} = \dfrac{(\sqrt{2})^2 - 3\sqrt{2}}{(\sqrt{2})^2 - 3^2} = \dfrac{2 - 3\sqrt{2}}{2-9} = \dfrac{2-3\sqrt{2}}{-7}$ *or* $\dfrac{3\sqrt{2}-2}{7}$

20. $\dfrac{\sqrt{6}(5+\sqrt{6})}{(5-\sqrt{6})(5+\sqrt{6})} = \dfrac{5\sqrt{6} + (\sqrt{6})^2}{5^2 - (\sqrt{6})^2} = \dfrac{5\sqrt{6}+6}{25-6} = \dfrac{5\sqrt{6}+6}{19}$

21. $\dfrac{(3-\sqrt{5})(2+\sqrt{5})}{(2-\sqrt{5})(2+\sqrt{5})} = \dfrac{6 + 3\sqrt{5} - 2\sqrt{5} - (\sqrt{5})^2}{2^2 - (\sqrt{5})^2} = \dfrac{6+\sqrt{5}-5}{4-5} = \dfrac{1+\sqrt{5}}{-1} = -1 - \sqrt{5}$

22. $\dfrac{(\sqrt{6}-1)(3-\sqrt{6})}{(3+\sqrt{6})(3-\sqrt{6})} = \dfrac{3\sqrt{6} - (\sqrt{6})^2 - 3 + \sqrt{6}}{3^2 - (\sqrt{6})^2} = \dfrac{3\sqrt{6} - 6 - 3 + \sqrt{6}}{9-6} = \dfrac{4\sqrt{6}-9}{3}$

23. $\dfrac{2(3\sqrt{2}+2)}{(3\sqrt{2}-2)(3\sqrt{2}+2)} = \dfrac{6\sqrt{2}+4}{(3\sqrt{2})^2 - 2^2} = \dfrac{6\sqrt{2}+4}{18-4} = \dfrac{6\sqrt{2}+4}{14} = \dfrac{2(3\sqrt{2}+2)}{2 \cdot 7} = \dfrac{3\sqrt{2}+2}{7}$

24. $\dfrac{6(3-2\sqrt{3})}{(3+2\sqrt{3})(3-2\sqrt{3})} = \dfrac{18 - 12\sqrt{3}}{3^2 - (2\sqrt{3})^2} = \dfrac{18-12\sqrt{3}}{9-12} = \dfrac{18-12\sqrt{3}}{-3} = \dfrac{6(3-2\sqrt{3})}{3(-1)} = \dfrac{2(3-2\sqrt{3})}{-1} = -2(3-2\sqrt{3}) = 4\sqrt{3} - 6$

Page 422 · **WRITTEN EXERCISES**

A **1.** $2y = 2^2 = 4$; $y = 2$. $\{2\}$ **2.** $3x = (\frac{1}{3})^2 = \frac{1}{9}$; $x = \dfrac{1}{9 \cdot 3} = \dfrac{1}{27}$. $\{\frac{1}{27}\}$

3. $5x = (\frac{5}{2})^2 = \frac{25}{4}$; $x = \dfrac{25}{4 \cdot 5} = \dfrac{5}{4}$. $\{\frac{5}{4}\}$ **4.** $7z = (\frac{7}{4})^2 = \frac{49}{16}$; $z = \dfrac{49}{16 \cdot 7} = \dfrac{7}{16}$. $\{\frac{7}{16}\}$

5. $\sqrt{s} = 2$; $s = 4$. $\{4\}$ **6.** $\sqrt{t} = 2\frac{1}{2} = \frac{5}{2}$; $t = \frac{25}{4}$. $\{\frac{25}{4}\}$

7. $\frac{r}{2} = 1; r = 2.$ $\{2\}$

8. $\frac{v}{3} = 4; v = 12.$ $\{12\}$

9. $x + 10 = 9; x = -1.$ $\{-1\}$

10. $x - 4 = 81; x = 85.$ $\{85\}$

11. $2y - 3 = 9; 2y = 12; y = 6.$ $\{6\}$

12. $3y + 4 = 1; 3y = -3; y = -1.$ $\{-1\}$

13. $\sqrt{5m} = 5; 5m = 25; m = 5.$ $\{5\}$

14. $\sqrt{10m} = 5; 10m = 25; m = \frac{5}{2}.$ $\{\frac{5}{2}\}$

15. $\sqrt{p} = -2; p = 4$
Check: $\sqrt{4} + 5 \stackrel{?}{=} 3$
$2 + 5 \neq 3.$ $\emptyset$

16. $\sqrt{p} = -1; p = 1$
Check: $\sqrt{1} + 2 \stackrel{?}{=} 1$
$1 + 2 \neq 1.$ $\emptyset$

17. $\sqrt{4y - 3} = 3; 4y - 3 = 9; 4y = 12; y = 3.$ $\{3\}$

18. $\sqrt{5y - 1} = 7; 5y - 1 = 49; 5y = 50; y = 10.$ $\{10\}$

19. $\sqrt{\frac{9x}{2}} = 9; \frac{9x}{2} = 81; \frac{x}{2} = 9; x = 18.$ $\{18\}$

20. $\sqrt{\frac{4x}{3}} = 4; \frac{4x}{3} = 16; \frac{x}{3} = 4; x = 12.$ $\{12\}$

B **21.** $\frac{3x - 5}{4} = 4; 3x - 5 = 16; 3x = 21; x = 7.$ $\{7\}$

22. $\frac{5y + 1}{6} = 1; 5y + 1 = 6; 5y = 5; y = 1.$ $\{1\}$

23. $(\sqrt{x})^2 = (2\sqrt{5})^2; x = 4 \cdot 5 = 20.$ $\{20\}$

24. $(3\sqrt{k})^2 = (4\sqrt{3})^2; 9k = 16 \cdot 3; k = \frac{16 \cdot 3}{9} = \frac{16}{3}.$ $\{\frac{16}{3}\}$

25. $\sqrt{5t^2 + 5} = 5; 5t^2 + 5 = 25; 5t^2 = 20; t^2 = 4; t = \pm 2.$ $\{2, -2\}$

26. $\sqrt{3s^2 - 12} = 6; 3s^2 - 12 = 36; 3s^2 = 48; s^2 = 16; s = \pm 4.$ $\{4, -4\}$

27. $v^2 = \frac{2E}{m}; 2E = mv^2; E = \frac{mv^2}{2}.$ $\left\{\frac{mv^2}{2}\right\}$

28. $r^2 = \frac{7v}{22h}; 22hr^2 = 7v; h = \frac{7v}{22r^2}.$ $\left\{\frac{7v}{22r^2}\right\}$

29. $a^2 + 9 = a^2 + 6a + 9; 6a = 0; a = 0.$ $\{0\}$

30. $b^2 - 16 = b^2 - 8b + 16; 8b = 32; b = 4.$ $\{4\}$

31. $n = n^2; n^2 - n = 0; n(n - 1) = 0; n = 0$ or $n - 1 = 0; n = 0$ or $n = 1;$ but since $\sqrt{n}$ means the principal (positive) root, $\sqrt{1} \neq -1; \therefore \{0\}$

32. $n = \frac{n^2}{4}; 4n = n^2; n^2 - 4n = 0; n(n - 4) = 0; n = 0$ or $n - 4 = 0; n = 0$ or $n = 4;$ but since $\sqrt{n}$ means the principal root, $\sqrt{4} \neq \frac{-4}{2}; \therefore \{0\}$

C **33.** $\sqrt{x^2 + 4} = x + 1; x^2 + 4 = x^2 + 2x + 1; 2x = 3; x = \frac{3}{2}.$ $\{\frac{3}{2}\}$

34. $\sqrt{x^2 - 4} = x + 2; x^2 - 4 = x^2 + 4x + 4; 4x = -8; x = -2.$ $\{-2\}$

35. $2k - 4 = k^2 - 4k + 4; k^2 - 6k + 8 = 0; (k - 4)(k - 2) = 0; k - 4 = 0$ or $k - 2 = 0;$ $k = 4$ or $k = 2.$ $\{4, 2\}$

36. $3r - 9 = r^2 - 6r + 9; r^2 - 9r + 18 = 0; (r - 6)(r - 3) = 0; r - 6 = 0$ or $r - 3 = 0;$ $r = 6$ or $r = 3.$ $\{6, 3\}$

37. $19 - y = y^2 - 14y + 49; y^2 - 13y + 30 = 0; (y - 10)(y - 3) = 0; y - 10 = 0$ or $y - 3 = 0;$ $y = 10$ or $y = 3;$ Check: $\sqrt{19 - 10} \stackrel{?}{=} 10 - 7$ $\sqrt{19 - 3} \stackrel{?}{=} 3 - 7$
$3 = 3$ $4 \neq -4.$ $\{10\}$

38. $y + 3 = y^2 - 18y + 81; y^2 - 19y + 78 = 0; (y - 13)(y - 6) = 0; y - 13 = 0$ or $y - 6 = 0;$ $y = 13$ or $y = 6;$ Check: $\sqrt{13 + 3} \stackrel{?}{=} 13 - 9$ $\sqrt{6 + 3} \stackrel{?}{=} 6 - 9$
$4 = 4$ $3 \neq -3.$ $\{13\}$

39. $\sqrt{x + 2} = -3; x + 2 = 9; x = 7;$ but since $\sqrt{x + 2}$ means the principal root $\sqrt{7 + 2} + 3 \neq; \therefore \emptyset$

40. $\sqrt{x+4} = -1; x+4 = 1; x = -3;$ but since $\sqrt{x+4}$ means the principal root, $1 + \sqrt{-3+4} \neq 0;$ $\therefore \emptyset$

Pages 422–423 · **PROBLEMS**

A **1.** $3\sqrt{n} = 30; 9n = 900; n = 100,$ *Ans.*

2. $\frac{1}{3}\sqrt{n} = 5; \sqrt{n} = 15; n = 225,$ *Ans.*

3. $\sqrt{2n - 3} = 3; 2n - 3 = 9; 2n = 12; n = 6,$ *Ans.*

4. $\sqrt{5n + 9} = 8; 5n + 9 = 64; 5n = 55; n = 11,$ *Ans.*

5. $p^2 = (4\sqrt{A})^2; p^2 = 16A; A = \frac{p^2}{16}; A = \frac{32 \cdot 32}{16} = 2 \cdot 32 = 64$ sq in, *Ans.*

6. $d^2 = \frac{3h}{2}$; $3h = 2d^2$; $h = \frac{2d^2}{3}$; $h = \frac{2 \cdot 9 \cdot 9}{3} = 2 \cdot 9 \cdot 3 = 54$ ft, *Ans.*

7. $c^2 = a^2 + b^2$; $a^2 = c^2 - b^2$; $a = \sqrt{c^2 - b^2}$; $a = \sqrt{17^2 - 15^2} = \sqrt{289 - 225} = \sqrt{64} = 8$, *Ans.*

8. $4s = \sqrt{f}$; $f = 16s^2$; $f = 16 \cdot 6^2 = 16 \cdot 36 = 576$ ft, *Ans.*

B **9.** $3 = \sqrt{\frac{H}{1.6(6)}}$; $9 = \frac{H}{9.6}$; $H = 86.4$ horsepower, *Ans.*

10. $60 = \sqrt{\frac{h}{.0336}}$; $3600 = \frac{h}{.0336}$; $h = 120.96 = 121$ ft, *Ans.*

11. $11{,}988 = 333\sqrt{14.0037t}$; $\sqrt{14.0037t} = 36$; $14.0037t = 1296$; $t \doteq 92.5°$, *Ans.*

12. $1.21 = \left(1 + \frac{r}{100}\right)^2$; $\sqrt{1.21} = 1 + \frac{r}{100}$; $1.1 = 1 + \frac{r}{100}$; $\frac{r}{100} = .1$; $r = 10\%$, *Ans.*

13. Let x and $x + 1$ be a pair of consecutive positive integers; $\sqrt{x(x+1)} = 6\sqrt{2}$; $x(x+1) = 72$; $x^2 + x = 72$; $x^2 + x - 72 = 0$; $(x + 9)(x - 8) = 0$; $x + 9 = 0$ or $x - 8 = 0$; $x = -9$ (reject); $x = 8$, $x + 1 = 9$, *Ans.*

14. $i = \frac{6}{10 \cdot 10} = \frac{6}{100} = \frac{3}{50}$; $\frac{3}{50} = \frac{54}{f^2}$; $3f^2 = 2700$; $f^2 = 900$; $f = -30$ (reject); $f = 30$ ft, *Ans.*

Pages 426–427 · CHAPTER TEST

1. $8(16)$? $7(15)$; $128 > 105$; $\frac{8}{15} > \frac{7}{16}$
$8(17)$? $9(15)$; $136 > 135$; $\frac{8}{15} > \frac{9}{17}$
$7(17)$? $9(16)$; $119 < 144$; $\frac{7}{16} < \frac{9}{17}$. $\frac{8}{15}, \frac{9}{17}, \frac{7}{16}$

2. $\frac{117}{72} = 1\frac{5}{8}$; $\frac{79}{48} = 1\frac{31}{48}$; $\frac{470}{288} = 1\frac{91}{144}$;
$5(48)$? $31(8)$; $240 < 248$; $\frac{5}{8} < \frac{31}{48}$
$5(144)$? $91(8)$; $720 < 728$; $\frac{5}{8} < \frac{91}{144}$
$31(144)$? $91(48)$; $4464 > 4368$; $\frac{31}{48} > \frac{91}{144}$;
$\frac{31}{48}, \frac{91}{144}, \frac{5}{8}$. $\frac{79}{48}, \frac{470}{288}, \frac{117}{72}$

3. $\frac{15}{14} - 1 = \frac{15}{14} - \frac{14}{14} = \frac{1}{14}$; $1 + \frac{1}{2}(\frac{1}{14}) = \frac{28}{28} + \frac{1}{28} = \frac{29}{28}$

4. $\frac{10}{9} - (-\frac{34}{7}) = \frac{70}{63} + \frac{306}{63} = \frac{376}{63}$; $-\frac{34}{7} + \frac{1}{2}(\frac{376}{63}) = -\frac{306}{63} + \frac{188}{63} = -\frac{118}{63}$

5. True; for any rational number >0 it is possible to find another rational number between it and 0.

6. False; integers do not possess the property of density.

7. $.\overline{714285}$

8. $-.8\overline{6}$

9. 12.375

10.
$100N = 72.72\overline{72}$
$N = \quad .72\overline{72}$
$99N = 72.0000$
$N = \frac{72}{99} = \frac{8}{11}$

11.
$10N = 8.\overline{8}$
$N = \quad .\overline{8}$
$9N = 8.0$
$N = \frac{8}{9}$

12.
$1000N = 2345.\overline{345}$
$N = \quad 2.\overline{345}$
$999N = 2343.000$
$N = \frac{2343}{999} = \frac{781}{333}$

13. $\sqrt{9 \cdot 100} = \sqrt{9} \cdot \sqrt{100} = 3 \cdot 10 = 30$

14. $-\frac{\sqrt{1}}{\sqrt{576}} = -\frac{1}{24}$

15. $x = \pm\sqrt{400} = \pm 20$. $\{20, -20\}$

16. $3y^2 = 363$; $y^2 = 121$; $y = \pm\sqrt{121} = \pm 11$. $\{11, -11\}$

17. $\sqrt{4.72} = \sqrt{\frac{472}{100}} = \frac{\sqrt{4 \cdot 118}}{10} = \frac{2\sqrt{59 \cdot 2}}{10} \doteq \frac{(7.681)(1.414)}{5} \doteq 2.17$

18. $-\sqrt{8.2314} = -\sqrt{\frac{82314}{10{,}000}} = -\frac{\sqrt{9 \cdot 9146}}{100} = -\frac{3\sqrt{9146}}{100} \doteq -\frac{3(95.63)}{100} = -\frac{286.89}{100} \doteq -2.87$

To find $\sqrt{9146}$:
Let $a^2 \doteq 9146$
$a = 96$
$9146 \div 96 \doteq 95$
$2)191$
$a = 95.5$
$9146 \div 95.5 \doteq 95.76$
$2)191.26$
$a = 95.63$
$9146 \div 95.63 \doteq 95.639$
$\sqrt{9146} \doteq 95.63$

19. $c^2 = a^2 + b^2$; $25^2 = 20^2 + b^2$;
$b^2 = 625 - 400 = 225$; $b = \sqrt{225} = 15$, *Ans.*

20. $c^2 = 16^2 + 12^2$; $c^2 = 256 + 144 = 400$;
$c = \sqrt{400} = 20$, *Ans.*

21. $(4 \cdot 2)\sqrt{13^2} = 8 \cdot 13 = 104$

22. $\frac{2}{3}\sqrt{\frac{49}{40} \cdot \frac{10}{10}} = \frac{2}{3} \cdot \frac{7}{20}\sqrt{10} = \frac{7}{30}\sqrt{10}$

23. $(3 \cdot 2)\sqrt{7 \cdot 12 \cdot 21} = 6\sqrt{7^2 \cdot 2^2 \cdot 3^2}$
$= 6 \cdot 7 \cdot 2 \cdot 3 = 252$

24. $2\sqrt{\frac{a^3b^7}{8a^5b^5}} = 2\sqrt{\frac{b^2}{8a^2} \cdot \frac{2}{2}} = \frac{2b}{4a}\sqrt{2} = \frac{b\sqrt{2}}{2a}$

25. $4\sqrt{5}$

26. $10\sqrt{\frac{1}{25\cdot 2}} - 6\sqrt{\frac{1}{2}} = \frac{10}{5}\sqrt{\frac{1}{2}} - 6\sqrt{\frac{1}{2}} = -4\sqrt{\frac{1}{2}\cdot\frac{2}{2}} = -2\sqrt{2}$

27. $3\sqrt{3^2\cdot 3} - \sqrt{\frac{1}{3}\cdot\frac{3}{3}} = 9\sqrt{3} - \frac{\sqrt{3}}{3} = \frac{26}{3}\sqrt{3}$

28. $\frac{1}{2}\sqrt{\frac{3}{8}\cdot\frac{2}{2}} + \frac{3}{4}\sqrt{\frac{1}{6}\cdot\frac{6}{6}} = \frac{1}{2}\cdot\frac{1}{4}\sqrt{6} + \frac{3}{4}\cdot\frac{1}{6}\sqrt{6} = \frac{1}{8}\sqrt{6} + \frac{1}{8}\sqrt{6} = \frac{1}{4}\sqrt{6}$

29. $3^2 - (\sqrt{2})^2 = 9 - 2 = 7$

30. $\frac{1(3\sqrt{3}-2)}{(3\sqrt{3}+2)(3\sqrt{3}-2)} = \frac{3\sqrt{3}-2}{(3\sqrt{3})^2 - 2^2} = \frac{3\sqrt{3}-2}{27-4} = \frac{3\sqrt{3}-2}{23}$

31. $4^2 + 2(4\sqrt{5}) + (\sqrt{5})^2 = 16 + 8\sqrt{5} + 5 = 21 + 8\sqrt{5}$

32. $\frac{(\sqrt{6}+1)(3+\sqrt{6})}{(3-\sqrt{6})(3+\sqrt{6})} = \frac{3\sqrt{6}+6+3+\sqrt{6}}{3^2-(\sqrt{6})^2} = \frac{9+4\sqrt{6}}{9-6} = \frac{9+4\sqrt{6}}{3}$

33. $(\sqrt{3y})^2 = (-12y)^2$; $3y = 144y^2$; $y = 48y^2$; $48y^2 - y = 0$; $y(48y-1) = 0$; $y = 0$ or $48y - 1 = 0$; $y = \frac{1}{48}$ (reject). $\{0\}$

34. $(\sqrt{3z})^2 = (z+2)^2$; $3z = z^2 + 4z + 4$; $z^2 + z + 4 = 0$. $\emptyset$

Pages 427–429 · CHAPTER REVIEW

1. addition, multiplication

2. $\frac{37}{40}$, since $\frac{11}{12} = \frac{110}{120}$ and $\frac{37}{40} = \frac{111}{120}$

3. $\frac{29}{48}$; $\frac{5}{8} - \frac{7}{12} = \frac{1}{24}$; $\frac{7}{12} + \frac{1}{2}(\frac{1}{24}) = \frac{29}{48}$.

4. Infinite

5. True

6. False; between any two different rational numbers, there is an infinite number of others.

7. $2\frac{5}{16} = 2.3125$

8. $-\frac{11}{12} = -.91\overline{6}$

9. $\frac{35}{32} = 1.09375$

10.
$100N = 63.63\overline{63}$
$N = \quad .63\overline{63}$
$99N = 63$
$N = \frac{63}{99} = \frac{7}{11}$

11.
$10N = 14.6\overline{6}$
$N = \quad 1.4\overline{6}$
$9N = 13.2$
$N = \frac{13.2}{9} = \frac{132}{90} = \frac{22}{15}$

12.
$1000N = 297.\overline{297}$
$N = \quad .\overline{297}$
$999N = 297$
$N = \frac{297}{999} = \frac{11}{37}$

13. 8.4 **14.** 1.1 **15.** .7 **16.** 5.0 **17.** 5, radicand **18.** Two **19.** 3; −3

20. $\sqrt{1024} = \sqrt{2^{10}} = 2^5 = 32$ **21.** $\pm\sqrt{\frac{676}{25}} = \pm\frac{26}{5}$ **22.** $-\sqrt{\frac{1}{361}} = -\frac{1}{19}$

23. $n^2 = 64$; $n = \pm\sqrt{64} = \pm 8$. $\{8, -8\}$

24. $s^2 = \frac{1}{4}$; $s = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$. $\{\frac{1}{2}, -\frac{1}{2}\}$

25. $r - 5 = \pm\sqrt{25}$; $r - 5 = 5$ or $r - 5 = -5$; $r = 10$ or $r = 0$. $\{10, 0\}$

26. y, v **27.** real, rational, irrational **28.** terminating, repeating

29. $\sqrt{35}$

Let a be a no. such that
$a^2 \doteq 35$
$a = 6$
$35 \div 6 \doteq 5.8$
$2)\underline{11.8}$
$a = 5.9$
$35 \div 5.9 \doteq 5.932$
$2)\underline{11.832}$
$a = 5.916$
$35 \div 5.916 \doteq 5.916$
$a = 5.92$ to the nearest hundredth, *Ans.*

30. $\sqrt{9.641}$

Let a be a no. such that
$a^2 \doteq 9.641$
$a = 3.1$
$9.641 \div 3.1 \doteq 3.11$
$2)\underline{6.21}$
$a = 3.105$
$9.641 \div 3.105 \doteq 3.1049$
$2)\underline{6.2099}$
$a = 3.10495$
$a = 3.10$ to the nearest hundredth, *Ans.*

31. $-\sqrt{.6803}$

Let a be a no. such that
$a^2 \doteq .6803$
$a = .82$
$.6803 \div .82 \doteq .8296$
$2)\underline{1.6496}$
$a = .8248$
$.6803 \div .8248 \doteq .8248$
$-\sqrt{.6803} \doteq -.82$, *Ans.*

32. Let s = the no. of feet in the side of the square;
$s^2 = 250$; $s = \sqrt{250}$.
Let a be a number such that
$a^2 \doteq 250$
$a = 15$
$250 \div 15 \doteq 16.67$
$2)\underline{31.67}$
$a = 15.84$
$250 \div 15.84 \doteq 15.782$
$2)\underline{31.622}$
$a = 15.811$
$250 \div 15.811 \doteq 15.811$
$\sqrt{250} \doteq 15.81$ ft, *Ans.*

33. hypotenuse

34. $(AB)^2 = (AC)^2 + (BC)^2$; $(AB)^2 = 8^2 + 15^2$; $(AB)^2 = 289$; $AB = 17$ cm, *Ans.*

35. $(41)^2 = (40)^2 + (BC)^2$; $1681 = 1600 + (BC)^2$; $81 = (BC)^2$; $BC = 9$ in, *Ans.*

36. 39, 36, 15 are the sides of a right triangle if $(39)^2 \stackrel{?}{=} (36)^2 + (15)^2$

$$1521 \stackrel{?}{=} 1296 + 225$$
$$1521 = 1521$$

39, 36, 15 are the sides of a right triangle, *Ans.*

37. 18, 30, 24 are the sides of a right triangle if $(30)^2 \stackrel{?}{=} (18)^2 + (24)^2$

$$900 \stackrel{?}{=} 324 + 576$$
$$900 = 900$$

18, 30, 24 are the sides of a right triangle, *Ans.*

38. $2\sqrt{7} \cdot 3\sqrt{7} = 42$

39. $\dfrac{6\sqrt{81}}{2\sqrt{27}} = 3\sqrt{3}$

40. $5\sqrt{12} \div 2\sqrt{3} = 5$

41. $\dfrac{\sqrt{4r^2s^3t^4}}{2\sqrt{9r^6st^6}} = \dfrac{\sqrt{r^2s^3t^4}}{3\sqrt{r^6st^6}}$

$$= \frac{1}{3}\sqrt{\frac{r^2s^3t^4}{r^6st^6}}$$
$$= \frac{1}{3}\sqrt{\frac{s^2}{r^4t^2}}$$
$$= \frac{1}{3}\left(\frac{s}{r^2t}\right)$$
$$= \frac{s}{3r^2t}$$

42. $\sqrt{3}\,(2\sqrt{2} - 3\sqrt{3}\,) = 2\sqrt{6} - 9$

43. $\dfrac{\sqrt{8} - 2\sqrt{28}}{2\sqrt{2}} = \dfrac{2\sqrt{2} - 4\sqrt{7}}{2\sqrt{2}}$

$$= \frac{\sqrt{2} - 2\sqrt{7}}{\sqrt{2}}$$
$$= \frac{(\sqrt{2} - 2\sqrt{7})\sqrt{2}}{\sqrt{2} \cdot \sqrt{2}}$$
$$= \frac{2 - 2\sqrt{14}}{2}$$
$$= 1 - \sqrt{14}$$

44. denominator

45. rationalize, rational

46. $\frac{2}{3}\sqrt{162} = \frac{2}{3}\sqrt{81 \cdot 2}$

$$= \tfrac{2}{3}\sqrt{81} \cdot \sqrt{2}$$
$$= \tfrac{2}{3} \cdot 9\sqrt{2}$$
$$= 6\sqrt{2}$$

47. $-2\sqrt{\frac{4}{5}} = -2\sqrt{\frac{4}{5} \cdot \frac{5}{5}}$

$$= -\tfrac{2}{5}\sqrt{4} \cdot \sqrt{5}$$
$$= -\tfrac{4}{5}\sqrt{5}$$

48. $\dfrac{5}{\sqrt{8}} = \dfrac{5}{\sqrt{8}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}}$

$$= \frac{5\sqrt{2}}{\sqrt{16}}$$
$$= \tfrac{5}{4}\sqrt{2}$$

49. $\dfrac{6}{5\sqrt{12}} = \dfrac{6}{5\sqrt{12}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}$

$$= \frac{6\sqrt{3}}{5\sqrt{36}}$$
$$= \frac{6\sqrt{3}}{5 \cdot 6}$$
$$= \frac{\sqrt{3}}{5}$$

50. $6\sqrt{18} - 2\sqrt{98} + 2\sqrt{9}$

$$= 6 \cdot \sqrt{9} \cdot \sqrt{2} - 2\sqrt{49} \cdot \sqrt{2} + 2 \cdot 3$$
$$= 18\sqrt{2} - 14\sqrt{2} + 6$$
$$= 4\sqrt{2} + 6$$

51. $3\sqrt{6} + 6\sqrt{\frac{1}{6}} - 8\sqrt{1.5}$

$$= 3\sqrt{6} + \sqrt{6} - 4\sqrt{6}$$
$$= 0$$

52. distributive

53. Let x = no. of inches in width;
then $3x$ = no. of inches in length;
$x^2 + (3x)^2 = 39^2$; $x^2 + 9x^2 = 1521$;
$10x^2 = 1521$; $x^2 = 152.1$; $x = \sqrt{152.1}$.
Let a be a number such that

$$a^2 \doteq 152.1$$
$$a = 12$$
$$152.1 \div 12 \doteq \underline{12.68}$$
$$2\overline{)24.68}$$
$$a = 12.34$$
$$152.1 \div 12.34 \doteq 12.33$$

$\sqrt{152.1} \doteq 12.3$ in, *Ans.*

54. $(2\sqrt{5} + 5)^2 = (2\sqrt{5})^2 + 2(2\sqrt{5})(5) + 5^2$

$$= 20 + 20\sqrt{5} + 25$$
$$= 45 + 20\sqrt{5}$$

55. $(3\sqrt{2} - 1)(3\sqrt{2} + 1) = (3\sqrt{2})^2 - 1^2$

$$= 18 - 1$$
$$= 17$$

56. $\dfrac{2}{\sqrt{6}+2} = \dfrac{2}{(\sqrt{6}+2)}\dfrac{(\sqrt{6}-2)}{(\sqrt{6}-2)}$

$= \dfrac{2\sqrt{6}-4}{6-4}$

$= \dfrac{2\sqrt{6}-4}{2}$

$= \sqrt{6}-2$

57. $\dfrac{10}{2\sqrt{3}-\sqrt{2}} = \dfrac{10}{(2\sqrt{3}-\sqrt{2})}\dfrac{(2\sqrt{3}+\sqrt{2})}{(2\sqrt{3}+\sqrt{2})}$

$= \dfrac{10(2\sqrt{3}+\sqrt{2})}{12-2}$

$= \dfrac{10(2\sqrt{3}+\sqrt{2})}{10}$

$= 2\sqrt{3}+\sqrt{2}$

58. equivalent

59. $\sqrt{2x} = 6$; $(\sqrt{2x})^2 = 6^2$; $2x = 36$; $x = 18$. $\{18\}$

60. $2\sqrt{3n} = 9$; $(2\sqrt{3n})^2 = 9^2$; $12n = 81$; $n = \frac{81}{12} = \frac{27}{4}$. $\{\frac{27}{4}\}$

61. $2 + \sqrt{r} = 5$; $\sqrt{r} = 3$; $(\sqrt{r})^2 = 3^2$; $r = 9$. $\{9\}$

62. $\sqrt{2s-1} + 3 = 2$; $\sqrt{2s-1} = -1$; $(\sqrt{2s-1})^2 = (-1)^2$; $2s - 1 = 1$; $2s = 2$; $s = 1$
Check: $\sqrt{2\cdot 1 - 1} + 3 \stackrel{?}{=} 2$; $\sqrt{1} + 3 \stackrel{?}{=} 2$; $1 + 3 \stackrel{?}{=} 2$; $4 \neq 2$. $\emptyset$

63. $\sqrt{2x-2} - x = 1$; $\sqrt{2x-2} = x + 1$; $(\sqrt{2x-2})^2 = (x+1)^2$; $2x - 2 = x^2 + 2x + 1$; $-3 = x^2$; $\sqrt{-3} = x$; $\sqrt{-3}$ is not a real no. $\emptyset$

64. $\sqrt{3y+13} = y + 3$; $(\sqrt{3y+13})^2 = (y+3)^2$; $3y + 13 = y^2 + 6y + 9$; $0 = y^2 + 3y - 4$; $(y+4)(y-1) = 0$; $y + 4 = 0$ or $y - 1 = 0$; $y = -4$ or $y = 1$

Check: $y = -4$: $\sqrt{3(-4)+13} \stackrel{?}{=} -4 + 3$; $\sqrt{1} \stackrel{?}{=} -1$; $\sqrt{1} \neq -1$

$y = 1$: $\sqrt{3(1)+13} \stackrel{?}{=} 1 + 3$; $\sqrt{16} \stackrel{?}{=} 4$; $4 = 4$. $\{1\}$

Pages 429–431 · CUMULATIVE REVIEW

1. If $x \neq 0$,

$x - \dfrac{x^2 - y^2}{x} = \dfrac{x^2 - (x^2 - y^2)}{x}$

$= \dfrac{x^2 - x^2 + y^2}{x}$

$= \dfrac{y^2}{x}$

2. If $x \neq 3$ and $x \neq -3$,

$\dfrac{2x+6}{x^2-9} \cdot \dfrac{x^2 - 6x + 9}{2} \div (x - 3)$

$= \dfrac{2(x+3)}{(x-3)(x+3)} \cdot \dfrac{(x-3)(x-3)}{2} \cdot \dfrac{1}{x-3}$

$= \dfrac{2(x+3)(x-3)(x-3)}{2(x+3)(x-3)(x-3)}$

$= 1$

3. $(a - b)^2 = a^2 - 2ab + b^2$

4. $x^2 - 2x = 3$; $x^2 - 2x - 3 = 0$; $(x-3)(x+1) = 0$; $x - 3 = 0$ or $x + 1 = 0$; $x = 3$, $x = -1$. $\{3, -1\}$

5. x^2z^2 **6.** (10,000 − x) dollars **7.** $r - c$ **8.** subset **9.** multiplicative inverse

10. (a) **11.** (d) **12.** (b) **13.** (c) **14.** (e)

15. $3 = m^2 + 2m$; $m^2 + 2m - 3 = 0$; $(m+3)(m-1) = 0$; $m + 3 = 0$ or $(m - 1) = 0$; $m = -3$, $m = 1$. $\{-3, 1\}$

16. $\dfrac{x+1}{5} + \dfrac{x-1}{2} = 9 - \dfrac{x}{3}$; $30\dfrac{(x+1)}{5} + 30\dfrac{(x-1)}{2} = 30(9) - 30\dfrac{x}{3}$;
$6(x+1) + 15(x-1) = 270 - 10x$; $6x + 6 + 15x - 15 = 270 - 10x$; $31x = 279$; $x = 9$. $\{9\}$

17. $y^2 - 3y = 0$; $y(y-3) = 0$; $y = 0$ or $y - 3 = 0$, $y = 3$. $\{0, 3\}$

18. $9x + 4y = -21$ $\quad 27x + 12y = -63$
$7x - 3y = \quad 2$ $\quad 28x - 12y = \quad 8$
$55x \qquad = -55$; $x = -1$; $9(-1) + 4y = -21$; $-9 + 4y = -21$; $4y = -12$; $y = -3$. $\{(-1, -3)\}$

19. $4 - 2(3r - 1) \leq 12$; $4 - 6r + 2 \leq 12$; $-6r \leq 6$; $r \geq -1$

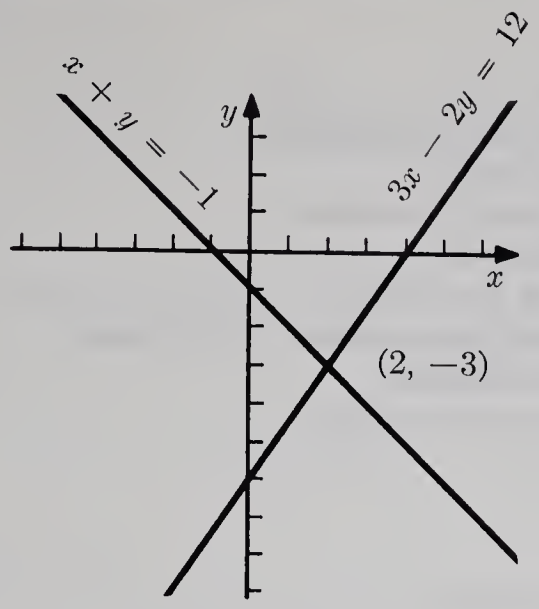
20. $3x - 2y = 12$;
$-2y = -3x + 12$;
$y = \frac{3}{2}x - 6$

$x + y = -1$;
$y = -x - 1$.
$\{(2, -3)\}$

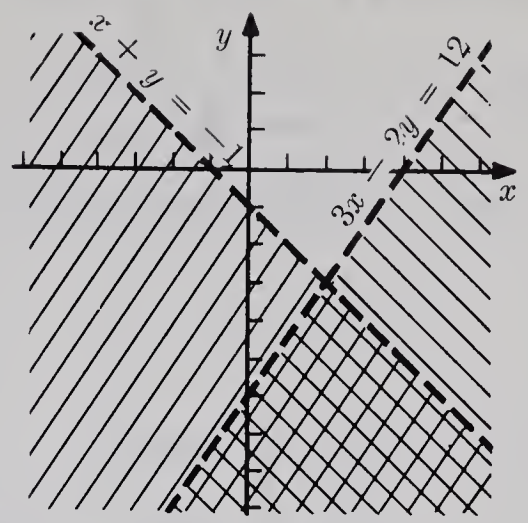
21. $3x - 2y > 12$;
$-2y > -3x + 12$;
$y < \frac{3}{2}x - 6$

$x + y < -1$;
$y < -x - 1$

20.



21.



22. $\frac{2}{t}$ decreases; $\therefore y$ increases

23. $\frac{1}{2} = \frac{3}{6}$ and $\frac{2}{3} = \frac{4}{6}$; then $\frac{4}{6} - \frac{3}{6} = \frac{1}{6}$; $\frac{1}{6} \div 2 = \frac{1}{12}$; $\frac{1}{2} + \frac{1}{12} = \frac{6}{12} + \frac{1}{12} = \frac{7}{12}$.

24. $\frac{3}{25} = .12$

25. $\frac{2}{3}$

26.
$$\begin{aligned}\sqrt{5n} \cdot \sqrt{10n^3} &= \sqrt{50n^4} \\ &= \sqrt{25n^4 \cdot 2} \\ &= \sqrt{25n^4} \cdot \sqrt{2} \\ &= 5n^2\sqrt{2}\end{aligned}$$

27.
$$\begin{aligned}\frac{\sqrt{216}}{2\sqrt{2}} &= \tfrac{1}{2}\sqrt{\tfrac{216}{2}} \\ &= \tfrac{1}{2}\sqrt{108} \\ &= \tfrac{1}{2}\sqrt{36 \cdot 3} \\ &= \tfrac{1}{2} \cdot 6\sqrt{3} \\ &= 3\sqrt{3}\end{aligned}$$

28.
$$\begin{aligned}(\sqrt{2} + 1)(\sqrt{2} - 1) &= (\sqrt{2})^2 - 1^2 \\ &= 2 - 1 \\ &= 1\end{aligned}$$

29. $x + 3\sqrt{x} = 10$; $3\sqrt{x} = 10 - x$; $(3\sqrt{x})^2 = (10 - x)^2$; $9x = 100 - 20x + x^2$; $0 = x^2 - 29x + 100$; $(x - 25)(x - 4) = 0$; $x - 25 = 0$ or $x - 4 = 0$; $x = 25$ or $x = 4$
Check: $25 + 3\sqrt{25} \stackrel{?}{=} 10$; $25 + 15 \stackrel{?}{=} 10$; $40 \neq 10$. $4 + 3\sqrt{4} \stackrel{?}{=} 10$; $4 + 6 \stackrel{?}{=} 10$; $10 = 10$. $\{4\}$

30. Let x = no. of inches in one edge; then x^2 = area of each face;
$6x^2 = 540$; $x^2 = 90$; $x = \sqrt{90}$; $x = 9.5$ in, *Ans.*

31. Let x = measure of third side; $17^2 = x^2 + 8^2$; $289 = x^2 + 64$; $225 = x^2$; $x = 15$, *Ans.*

32. Let x = cost in dollars of 1 ton hard coal and y = cost in dollars of 1 ton soft coal;
$3x + 2y = 112$; $2x + 6y = 168$; $9x + 6y = 336$; $2x + 6y = 168$; $7x = 168$; $x = 24$.
$3(24) + 2y = 112$; $72 + 2y = 112$; $2y = 40$; $y = 20$. Hard coal, \$24/lb, soft coal \$20/lb, *Ans.*

33. Let x = John's rate in mph

	r	t	d
John	x	2	$2x$
Scouts	3	3	9

$2x = 9$
$x = 4\frac{1}{2}$ mph, *Ans.*

34. Let y = no. of votes Ruth received; then $500 - y$ = no. of votes Alan received; $y - x$ = no. of votes Ruth would have received if x students voted for Alan instead of Ruth; $500 - y + x$ = no. of votes Alan would have received if x students voted for Alan instead of Ruth; $y + 2x$ = no. of votes Ruth would have received if $2x$ students voted for her instead of for Alan; $500 - y - 2x$ = no. of votes Alan would have received if $2x$ students voted for Ruth instead of him;

$$\begin{aligned} 500 - y + x &= 4(y - x) \\ 5x - 5y &= -500 \\ -20x + 20y &= 2000 \end{aligned} \qquad \begin{aligned} y + 2x &= 500 - y - 2x \\ 4x + 2y &= 500 \\ 20x + 10y &= 2500 \end{aligned}$$

$$30y = 4500$$
$$y = 150$$

Ruth, 150; Alan, 350, *Ans.*

Pages 431–432 · **EXTRA FOR EXPERTS**

1.

	$a = b + 1$	Given
Then,	$(a - b)a = (a - b)(b + 1)$	Mult. prop. of equality
	$a^2 - ab = ab + a - b^2 - b$	Distrib. prop.
	$a^2 - ab - a = ab + a - a - b^2 - b$	Subt. prop. of equality
	$a(a - b - 1) = b(a - b - 1)$	Distrib. prop.
	$a = b$	Invalid step. Since $a = b + 1$, $a - b - 1 = 0$. Dividing each member by 0 is invalid.
	$b + 1 = b$	Subst. principle
	$\therefore 1 = 0$	Subt. prop. of equality

2.

$\frac{y + 9}{y - 1} - 6 = \frac{15 - 5y}{y - 8}$	Given
$\frac{15 - 5y}{y - 1} = \frac{15 - 5y}{y - 8}$	Mixed expression written as a fraction.
$\frac{1}{y - 1} = \frac{1}{y - 8}$	Invalid step. Since $y = 3$, $15 - 5y = 0$. Dividing each member by 0 is invalid.
$y - 1 = y - 8$	Mult. prop. of equality
$-1 = -8$	Subt. prop. of equality
$\therefore 1 = 8$	Mult. prop. of equality and mult. prop. of -1.

3.

$a > 3$	Given
$3a > 3(3)$	Mult. prop. of inequality
$3a - a^2 > 9 - a^2$	Subt. prop. of inequality
$a(3 - a) > (3 - a)(3 + a)$	Distrib. prop.
$a > 3 + a$	Invalid step. Since $a > 3$, $3 - a$ is negative. Hence, dividing each member of the inequality by $3 - a$ reverses the order of the inequality.
$\therefore 0 > 3$	Subt. prop. of inequality

4.

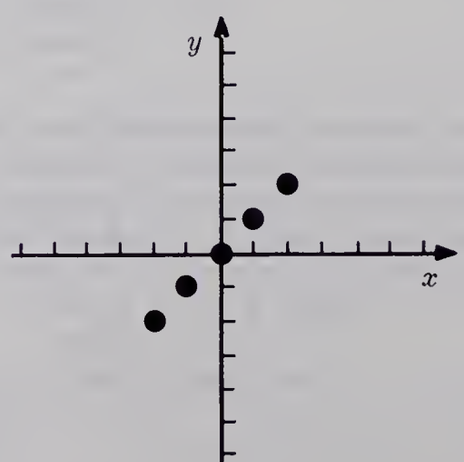
	$c > 1$ and $c = d$	Given
Then,	$c - 1 = d - 1$	Subt. prop. of equality
	$c - 1 = -1(1 - d)$	Mult. prop. of -1
	$(c - 1)^2 = (-1)^2(1 - d)^2$	Squaring each member equality
	$(c - 1)^2 = (1 - d)^2$	Mult. prop. of 1
	$c - 1 = 1 - d$	Invalid step. It is not correct to assume that nos. whose squares are equal must equal each other. Since $c - 1 = -1(1 - d)$, $c - 1 \neq 1 - d$.
	$\therefore c - 1 = 1 - c$	Subst. principle
	$2c = 2$	Add. prop. of equality
	$c = 1$	Div. prop. of equality
	$\therefore 1 > 1$	Subst. principle ($c > 1$)

CHAPTER 12. Functions and Variation

Pages 437–438 · **WRITTEN EXERCISES**

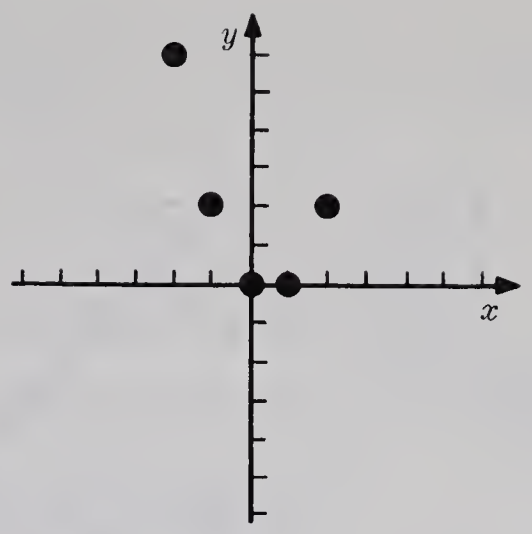
A **1.** $y = x$

x	-2	-1	0	1	2
y	-2	-1	0	1	2



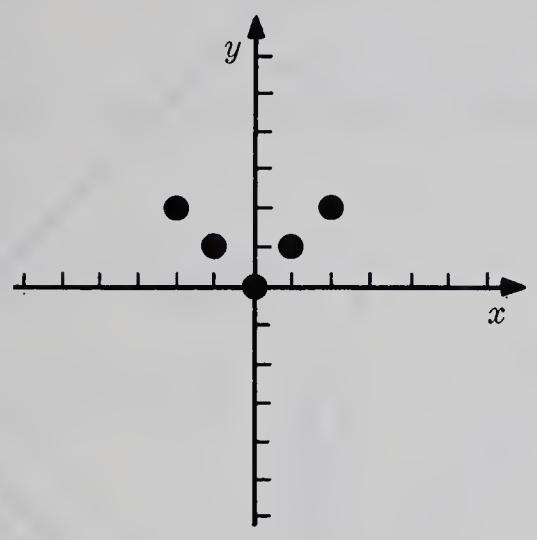
2. $y = x^2 - x$

x	-2	-1	0	1	2
y	6	2	0	0	2



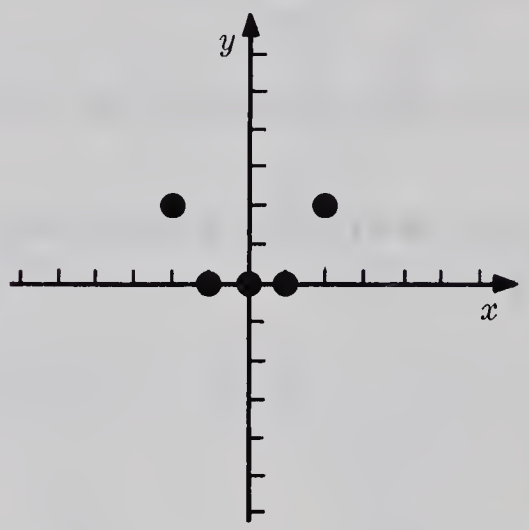
3. $y = |x|$

x	-2	-1	0	1	2
y	2	1	0	1	2



4. $y = x^2 - |x|$

x	-2	-1	0	1	2
y	2	0	0	0	2



5. Domain: {0, 1, 2}
Range: {2, 3, 4}

6. Domain: {1, 2, 3}
Range: {−1, 0, 1}

7. Domain: {−1, 3, 0}
Range: {4, −2, 0}

8. Domain: {5, −2, 3}
Range: {0, 3, 1, 6}

9. Domain: {−5, 4, 6, −7}
Range: {4, −4}

10. Domain: {−6, −8, −9, −10}
Range: {−1, −2}

11. $c = 32n$

Gallons	2	**3**	4	5	10	**12**	13
Cents	64	96	**128**	160	320	384	**416**

12. $A = 7h$

Height	4	5	**7**	9	10	**15**	20
Area	28	35	49	**63**	70	105	**140**

13. $f = 3.28m$

Meters	1	2	3	4	**6**
Feet	**3.28**	6.56	9.84	**13.12**	19.68

14. $p = 2.2k$

Kilograms	1	2	3	7	**15.4**
Pounds	**2.2**	**4.4**	6.6	15.4	33.88

B **15.** $y = 0 \cdot x$ or $y = \dfrac{0}{x}$

x	1	2	3	4	**5**
y	0	0	**0**	0	0

(Any value for x except 0 will complete the chart.)

16. $y = |x|$

x	-3	-2	-1	4	5
y	3	2	1	4	5

17. $q = |p|$

p	-2	-1	1	3	7
q	2	1	1	3	7

18. $y = 2x + 3$

x	1	3	5	7
y	5	9	13	17

19.

n	4	5	6	7
t	2	3	4	5

$t = n - 2$

20.

n	4	5	6	7
d	1	2	3	4

$d = n - 3$

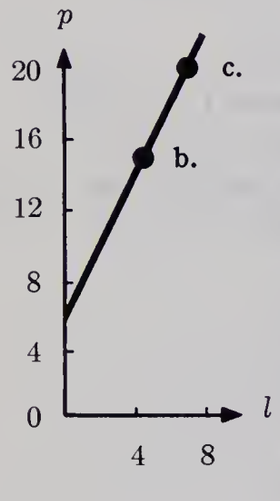
21. $d = n - 3$; $d + 3 = n$; $t = n - 2$; $t = (d + 3) - 2$; $t = d + 1$, *Ans.*

22. Sum of angles of a triangle is 180°. No. of triangles times 180° $= s$; $s = t(180°)$; $s = (n - 2)180°$, *Ans.*

Pages 441–442 · **WRITTEN EXERCISES**

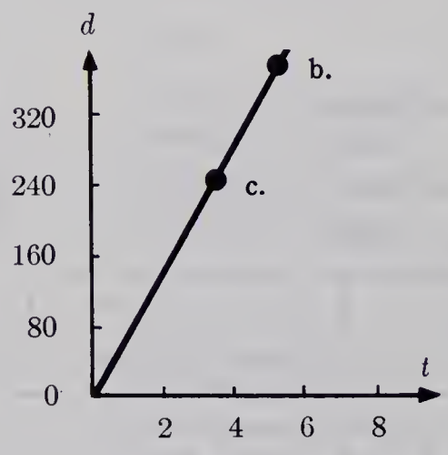
A **1.** a. $p = 2l + 6$

p	6	8	10	15	20
l	0	1	2	4.5	7

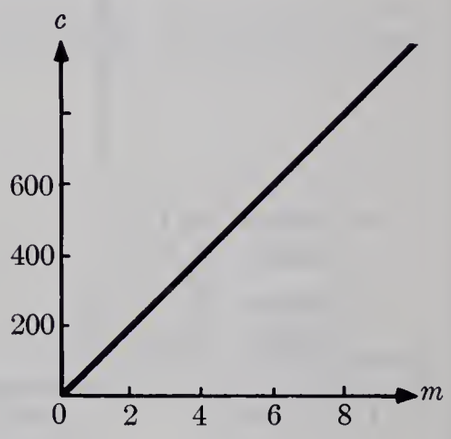


2. a. $d = 70t$

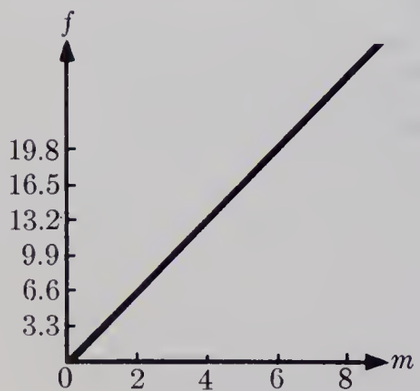
d	70	140	210	280
t	1	2	3	4



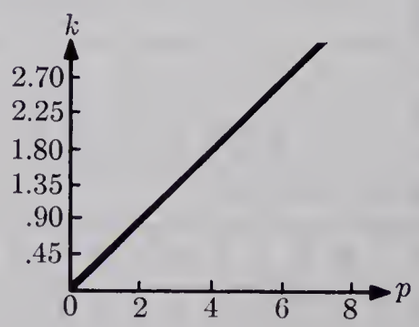
3. $c = 100m$



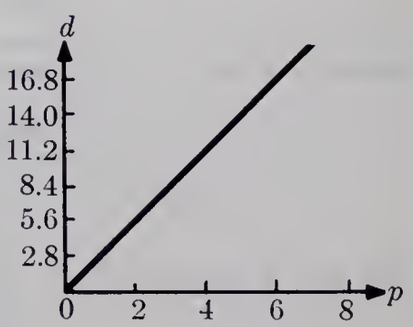
4. $f = 3.3m$



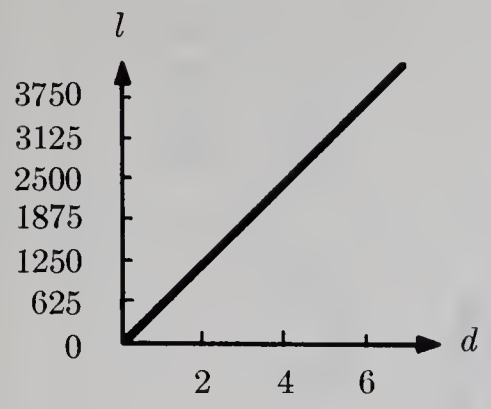
5. $k = .45p$



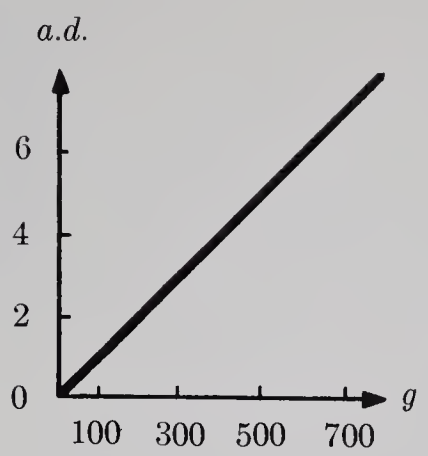
6. $d = 2.8p$



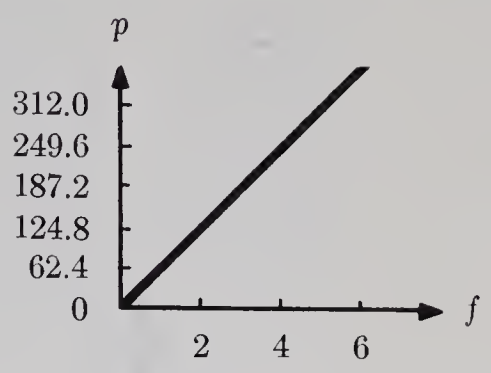
7. $l = 625d$



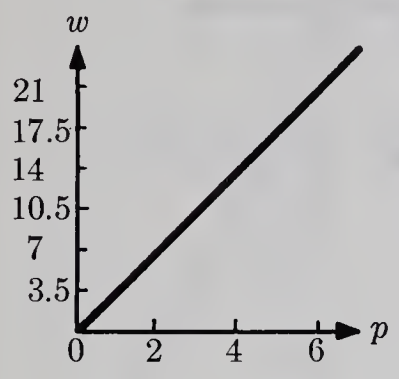
8. $a.d. = \frac{g}{100}$



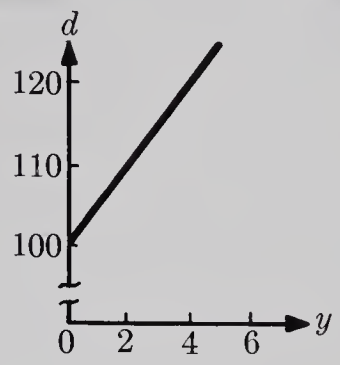
9. $p = 62.4f$



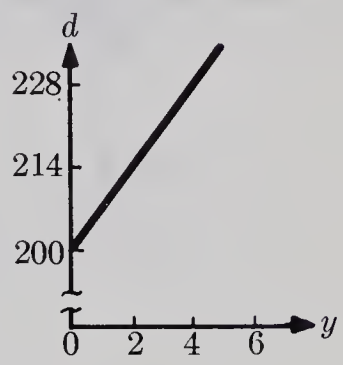
10. $w = 3\frac{1}{2}p$



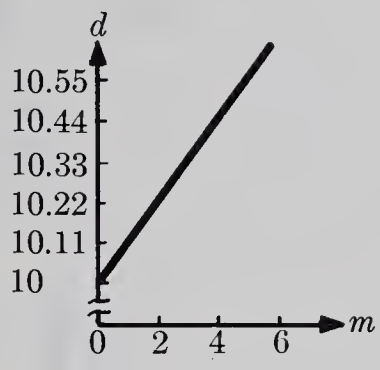
11. $d = 5y + 100$



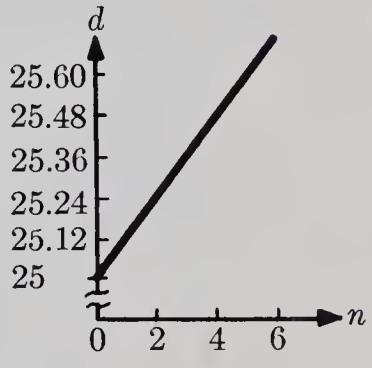
12. $d = 7y + 200$



13. $d = 10 + .11m$

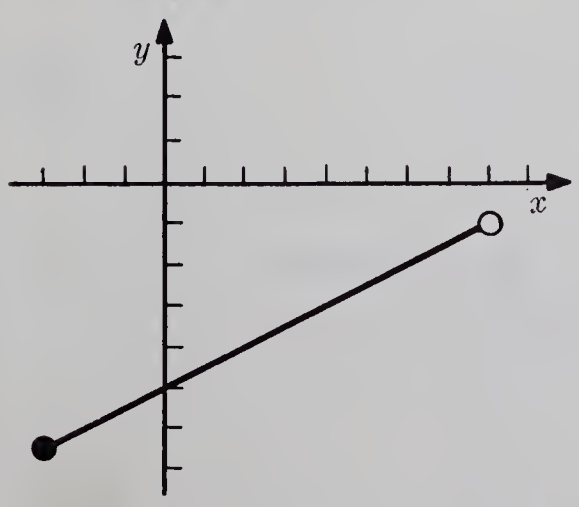


14. $d = 25 + .12n$



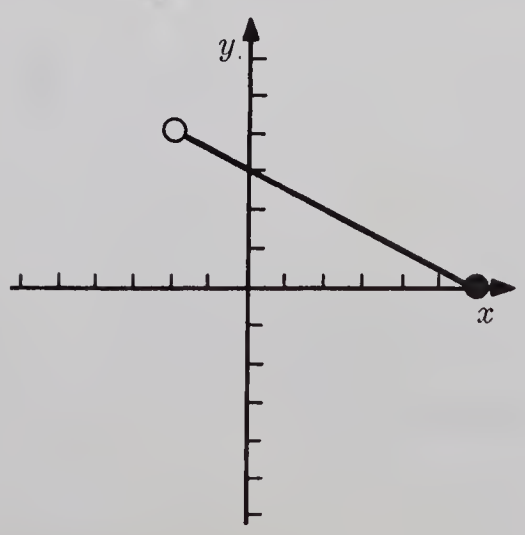
B **15.** $y = \frac{1}{2}x - 5, -3 \leq x < 8$

x	y
-3	$-6\frac{1}{2}$
0	-5
2	-4
4	-3



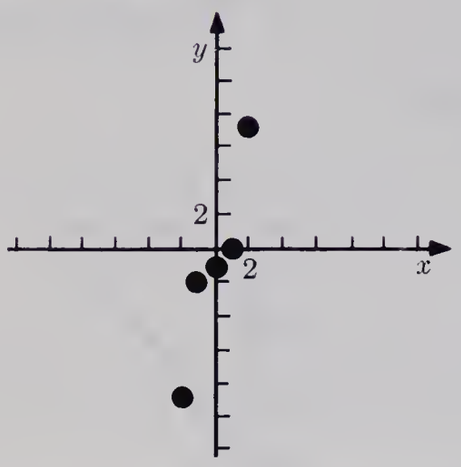
16. $y = 3 - \frac{x}{2}; -2 < x \leq 6$

x	y
0	3
2	2
4	1
6	0



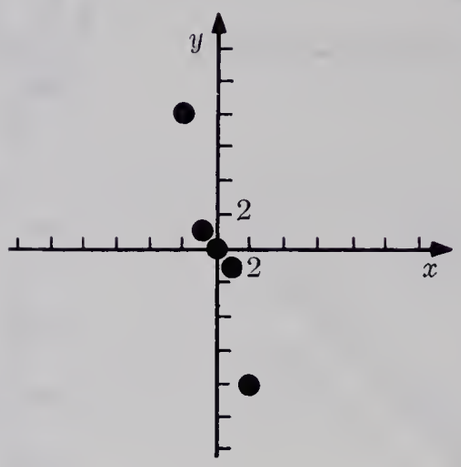
17. $y = x^3 - 1; x \in \{-2, -1, 0, 1, 2\}$

x	y
-2	-9
-1	-2
0	-1
1	0
2	7



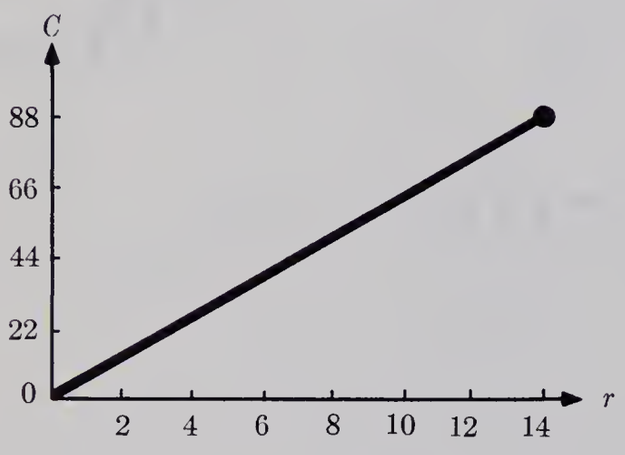
18. $y = -x^3; x \in \{-2, -1, 0, 1, 2\}$

x	y
-2	8
-1	1
0	0
1	-1
2	-8



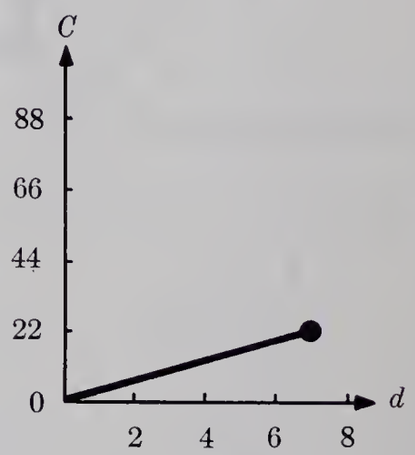
19. $C = \frac{44}{7}r; 0 \leq r \leq 14$

r	C
0	0
7	44
14	88



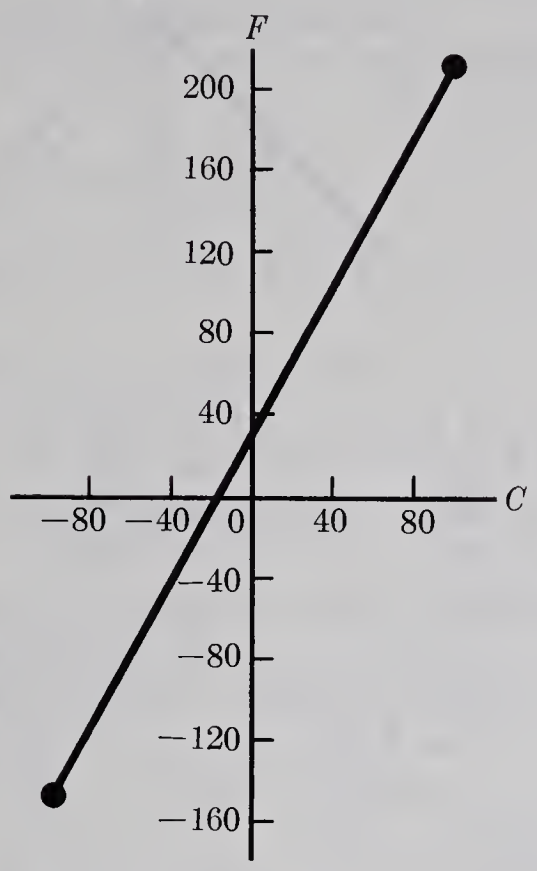
20. $C = \frac{22}{7}d; 0 \leq d \leq 7$

d	C
0	0
7	22



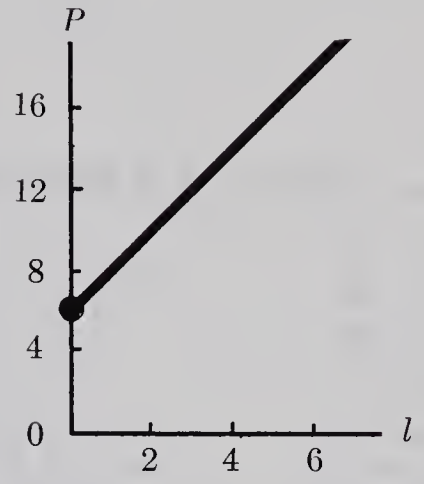
21. $F = \frac{9}{5}C + 32; -100 \leq C \leq 100$

C	F
-100	-148
0	32
100	212



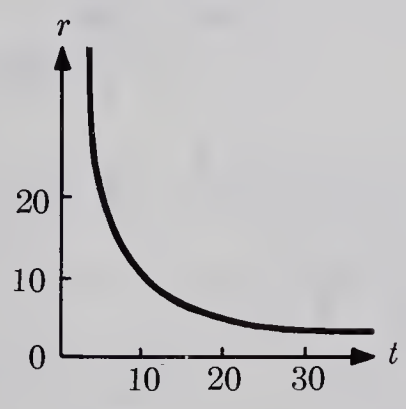
22. $P = 2(l + 3) = 2l + 6; l \geq 0$

l	P
0	6
1	8
2	10
3	12
4	14
5	16
6	18



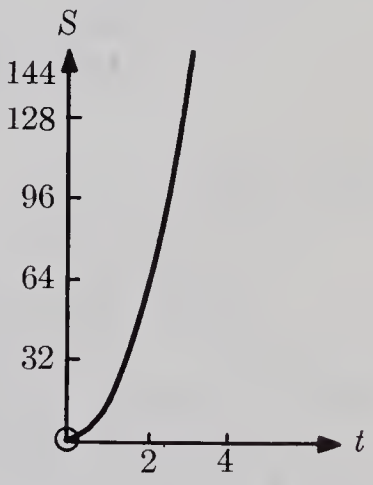
23. $r = \dfrac{100}{t}; t > 0$

t	r
10	10
5	20
20	5



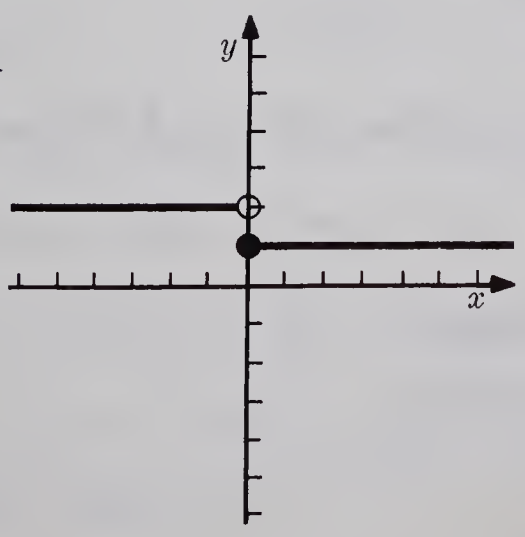
24. $S = 16t^2; t > 0$

t	S
1	16
2	64
3	144



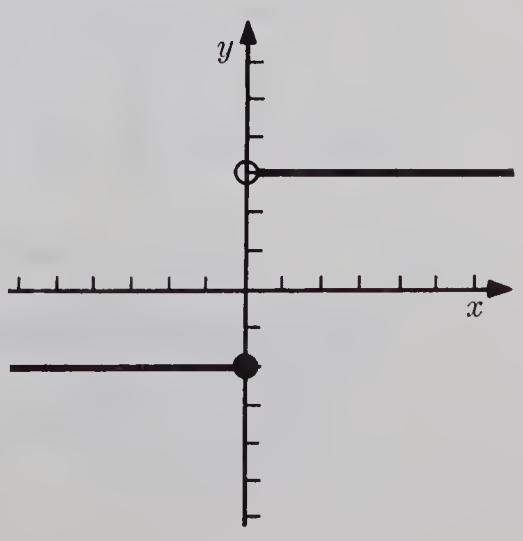
C **25.**

x	y
0	1
1	1
2	1
-1	2
-2	2
-3	2



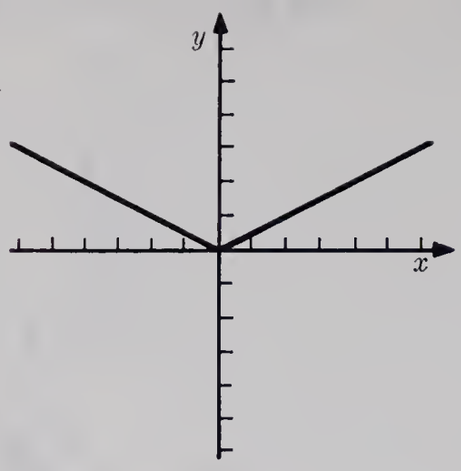
26.

x	y
1	3
2	3
3	3
0	-2
-1	-2
-2	-2



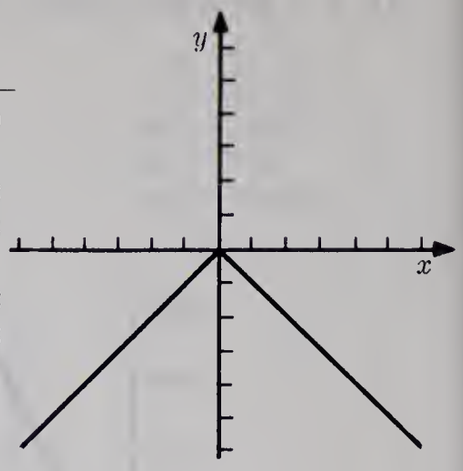
27. $y = \frac{1}{2}|x|$

x	y
2	1
4	2
6	3
−2	1
−4	2
−6	3
0	0



28. $y = -|x|$

x	y
0	0
1	−1
2	−2
3	−3
−1	−1
−2	−2
−3	−3



Pages 445–446 · **WRITTEN EXERCISES**

A

1. $\frac{y_1}{12} = \frac{14}{8}$
$8y_1 = 168$
$y_1 = 21$

2. $\frac{8}{18} = \frac{6}{x_2}$
$8x_2 = 108$
$x_2 = 13\frac{1}{2}$

3. $\frac{15}{35} = \frac{8}{z_2}$
$15z_2 = 280$
$z_2 = 18\frac{2}{3}$

4. $\frac{13}{n_1} = \frac{9}{6}$
$9n_1 = 78$
$n_1 = 8\frac{2}{3}$

5. $\frac{2.6}{6.5} = \frac{r_2}{4.5}$
$6.5r_2 = 11.70$
$r_2 = 1.8$

6. $\frac{2\frac{1}{2}}{4\frac{1}{2}} = \frac{6\frac{2}{3}}{v_2}$
$2\frac{1}{2}v_2 = 30$
$v_2 = 12$

7. $7(x - 3) = 4x$
$7x - 21 = 4x$
$3x = 21$
$x = 7$. {7}

8. $9(w - 4) = 5w$
$9w - 36 = 5w$
$4w = 36$
$w = 9$. {9}

9. $3y = 5(y - 16)$
$3y = 5y - 80$
$-2y = -80$
$y = 40$. {40}

10. $4z = 7(z - 3)$
$4z = 7z - 21$
$-3z = -21$
$z = 7$. {7}

11. $9(3x + 2) = 60$
$27x + 18 = 60$
$27x = 42$
$x = 1\frac{15}{27} = 1\frac{5}{9}$. $\{1\frac{5}{9}\}$

12. $6(3x - 2) = 297$
$18x - 12 = 297$
$18x = 309$
$x = 17\frac{1}{6}$. $\{17\frac{1}{6}\}$

B

13. $x^2 = 36$
$x = \pm 6$. {6, −6}

14. $(x + 1)(x - 1) = x(x + 2)$
$x^2 - 1 = x^2 + 2x$
$2x = -1$
$x = -\frac{1}{2}$. $\{-\frac{1}{2}\}$

15. $x(x - 1) = (x + 3)^2$
$x^2 - x = x^2 + 6x + 9$
$-7x = 9$
$x = -\frac{9}{7}$. $\{-1\frac{2}{7}\}$

16. $\frac{x}{y - 2} = k$; $k = \frac{6}{11 - 2} = \frac{6}{9} = \frac{2}{3}$; $\frac{4}{y - 2} = \frac{2}{3}$; $2y - 4 = 12$; $2y = 16$; $y = 8$, *Ans.*

17. $\frac{y}{2x + 3} = k$; $k = \frac{12}{15} = \frac{4}{5}$; $\frac{36}{2x + 3} = \frac{4}{5}$; $4(2x + 3) = 180$; $8x + 12 = 180$; $8x = 168$; $x = 21$, *Ans.*

18. $4x + 10x = 42$; $14x = 42$; $x = 3$; $5x = 15$; $b = 15$, *Ans.*

19. $\frac{V}{R^3} = \frac{\frac{9\pi}{2}}{(\frac{3}{2})^3} = \frac{\frac{9\pi}{2}}{\frac{27}{8}} = \frac{9\pi}{2} \cdot \frac{8}{27} = \frac{4\pi}{3}$; $\frac{V}{R^3} = \frac{4\pi}{3}$; $3V = 4\pi R^3$; $V = \frac{4}{3}\pi R^3$, *Ans.*

20.

$\frac{y_1}{x_1} = \frac{y_2}{x_2}$; $y_2 \neq 0$	Given
$\left(\frac{x_1}{y_2}\right)\left(\frac{y_1}{x_1}\right) = \left(\frac{x_1}{y_2}\right)\left(\frac{y_2}{x_2}\right)$	Mult. prop. of equality
$\frac{y_1}{y_2}\left(\frac{x_1}{x_1}\right) = \frac{x_1}{x_2}\left(\frac{y_2}{y_2}\right)$	Comm. and assoc. props.
$\therefore \frac{y_1}{y_2} = \frac{x_1}{x_2}$	Mult. prop. of 1

21.

$\frac{y_1}{x_1} = \frac{y_1}{x_2}\,;\, y_1 \neq 0$	Given
$\left(\frac{x_1x_2}{y_1}\right)\left(\frac{y_1}{x_1}\right) = \left(\frac{x_1x_2}{y_1}\right)\left(\frac{y_1}{x_2}\right)$	Mult. prop. of equality
$\left(\frac{x_1y_1}{x_1y_1}\right)x_2 = \left(\frac{x_2y_1}{x_2y_1}\right)x_1$	Comm. and assoc. props.
$\therefore x_2 = x_1$ *or* $x_1 = x_2$	Mult. prop. of 1

C **22.** Divide both numerator and denominator of the given expression by ny: $\dfrac{4 \cdot \frac{m}{n} \cdot \frac{x}{y} - 1}{2 \cdot \frac{x}{y} - \frac{m}{n}}$; substitute the ratios: $\dfrac{4 \cdot \frac{1}{3} \cdot \frac{3}{4} - 1}{2 \cdot \frac{3}{4} - \frac{1}{3}} = \frac{0}{\frac{7}{6}} = 0.$

23. $\frac{r}{s} \div \frac{r}{t} = \frac{t}{s}\,;\, \frac{3}{5} \div \frac{9}{10} = \frac{2}{3};\, \frac{s+t}{s} = 1 + \frac{t}{s} = \frac{3}{3} + \frac{2}{3} = \frac{5}{3}$

24. $5y - 5 = x + 1$ $\quad 2 + 1 = x - 1;\, x = 4;\, \frac{x}{y} = \frac{4}{2} = 2$

$y + 1 = x - 1$

$4y - 6 = \quad + 2;\, 4y = 8;\, y = 2$

Pages 446–447 · **PROBLEMS**

A **1.** $\frac{5}{150} = \frac{7}{y_2}\,;\, 5y_2 = 1050;\, y_2 = 210$ lb, *Ans.*

2. $\frac{2}{14} = \frac{5}{y_2}\,;\, 2y_2 = 70;\, y_2 = 35$ bags, *Ans.*

3. $\frac{3}{2} = \frac{y_1}{78}\,;\, 2y_1 = 234;\, y_1 = 117$ tons, *Ans.*

4. $\frac{16}{x_2} = \frac{49}{35}\,;\, 49x_2 = 560;\, x_2 = 11\frac{3}{7}$ tons, *Ans.*

5. $\frac{18}{1080} = \frac{20}{y_2}\,;\, 18y_2 = 21{,}600;\, y_2 = 1200$ g, *Ans.*

6. $\frac{20}{50} = \frac{100}{y_2}\,;\, 20y_2 = 5000;\, y_2 = 250$ volts, *Ans.*

7. $\frac{.3}{50} = \frac{1}{y_2}\,;\, .3y_2 = 50;\, y_2 = 166.7$ ft, *Ans.*

8. $\frac{1}{300} = \frac{.00027}{y_2}\,;\, y_2 = 300 \times .00027;\, y_2 = .081$ ohms, *Ans.*

9. $\frac{1}{5\frac{1}{3}} = \frac{\frac{3}{16}}{y_2}\,;\, y_2 = 5\frac{1}{3} \times \frac{3}{16} = 1$ in, *Ans.*

10. $\frac{1}{28} = \frac{y_1}{1\frac{3}{4}}\,;\, 28y_1 = 1\frac{3}{4};\, y_1 = \frac{\frac{7}{4}}{28}\,;\, \frac{1}{16}$ in $= 1$ in, *Ans.*

B **11.** $\frac{p_1}{5 \cdot 3 \cdot \frac{2}{3}} = 170\,;\, \frac{p_1}{10} = 170\,;\, p_1 = 1700$ lb, *Ans.*

$\therefore$ limestone is heavier by 440 lb, *Ans.*

$\frac{p_2}{4\frac{1}{2} \cdot 4 \cdot \frac{1}{2}} = 140\,;\, \frac{p_2}{9} = 140\,;\, p_2 = 1260$ lb, *Ans.*

12. $V = \frac{4}{3}\pi r^3$ and $r = 21$ mm $= 2.1$ cm; $\frac{p_1}{\frac{4}{3} \cdot \frac{22}{7}(2.1)^3} = 1.4;\, \frac{p_1}{38.8} = 1.4;\, p_1 = 54.3$ g;

$\frac{p_2}{\frac{4}{3} \cdot \frac{22}{7}(2.1)^3} = 1.9;\, \frac{p_2}{38.8} = 1.9;\, p_2 = 73.7$ g; $54.3 - 73.7 = -19.4$ g, *Ans.*

13. $\frac{625}{(\frac{7}{8})^2} = \frac{y_1}{(1\frac{2}{5})^2}\,;\, \frac{49}{64}y_1 = (\frac{7}{5})^2 \times 625;\, \frac{49}{64}y_1 = 49 \cdot 25;\, y_1 = 49 \cdot 25 \cdot \frac{64}{49} = \$1600$, *Ans.*

14. $\frac{80}{100} = \frac{y_1}{60}\,;\, 100y_1 = 4800;\, y_1 = 48$ divisions, *Ans.*

15. $\frac{60}{100} = \frac{48}{y_2}\,;\, 60y_2 = 4800;\, y_2 = 80$ volts, *Ans.*

C **16.** $\frac{1}{5\frac{1}{2}} = \frac{50}{w}\,;\, w = 275$ mi; $\frac{1}{7\frac{1}{8}} = \frac{50}{l}\,;\, l = 356.25$ mi; $A = 275 \times 356.25 \doteq 97{,}969$ sq mi, *Ans.*

17. $\frac{1}{8} = \frac{4}{l}\,;\, l = 4 \cdot \frac{7}{8} = \frac{7}{2} = 3\frac{1}{2}$ mi; $\frac{1}{\frac{1}{2}} = \frac{4}{w}\,;\, w = 4 \cdot \frac{1}{2} = 2$ mi; $A = 2 \cdot 3\frac{1}{2} = 7$ sq mi;

7×175 men $= 1225$ men, *Ans.*

18. $\frac{50{,}000 \times 2000}{(1000)^3} = \frac{100}{l^3}$; $\frac{50{,}000 \times 2000}{1000 \times 1000 \times 1000} = \frac{100}{l^3}$; $\frac{1}{10} = \frac{100}{l^3}$; $l^3 = 1000$; $l = 10$ ft; $\frac{10}{1000} = \frac{1}{100}$. Length, 10 ft; scale, 1 ft = 100 ft, *Ans.*

Pages 450–451 · **WRITTEN EXERCISES**

A **1.** $12 \cdot 20 = 15y_2$; $15y_2 = 240$; $y_2 = 16$

2. $\frac{2}{3} \cdot \frac{3}{4} = 6x_2$; $\frac{6}{12} = 6x_2$; $6x_2 = \frac{1}{2}$; $x_2 = \frac{1}{12}$

3. $(1.6)(36) = 1.8v_2$; $1.8v_2 = 57.6$; $v_2 = 32$

4. $(.75)(.4) = .5v_2$; $.5v_2 = .3$; $v_2 = .6$

5. Let $d = 8$ and $r = 2$; $2t = 8$; $t = 4$; if r is doubled, $4t = 8$; $t = 2$. t is halved when r is doubled, *Ans.*

6. Let $k = 12$ and $P = 2$; $2V = 12$; $V = 6$; if P is trebled, $6V = 12$; $V = 2$. V is divided by 3 when P is trebled, *Ans.*

7. $x(t+3) = k$; $6(7+3) = k$; $6(10) = k$; $k = 60$; $15(t+3) = 60$; $15t + 45 = 60$; $15t = 15$; $t = 1$, *Ans.*

8. $y(2t-1) = k$; $8(15-1) = k$; $8(14) = k$; $k = 112$; $y\big((2 \times 8.5) - 1\big) = 112$; $16y = 112$; $y = 7$, *Ans.*

B **9.** $Id_1^2 = k$; $1 \cdot 3^2 = k$; $k = 9$; $4d_2^2 = 9$; $d_2^2 = \frac{9}{4}$; $d_2 = \frac{3}{2} = 1\frac{1}{2}$ ft, *Ans.*

10. $Fs^2 = k$; $80(100)^2 = k$; $k = 800{,}000$; $F(40)^2 = 800{,}000$; $F = \dfrac{800{,}000}{1600} = 500$, *Ans.*

11. $Td^2 = k$; let $T = 2$ and $d = 3$; $2(3)^2 = k$; $2 \cdot 9 = k$; $k = 18$; $2d = 6$; $T(6)^2 = 18$; $36T = 18$; $T = \frac{18}{36} = \frac{1}{2}$. When T is divided by 4, d is doubled, *Ans.*

12. $H_1R_1^2 = H_2R_2^2$; let $H_1 = 9$; H_2 will then $= 1$; $9(8)^2 = R_2^2$; $R_2 = \sqrt{9(8)^2} = 24$, *Ans.*

13. $SL^3 = k$; $20(3)^3 = k$; $k = 540$; $SL^3 = 540$; $S = \dfrac{540}{L^3}$, *Ans.*

14. $RT^2 = k$; $5.5(2)^2 = k$; $k = 22.0$; $RT^2 = 22$; $R = \dfrac{22}{T^2}$, *Ans.*

Pages 451–452 · **PROBLEMS**

A **1.** $d = rt = 6 \cdot 30 = 180$ mi; $180 = 40t$; $t = 4\frac{1}{2}$ hr, *Ans.*

2. $\frac{8}{24} = \frac{y_2}{9}$; $24y_2 = 72$; $y_2 = 3$ days, *Ans.*

3. $4 \cdot 100 = 80d_2$; $80d_2 = 400$; $d_2 = 5$ ft, *Ans.*

4. $5w_1 = 6 \cdot 110$; $5w_1 = 660$; $w_1 = 132$ lb, *Ans.*

5. $40 \cdot 200 = 400d_2$; $8000 = 400d_2$; $d_2 = 20$ cm, *Ans.*

6. $\frac{5}{6} = \frac{y_2}{36}$; $6y_2 = 180$; $y_2 = \$30$, *Ans.*

7. $1400r = .02(2100)$; $r = \dfrac{.02(2100)}{1400} = \dfrac{.06}{2} = .03 = 3\%$, *Ans.*

8. $.03x = .04(1500)$; $x = \dfrac{.04(1500)}{.03} = \dfrac{4(1500)}{3} = \$2000$, *Ans.*

9. Area of $\triangle = \frac{1}{2}ab$; $\dfrac{15 \cdot 6}{2} = \dfrac{a \cdot 2}{2}$; $a = 45$ in, *Ans.*

10. $c = \pi d$; $42(30\pi) = (5\pi)r_2$; $5r_2 = 1260$; $r_2 = 252$ rev., *Ans.*

11. $\frac{3}{8} = \frac{V_2}{40}$; $8V_2 = 120$; $V_2 = 15$ cu ft, *Ans.*

12. $\frac{5}{3} = \frac{a_2}{18}$; $3a_2 = 90$; $a_2 = 30$ amp, *Ans.*

13. $\frac{10}{5} = \frac{r_2}{150}$; $5r_2 = 1500$; $r_2 = 300$ rpm, *Ans.*

14. $\frac{18}{12} = \frac{r_2}{100}$; $12r_2 = 1800$; $r_2 = 150$ rpm, *Ans.*

B. **15.** Let $x =$ Jack's distance in feet from support; then $13.2 - x =$ Jill's distance.
$\dfrac{x}{13.2 - x} = \dfrac{116}{148}$; $148x = 116(13.2 - x)$; $148x = 1531.2 - 116x$; $264x = 1531.2$; $x = 5.8$ ft, *Ans.*

16. $d_1p_1 = d_2p_2$; let $p_1 = 1$, then $p_2 = 2$; $d_1 = 2d_2$ *or* $d_2 = \frac{1}{2}d_1$. Piston is moved $\frac{1}{2}$ the distance to double pressure, *Ans.*

17. $\dfrac{w_1}{w_2} = \dfrac{d_2^2}{d_1^2}$; $\dfrac{540}{w_2} = \dfrac{(4800)^2}{(4000)^2} = \dfrac{6^2}{5^2}$; $36w_2 = 13{,}500$; $w_2 = 375$ lb, *Ans.*

18. $\dfrac{r_1}{r_2} = \dfrac{d_2^2}{d_1^2}$; $\dfrac{12}{r_2} = \dfrac{(\frac{1}{2})^2}{(\frac{3}{4})^2}$; $\frac{1}{4}r_2 = \frac{9}{16} \cdot 12 = \frac{27}{4}$; $r_2 = 27$ ohms, *Ans.*

Pages 454–455 · WRITTEN EXERCISES

1. $\frac{A}{h(a+b)} = k$ *or* $A = kh(a+b)$

2. $\frac{A}{lw} = k$ *or* $A = klw$

3. $\frac{V}{hB} = k$ *or* $V = khB$

4. $\frac{V}{hr^2} = k$ *or* $V = khr^2$

5. $\frac{nt}{w} = k$ *or* $n = \frac{kw}{t}$

6. $\frac{ts}{d} = k$ *or* $t = \frac{kd}{s}$

7. $\frac{g}{hr^2} = k$ *or* $g = khr^2$

8. $\frac{c}{md} = k$ *or* $c = kmd$

9. $\frac{fr}{s^2} = k$ *or* $f = \frac{ks^2}{r}$

10. $\frac{pd}{s^2} = k$ *or* $p = \frac{ks^2}{d}$

Pages 455–456 · PROBLEMS

A **1.** Let I and t each $= 1$, then $H = \frac{R}{4} = \frac{1}{4}R$. When I is doubled and t is made 3 times as large, $H = \frac{(2)^2 \times 3R}{4} = \frac{12R}{4} = 3R$; $3R$ is 12 times as large as $\frac{1}{4}R$. H is made 12 times as large, *Ans.*

2. Let v and r each $= 1$, then $F = m$. When v is halved and r is doubled, $F = \frac{(\frac{1}{2})^2 m}{2} = \frac{\frac{1}{4}m}{2} = \frac{1}{8}m$. F is made $\frac{1}{8}$ as large, *Ans.*

3. $\frac{wz^2}{xy} = k$ (a) $\frac{150(9)^2}{15 \cdot 18} = 45 = k$ (b) $wz^2 = 45xy$ (c) $\frac{(6)^2 w}{21 \cdot 12} = 45$; $3w = 945$; $w = 315$

4. (a) $\frac{rtu^2}{s^3} = k$; $\frac{4 \cdot 2(3)^2}{6^3} = \frac{1}{3} = k$ (b) $rtu^2 = \frac{s^3}{3}$ (c) $\frac{9(5)^2 t}{(15)^3} = \frac{1}{3}$; $\frac{t}{15} = \frac{1}{3}$; $3t = 15$; $t = 5$

5. No. of boxes picked varies jointly as no. of boys and hr. $\frac{360}{4 \cdot 15} = \frac{540}{3b}$; $6b = 180$; $b = 30$ boys, *Ans.*

6. $\frac{w}{lA} = k$; $\frac{4.8}{4\frac{1}{2} \cdot \frac{1}{4}} = \frac{\frac{24}{5}}{\frac{9}{8}} = \frac{64}{15} = k$; $\frac{w_2}{7\frac{1}{2} \cdot \frac{1}{16}} = \frac{64}{15}$; $15w_2 = 30$; $w_2 = 2$ lb, *Ans.*

7. Centrifugal force varies directly as square of speed of moving mass and inversely as radius of its circular path.

$f = \frac{ks^2}{r}$; $1260 = \frac{k(30)^2}{6}$; $k = \frac{6 \cdot 1260}{900} = 8.4$; $f = \frac{8.4(40)^2}{8} = \frac{8.4(1600)}{8} = 1680$ lb, *Ans.*

8. Let L = the load; l = length of beam between supports; d = beam's depth;

$\frac{Ll}{d^2} = k$; $\frac{1350 \cdot 10}{(\frac{1}{2})^2} = 54{,}000 = k$; $\frac{16L}{(\frac{2}{3})^2} = 54{,}000$; $16L = 24{,}000$; $L = 1500$ lb, *Ans.*

9. Let C = operating cost; w = no. of watts; h = hr of operation; c = cost per kw-hr;

$\frac{C}{whc} = k$; $\frac{2}{1000 \cdot \frac{1}{2} \cdot 4} = \frac{1}{1000} = k$; $\frac{C}{720 \cdot 1 \cdot 4} = \frac{1}{1000}$; $1000C = 2880$; $C = 2.88¢ \doteq 3¢$, *Ans.*

10. Let h = heat developed; r = resistance; t = time current flows; c = current;

$\frac{h}{rtc^2} = k$; $\frac{1200}{8 \cdot 2(5)^2} = \frac{3}{1} = 3 = k$; $\frac{6000}{5(10)^2 r} = 3$; $3r = 12$; $r = 4$ ohms, *Ans.*

B **11.** Let p = wind pressure; A = surface area; v = wind's velocity;

$\frac{p}{Av^2} = k$; $\frac{\frac{1}{2}}{1(8)^2} = \frac{1}{128} = k$; $\frac{27}{6v^2} = \frac{1}{128}$; $6v^2 = 3456$; $v^2 = 576$; $v = 24$ mph, *Ans.*

12. Let h = heat lost; d = difference of temp.; A = window area; t = thickness of pane;

$\frac{ht}{dA} = k$; $\frac{180 \cdot \frac{1}{2}}{30 \cdot 50 \cdot 24} = \frac{1}{400} = k$; $\frac{.4h}{32 \cdot 50 \cdot 12} = \frac{1}{400}$; $400h = 48{,}000$; $h = 120$ heat calories, *Ans.*

Page 457 · **CHAPTER TEST**

1. Domain: {nonnegative real nos.}
Range: {nonnegative real nos.}
Formula: $p = 62.4n$

2. Domain: {0, 1, 2, 3, 4, 5}
Range: {−1, 1, 3, 5, 7, 9}
Formula: $y = 2x - 1$

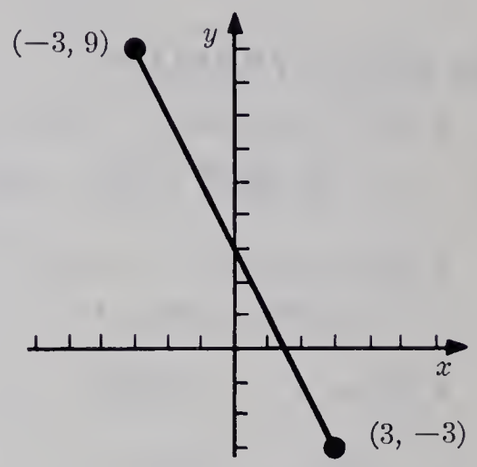
3.

x	$2x - x^2$	s
−3	$2(-3) - 9$	−15
−2	$2(-2) - 4$	−8
−1	$2(-1) - 1$	−3
0	$2(0) - 0$	0
1	$2(1) - 1$	1
2	$2(2) - 4$	0

Using only integral values for x, the roster is:
{(−3, −15), (−2, −8), (−1, −3), (0, 0), (1, 1), (2, 0)}

4. No, because the roster has one element of the domain appearing in two ordered pairs, e.g., both −1 and 1 of the range are assigned to the element 3 of the domain.

5. Domain: {real nos. between −3 and 3, inclusive}.
Range: {real nos. between −3 and 9, inclusive}.
This relation is a function because to each element of the domain there is assigned one and only one element of the range.



6. $t_1 = kr_1$; $t = 5.6$ when $r = 3.2$; $5.6 = k(3.2)$; $\frac{7}{4} = k$; $t_2 = kr_2$; $4.2 = \frac{7}{4}r_2$; $r_2 = 2.4$, *Ans.*

7. $d = kt$; $576 = k \cdot 6$; $96 = k$; $d = 96t$. If $t = 10$, $d = 96 \cdot 10 = 960$. $k = 96$; 960 ft, *Ans.*

8. Let $P_1 = 12$, $V_1 = 8$, $P_2 = 16$; $P_1V_1 = P_2V_2$; $12(8) = 16V_2$; $V_2 = 6$, *Ans.*

9. Let $H_1 = 2.7$, $d_1 = 4$, $d_2 = 3$; $H_1d_1^2 = H_2d_2^2$; $2.7(4)^2 = H_2(3)^2$; $2.7(16) = H_2 \cdot 9$; $H_2 = 4.8$, *Ans.*

10. Let x = no. of men needed to do the job in $h - 2$ hr; then $mh = x(h - 2)$; $x = \dfrac{mh}{h - 2}$, *Ans.*

11. (Optional) Let v = volume, b = area of base, h = altitude; $v_1 = 32$, $b_1 = (4)^2$, $h_1 = 6$, $b_2 = (3)^2$, $h_2 = 8$; $v_1 = kb_1h_1$; $32 = k \cdot 16 \cdot 6$; $\frac{1}{3} = k$; $v_2 = \frac{1}{3}b_2h_2$; $v_2 = \frac{1}{3} \cdot 9 \cdot 8$; $v_2 = 24$ cu in, *Ans.*

12. $y = \dfrac{kn}{t}$; $y_1 = 42$, $n_1 = 14$, $t_1 = 18$, $y_2 = 36$, $t_2 = 21$; $y_1 = \dfrac{kn_1}{t_1}$; $42 = \dfrac{k \cdot 14}{18}$; $54 = k$;
$y_2 = \dfrac{54n_2}{t_2}$; $36 = \dfrac{54n_2}{21}$; $n_2 = 14$, *Ans.*

Pages 458–459 · **CHAPTER REVIEW**

1. relation **2.** graph, roster, rule **3.** domain; range **4.** open **5.** domain and range

6.

t	$t^2 - 3t$	m
3	$3^2 - 3 \cdot 3$	0
1	$1^2 - 3 \cdot 1$	−2
0	$0^2 - 3 \cdot 0$	0
−1	$(-1)^2 - 3(-1)$	4
−3	$(-3)^2 - 3(-3)$	18

Roster: {(3, 0), (1, −2), (0, 0), (−1, 4), (−3, 18)}

7. Domain: {−2, −1, 0, 1, 2, 3}; range: {0, 1, 2, 3}; formula: $y = |x|$.

8. relation **9.** only once; many **10.** relation **11.** function

12. It is a function. To each element of the domain is assigned one and only one element of the range.

13. The relation is not a function because to the element 2 of the domain are assigned two elements of the range.

14. function **15.** ordinate, constant function **16.** kx

17. constant, proportionality **18.** $s_1 : s_2$ **19.** proportion

20. $b, c; a, d$ **21.** product, extremes **22.** straight line, constant, proportionality, 0

23. $c = kn$; $c_1 = 30$, $n_1 = 12$, $c_2 = 40$; $c_1 = kn_1$, $30 = k \cdot 12$, $\frac{5}{2} = k$; $c_2 = \frac{5}{2}n_2$; $40 = \frac{5}{2}n_2$; $16 = n_2$ *or* $\frac{c_1}{c_2} = \frac{n_1}{n_2}$; $\frac{30}{40} = \frac{12}{n_2}$; $30n_2 = 12 \cdot 40$; $n_2 = 16$, *Ans.*

24. $L = ks^2$ *or* $\frac{L_1}{L_2} = \frac{s_1^2}{s_2^2}$; $L_1 = 756$, $s_1 = 450$, $s_2 = 300$; $\frac{756}{L_2} = \frac{(450)^2}{(300)^2}$;

$(450)^2 L_2 = 756(300)^2$; $L_2 = \frac{756(300)^2}{(450)^2}$; $L_2 = 336$, *Ans.*

25. Let d = actual distance in mi; m = the no. of inches on the map; $d = km$; $d_1 = 30$, $m_1 = 1\frac{7}{8}$, $m_2 = 6\frac{1}{4}$; $d_1 = km_1$; $30 = k \cdot \frac{15}{8}$; $16 = k$; $d_2 = 16m_2$; $d_2 = 16 \cdot \frac{25}{4}$; $d_2 = 100$, *Ans.*

26. function **27.** $\frac{k}{x}$, 0 **28.** $s_2 : s_1$ **29.** hyperbola **30.** multiplicative inverse

31. Let $w_1 = 108$, $d_1 = 12$, $w_2 = 144$; $d_1 w_1 = d_2 w_2$; $12(108) = d_2(144)$; $\frac{12(108)}{144} = d_2$; $d_2 = 9$ ft, *Ans.*

32. Let $F_1 = 120$, $s_1 = 60{,}000$, $s_2 = 40{,}000$; $F_1 s_1^2 = F_2 s_2^2$; $120(60{,}000)^2 = F_2(40{,}000)^2$;
$\frac{120(60{,}000)^2}{(40{,}000)^2} = F_2$; $120 \cdot \frac{9}{4} = F_2$; $F_2 = 270$, *Ans.*

33. $xy = k$; $x_1 = 1.5$, $y_1 = 6$; $x_1 y_1 = k$; $1.5(6) = k$; $9 = k$; $xy = 9$; $y = \frac{9}{x}$

Rule: y is equal to 9 divided by x, $x \neq 0$, *Ans.*

34. directly, product

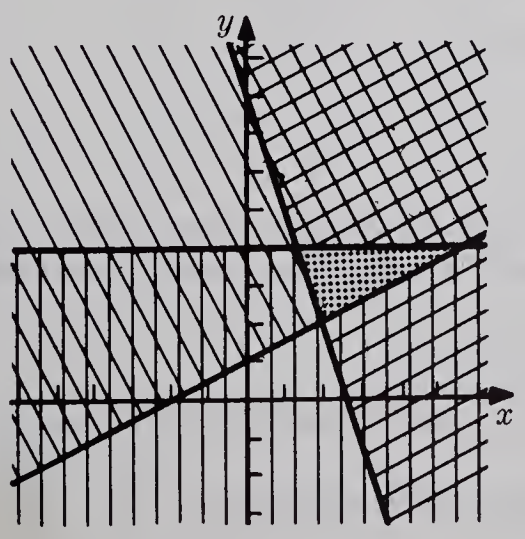
35. The centripetal force f on a body varies jointly as its mass m and the square of its speed v, i.e., $f = kmv^2$; $f_1 = 1{,}350{,}000$, $m_1 = 60$, $v_1 = 300$, $f_2 = 1{,}920{,}000$, $v_2 = 400$; $f_1 = km_1 v_1^2$;
$1{,}350{,}000 = k(60)(300)^2$; $\frac{1{,}350{,}000}{60(300)^2} = k$; $\frac{1}{4} = k$; $f_2 = \frac{1}{4}m_2 v_2^2$; $1{,}920{,}000 = \frac{1}{4}m_2(400)^2$;
$1{,}920{,}000 = 40{,}000m_2$; $m_2 = 48$, *Ans.*

36. directly, inversely

37. Let h = no. of hr, l = no. of ft in length of trench, m = no. of men;
$h = \frac{kl}{m}$; $h_1 = 3$, $l_1 = 150$, $m_1 = 12$, $l_2 = 200$, $m_2 = 8$; $h_1 = \frac{kl_1}{m_1}$; $3 = \frac{k \cdot 150}{12}$; $\frac{6}{25} = k$;
$h_2 = \frac{6}{25} \cdot \frac{l_2}{m_2}$; $h_2 = \frac{6}{25} \cdot \frac{200}{8}$; $h_2 = 6$ hr, *Ans.*

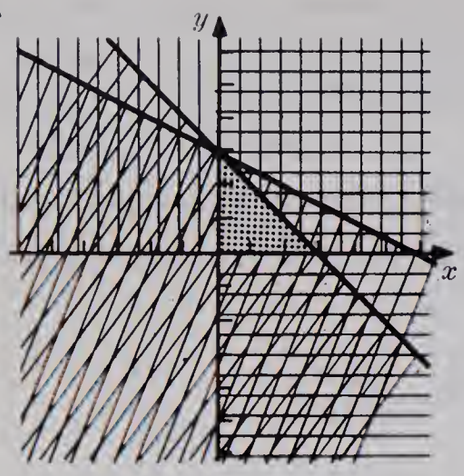
Page 462 · EXTRA FOR EXPERTS

1.



2. From the graph, the corner points are $(\frac{4}{3}, 4)$, $(6, 4)$, and $(2, 2)$.
$(\frac{4}{3}, 4)$ $2.5(\frac{4}{3}) + .8(4) = \frac{10}{3} + 3.2 = 6.5$
$(6, 4)$ $2.5(6) + .8(4) = 15 + 3.2 = 18.2$
$(2, 2)$ $2.5(2) + .8(2) = 5 + 1.6 = 6.6$
Maximum value at $(6, 4)$, minimum value at $(\frac{4}{3}, 4)$, *Ans.*

3.



The linear expression $R = 4x + y$ takes on its maximum value at a corner point. From the graph, the corner points are (3, 0), (0, 0) and (0, 3).

(3, 0) $R = 4(3) + 0 = 12$
(0, 0) $R = 4(0) + 0 = 0$
(0, 3) $R = 4(0) + 3 = 3$

The maximum value, 12, of R occurs at (3, 0), *Ans.*

4. Volume of shelf $= 18'' \times 10'' \times 10'' = 1800$ cu in.

Let $x =$ no. of boxes of Toanup displayed and $y =$ no. of boxes of Freshall displayed.

If $P =$ profit in cents, $P = 28x + 35y$. You are trying to maximize P within the limits of the following inequalities (constraints):

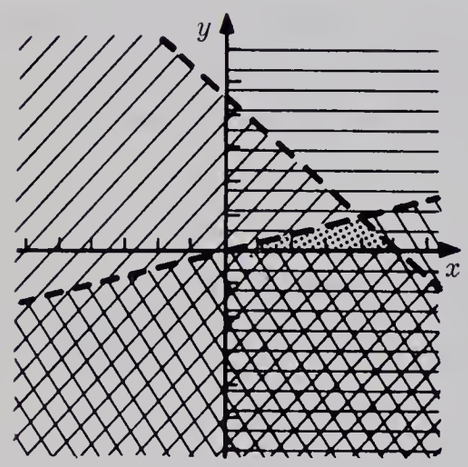
1. $35x + 40y \leq 1800$ The total volume must be less than or equal to the shelf space.
2. $x \geq 4y$ The number of boxes of Toanup sold will be at least 4 times the number of boxes of Freshall sold.
3. $x \geq 0$
4. $y \geq 0$

The druggist cannot sell a negative amount of either kind.

The maximum value of P lies at a corner point. From the graph, the corner points are (0, 0), (40, 10), and $(\frac{1800}{35}, 0)$.

(0, 0) $P = 28(0) + 35(0) = 0$
(40, 10) $P = 28(40) + 35(10) = 1120 + 350 = 1470$
$(\frac{1800}{35}, 0)$ $P = 28(\frac{1800}{35}) + 35(0) = 1440$

40 boxes of Toanup and 10 boxes of Freshall will realize the maximum profit of \$14.70.



5.

	Machine A	Machine B	Number Needed
Bolts	120	80	1000
Screws	50	80	750
Cost (\$)	1	1.20	

Let $x =$ the no. of hr Machine A runs and $y =$ the no. of hours Machine B runs. If $C =$ total cost in dollars, $C = 1x + 1.2y$. You are trying to minimize cost within the limits of the following inequalities (constraints):

1. $x + y \leq 15$ Combined running time cannot exceed 15.
2. $120x + 80y \geq 1000$
3. $50x + 80y \geq 750$

The total amount of each must equal at least the amount needed.

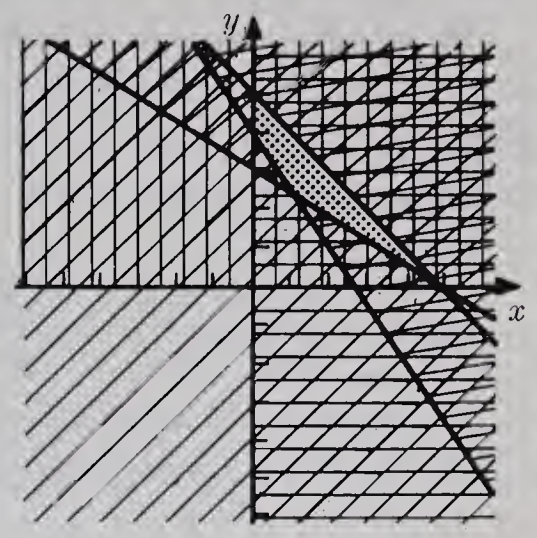
4. $x \geq 0$
5. $y \geq 0$

The machine cannot run a negative number of hr.

The minimum value of C lies at a corner point. From the graph, the corner points are $(0, 15)$, $(0, \frac{25}{2})$, $(\frac{25}{7}, \frac{50}{7})$, and $(15, 0)$.

$(0, 15)$ $\quad C = 0 + 1.2(15) = \$18.00$
$(0, \frac{25}{2})$ $\quad C = 0 + (1.2)(\frac{25}{2}) = \$15.00$
$(\frac{25}{7}, \frac{50}{7})$ $\quad C = \frac{25}{7} + (1.2)\frac{50}{7} = \frac{85}{7} = \$12.14$
$(15, 0)$ $\quad C = 15 + 1.2(0) = \$15.00$

The minimum cost is \$12.14; Machine A should run $\frac{25}{7}$ *or* $3\frac{4}{7}$ hr; Machine B should run $\frac{50}{7}$ *or* $7\frac{1}{7}$ hr, *Ans.*



CHAPTER 13. Quadratic Equations and Inequalities

Pages 466–467 · **WRITTEN EXERCISES**

A

1. $9x^2 = 25$; $x^2 = \frac{25}{9}$; $x = \pm\frac{5}{3}$. $\{\frac{5}{3}, -\frac{5}{3}\}$

2. $25x^2 = 9$; $x^2 = \frac{9}{25}$; $x = \pm\frac{3}{5}$. $\{\frac{3}{5}, -\frac{3}{5}\}$

3. $27 = y^2$; $y = \pm\sqrt{27} = \pm 3\sqrt{3}$. $\{3\sqrt{3}, -3\sqrt{3}\}$

4. $125 = y^2$; $y = \pm\sqrt{125} = \pm 5\sqrt{5}$. $\{5\sqrt{5}, -5\sqrt{5}\}$

5. $27m^2 = 3$; $m^2 = \frac{1}{9}$; $m = \pm\frac{1}{3}$. $\{\frac{1}{3}, -\frac{1}{3}\}$

6. $8m^2 = 2$; $m^2 = \frac{1}{4}$; $m = \pm\frac{1}{2}$. $\{\frac{1}{2}, -\frac{1}{2}\}$

7. $100v^2 = 1$; $v^2 = \frac{1}{100}$; $v = \pm\frac{1}{10}$. $\{\frac{1}{10}, -\frac{1}{10}\}$

8. $1 = 36v^2$; $v^2 = \frac{1}{36}$; $v = \pm\frac{1}{6}$. $\{\frac{1}{6}, -\frac{1}{6}\}$

9. $3w^2 = 16$; $w^2 = \frac{16}{3}$; $w = \pm\sqrt{\frac{16}{3}} = \pm\frac{4}{3}\sqrt{3}$. $\{\frac{4}{3}\sqrt{3}, -\frac{4}{3}\sqrt{3}\}$

10. $2w^2 = 121$; $w^2 = \frac{121}{2}$; $w = \pm\sqrt{\frac{121}{2}} = \pm\frac{11}{2}\sqrt{2}$. $\{\frac{11}{2}\sqrt{2}, -\frac{11}{2}\sqrt{2}\}$

11. $x - 1 = \pm\sqrt{4}$; $x - 1 = 2$ or $x - 1 = -2$; $x = 3$ or $x = -1$. $\{3, -1\}$

12. $x + 1 = \pm\sqrt{9}$; $x + 1 = 3$ or $x + 1 = -3$; $x = 2$ or $x = -4$. $\{2, -4\}$

13. $y + 2 = \pm\sqrt{1}$; $y + 2 = 1$ or $y + 2 = -1$; $y = -1$ or $y = -3$. $\{-1, -3\}$

14. $y - 2 = \pm\sqrt{1}$; $y - 2 = 1$ or $y - 2 = -1$; $y = 3$ or $y = 1$. $\{3, 1\}$

15. $(z + 3)^2 = \frac{25}{4}$; $z + 3 = \pm\sqrt{\frac{25}{4}}$; $z + 3 = \frac{5}{2}$ or $z + 3 = -\frac{5}{2}$; $z = -\frac{1}{2}$ or $z = -\frac{11}{2}$. $\{-\frac{1}{2}, -\frac{11}{2}\}$

16. $(z - 5)^2 = \frac{49}{9}$; $z - 5 = \pm\sqrt{\frac{49}{9}}$; $z - 5 = \frac{7}{3}$ or $z - 5 = -\frac{7}{3}$; $z = \frac{22}{3}$ or $z = \frac{8}{3}$. $\{\frac{22}{3}, \frac{8}{3}\}$

17. $r - \frac{1}{3} = \pm\sqrt{36}$; $r - \frac{1}{3} = 6$ or $r - \frac{1}{3} = -6$; $r = 6\frac{1}{3}$ or $r = -5\frac{2}{3}$. $\{\frac{19}{3}, -\frac{17}{3}\}$

18. $r + \frac{1}{7} = \pm\sqrt{25}$; $r + \frac{1}{7} = 5$ or $r + \frac{1}{7} = -5$; $r = 4\frac{6}{7}$ or $r = -5\frac{1}{7}$. $\{\frac{34}{7}, -\frac{36}{7}\}$

19. $s + \frac{3}{4} = \pm\sqrt{4}$; $s + \frac{3}{4} = 2$ or $s + \frac{3}{4} = -2$; $s = 1\frac{1}{4}$ or $s = -2\frac{3}{4}$. $\{\frac{5}{4}, -\frac{11}{4}\}$

20. $s - \frac{3}{5} = \pm\sqrt{25}$; $s - \frac{3}{5} = 5$ or $s - \frac{3}{5} = -5$; $s = 5\frac{3}{5}$ or $s = -4\frac{2}{5}$. $\{\frac{28}{5}, -\frac{22}{5}\}$

21. $(y + 5)^2 = 9$; $y + 5 = \pm\sqrt{9}$; $y + 5 = 3$ or $y + 5 = -3$; $y = -2$ or $y = -8$. $\{-2, -8\}$

22. $(y - 6)^2 = 4$; $y - 6 = \pm\sqrt{4}$; $y - 6 = 2$ or $y - 6 = -2$; $y = 8$ or $y = 4$. $\{8, 4\}$

23. $(t - \frac{1}{11})^2 = 1$; $t - \frac{1}{11} = \pm\sqrt{1}$; $t - \frac{1}{11} = 1$ or $t - \frac{1}{11} = -1$; $t = 1\frac{1}{11}$ or $t = -\frac{10}{11}$. $\{\frac{12}{11}, -\frac{10}{11}\}$

24. $(t + \frac{1}{9})^2 = 1$; $t + \frac{1}{9} = \pm\sqrt{1}$; $t + \frac{1}{9} = 1$ or $t + \frac{1}{9} = -1$; $t = \frac{8}{9}$ or $t = -1\frac{1}{9}$. $\{\frac{8}{9}, -\frac{10}{9}\}$

B

25. $(t - 6) = \pm\sqrt{5}$; $t - 6 = \sqrt{5}$ or $t - 6 = -\sqrt{5}$; $t = 6 + \sqrt{5}$ or $t = 6 - \sqrt{5}$. $\{6 + \sqrt{5}, 6 - \sqrt{5}\}$

26. $t + 6 = \pm\sqrt{7}$; $t + 6 = \sqrt{7}$ or $t + 6 = -\sqrt{7}$; $t = -6 + \sqrt{7}$ or $t = -6 - \sqrt{7}$. $\{-6 + \sqrt{7}, -6 - \sqrt{7}\}$

27. $k + 1 = \pm\sqrt{3}$; $k + 1 = \sqrt{3}$ or $k + 1 = -\sqrt{3}$; $k = -1 + \sqrt{3}$ or $k = -1 - \sqrt{3}$. $\{-1 + \sqrt{3}, -1 - \sqrt{3}\}$

28. $k - 1 = \pm\sqrt{2}$; $k - 1 = \sqrt{2}$ or $k - 1 = -\sqrt{2}$; $k = 1 + \sqrt{2}$ or $k = 1 - \sqrt{2}$. $\{1 + \sqrt{2}, 1 - \sqrt{2}\}$

29. $r(r^2 - 4) = 0$; $r = 0$ or $r^2 - 4 = 0$; $r = 0$ or $r^2 = 4$; $r = 0$ or $r = \pm 2$. $\{0, 2, -2\}$

30. $s(9 - s^2) = 0$; $s = 0$ or $9 - s^2 = 0$; $s = 0$ or $9 = s^2$; $s = 0$ or $s = \pm 3$. $\{0, 3, -3\}$

31. $z^3 - 25z = 0$; $z(z^2 - 25) = 0$; $z = 0$ or $z^2 - 25 = 0$; $z = 0$ or $z^2 = 25$; $z = 0$ or $z = \pm 5$. $\{0, 5, -5\}$

32. $16z - z^3 = 0$; $z(16 - z^2) = 0$; $z = 0$ or $16 - z^2 = 0$; $z = 0$ or $z^2 = 16$; $z = 0$ or $z = \pm 4$. $\{0, 4, -4\}$

33. $k(k^2 - 5) = 0$; $k = 0$ or $k^2 - 5 = 0$; $k = 0$ or $k^2 = 5$; $k = 0$ or $k = \pm\sqrt{5}$. $\{0, \sqrt{5}, -\sqrt{5}\}$

34. $k(k^2 - 7) = 0$; $k = 0$ or $k^2 - 7 = 0$; $k = 0$ or $k^2 = 7$; $k = 0$ or $k = \pm\sqrt{7}$. $\{0, \sqrt{7}, -\sqrt{7}\}$

35. $m^2(m^2 + 4m + 4) = 0$; $m^2(m + 2)^2 = 0$; $m^2 = 0$ or $(m + 2)^2 = 0$; $m = 0$ or $m + 2 = 0$; $m = 0$ or $m = -2$. $\{0, -2\}$

36. $m^2(m^2 - 6m + 9) = 0$; $m^2(m - 3)^2 = 0$; $m^2 = 0$ or $(m - 3)^2 = 0$; $m = 0$ or $m - 3 = 0$; $m = 0$ or $m = 3$. $\{0, 3\}$

C **37.** $(x - \sqrt{2})^2 = 0$; $x - \sqrt{2} = 0$; $x = \sqrt{2}$. $\{\sqrt{2}\}$

38. $(x + \sqrt{3})^2 = 0$; $x + \sqrt{3} = 0$; $x = -\sqrt{3}$. $\{-\sqrt{3}\}$

39. $(x - \sqrt{2})^2 = 9$; $x - \sqrt{2} = \pm\sqrt{9}$; $x - \sqrt{2} = +3$ or $x - \sqrt{2} = -3$; $x = 3 + \sqrt{2}$ or $x = -3 + \sqrt{2}$. $\{3 + \sqrt{2}, -3 + \sqrt{2}\}$

40. $(x - \sqrt{3})^2 = 4$; $x - \sqrt{3} = \pm\sqrt{4}$; $x - \sqrt{3} = +2$ or $x - \sqrt{3} = -2$; $x = 2 + \sqrt{3}$ or $x = -2 + \sqrt{3}$. $\{2 + \sqrt{3}, -2 + \sqrt{3}\}$

41. No; $x^2 = -7$ and negative numbers do not have square roots in the set of real nos.

42. No; $x^2 = -\frac{9}{25}$ and explanation of **41** applies.

Pages 468–469 · WRITTEN EXERCISES

A **1.** $t^2 - \frac{7}{2}t - 2 = 0$; $b = -\frac{7}{2}$, $c = -2$; $4 + (-\frac{1}{2}) = \frac{7}{2} = -b$; $4(-\frac{1}{2}) = -2 = c$; correct

2. $x^2 - \frac{1}{2}x - 3 = 0$; $b = -\frac{1}{2}$, $c = -3$; $2 + (-1\frac{1}{2}) = \frac{1}{2} = -b$; $2(-1\frac{1}{2}) = -3 = c$; correct

3. $x^2 - 2x - \frac{21}{4} = 0$; $b = -2$, $c = -\frac{21}{4}$; $\frac{7}{2} + (-\frac{3}{2}) = 2 = -b$; $\frac{7}{2}(-\frac{3}{2}) = -\frac{21}{4} = c$; correct

4. $x^2 - x - \frac{15}{4} = 0$; $b = -1$, $c = -\frac{15}{4}$; $\frac{3}{2} + (-\frac{5}{2}) = -1 \neq -b$; $\frac{3}{2}(-\frac{5}{2}) = -\frac{15}{4} = c$; incorrect

5. $u^2 - 18u - 7 = 0$; $b = -18$, $c = -7$; $9 + 2\sqrt{11} + 9 - 2\sqrt{11} = 18 = -b$; $(9 + 2\sqrt{11})(9 - 2\sqrt{11}) = 81 - 44 = 37$; incorrect

6. $v^2 - 20v - 25 = 0$; $b = -20$, $c = -25$; $10 + 5\sqrt{5} + 10 - 5\sqrt{5} = 20 = -b$; $(10 + 5\sqrt{5})(10 - 5\sqrt{5}) = 100 - 125 = -25 = c$; correct

7. $w^2 + 14w - 49 = 0$; $b = 14$, $c = -49$; $7 + (-7) = 0 \neq -b$; $7(-7) = -49 = c$; incorrect

8. $x^2 + 10x - 2 = 0$; $b = 10$, $c = -2$; $5 + 3\sqrt{3} + 5 - 3\sqrt{3} = 10 \neq -b$; $(5 + 3\sqrt{3})(5 - 3\sqrt{3}) = 25 - 27 = -2 = c$; incorrect

9. $x^2 + 6x - 11 = 0$; $b = 6$, $c = -11$; $-3 + 2\sqrt{5} - 3 - 2\sqrt{5} = -6 = -b$; $(-3 + 2\sqrt{5})(-3 - 2\sqrt{5}) = 9 - 20 = -11 = c$; correct

10. $x^2 + 12x - 9 = 0$; $b = 12$, $c = -9$; $6 + 9\sqrt{15} + 6 - 9\sqrt{15} = 12 \neq -b$; $(6 + 9\sqrt{15})(6 - 9\sqrt{15}) = 36 - 1215 = -1179 \neq c$; incorrect

11. $b = -1$, $c = -3$; $\dfrac{1 + \sqrt{13}}{2} + \dfrac{1 - \sqrt{13}}{2} = \dfrac{2}{2} = 1 = -b$;

$\left(\dfrac{1 + \sqrt{13}}{2}\right)\left(\dfrac{1 - \sqrt{13}}{2}\right) = \dfrac{1 - 13}{4} = -3 = c$; correct

12. $b = 1$, $c = -9$; $\dfrac{-1 + \sqrt{35}}{2} + \dfrac{-1 - \sqrt{35}}{2} = \dfrac{-2}{2} = -1 = -b$;

$\left(\dfrac{-1 + \sqrt{35}}{2}\right)\left(\dfrac{-1 - \sqrt{35}}{2}\right) = \dfrac{1 - 35}{4} = -\dfrac{17}{2} \neq c$; incorrect

B **13.** $5 + 2 = 7 = -b$; $b = -7$; $5(2) = 10 = c$; $x^2 - 7x + 10 = 0$

14. $-2 + 3 = 1 = -b$; $b = -1$; $-2(3) = -6 = c$; $x^2 - x - 6 = 0$

15. $-4 - 4 = -8 = -b$; $b = 8$; $-4(-4) = 16 = c$; $x^2 + 8x + 16 = 0$

16. $0 - 3 = -3 = -b$; $b = 3$; $0(-3) = 0 = c$; $x^2 + 3x = 0$

17. $\sqrt{7} - \sqrt{7} = 0 = -b$; $b = 0$; $\sqrt{7}\,(-\sqrt{7}) = -7 = c$; $x^2 - 7 = 0$

18. $-\sqrt{3} + \sqrt{3} = 0 = -b$; $b = 0$; $-\sqrt{3}\,(\sqrt{3}) = -3 = c$; $x^2 - 3 = 0$

19. $1 + \sqrt{2} + 1 - \sqrt{2} = 2 = -b$; $b = -2$; $(1 + \sqrt{2})(1 - \sqrt{2}) = 1 - 2 = -1 = c$; $x^2 - 2x - 1 = 0$

20. $3 - \sqrt{5} + 3 + \sqrt{5} = 6 = -b$; $b = -6$; $(3 - \sqrt{5})(3 + \sqrt{5}) = 9 - 5 = 4 = c$; $x^2 - 6x + 4 = 0$

21. $3\sqrt{5} + 3\sqrt{5} = 6\sqrt{5} = -b$; $b = -6\sqrt{5}$; $3\sqrt{5}\,(3\sqrt{5}) = 9 \cdot 5 = 45 = c$; $x^2 - 6\sqrt{5}\,x + 45 = 0$

22. $-2 - \frac{3}{2} = -\frac{7}{2} = -b$; $b = \frac{7}{2}$; $-2(-\frac{3}{2}) = 3 = c$; $x^2 + \frac{7}{2}x + 3 = 0$ *or* $2x^2 + 7x + 6 = 0$

Pages 471–472 · WRITTEN EXERCISES

A **1.** $x^2 + 2x + 1 = 7 + 1$; $(x + 1)^2 = 8$; $x + 1 = \pm\sqrt{8}$; $x + 1 = 2\sqrt{2}$ or $x + 1 = -2\sqrt{2}$; $x = -1 + 2\sqrt{2}$ or $x = -1 - 2\sqrt{2}$; $x \doteq -1 + 2(1.41) \doteq 1.8$ or $x \doteq -1 - 2(1.41) \doteq -3.8$. $\{-1 + 2\sqrt{2}, -1 - 2\sqrt{2}\}$; $\{1.8, -3.8\}$

2. $x^2 + 4x + 4 = 14 + 4$; $(x + 2)^2 = 18$; $x + 2 = \pm\sqrt{18}$; $x + 2 = 3\sqrt{2}$ or $x + 2 = -3\sqrt{2}$; $x = -2 + 3\sqrt{2}$ or $x = -2 - 3\sqrt{2}$; $x \doteq -2 + 3(1.41) \doteq 2.2$ or $x \doteq -2 - 3(1.41) \doteq -6.2$. $\{-2 + 3\sqrt{2}, -2 - 3\sqrt{2}\}$; $\{2.2, -6.2\}$

3. $y^2 + y + \frac{1}{4} = 6 + \frac{1}{4}$; $(y + \frac{1}{2})^2 = \frac{25}{4}$; $y + \frac{1}{2} = \pm\sqrt{\frac{25}{4}}$; $y + \frac{1}{2} = \frac{5}{2}$ or $y + \frac{1}{2} = -\frac{5}{2}$; $y = 2$ or $y = -3$. $\{2, -3\}$

4. $y^2 - 4y + 4 = -3 + 4$; $(y - 2)^2 = 1$; $y - 2 = \pm\sqrt{1}$; $y - 2 = 1$ or $y - 2 = -1$; $y = 3$ or $y = 1$. $\{3, 1\}$

5. $m^2 - 8m + 16 = -2 + 16$; $(m - 4)^2 = 14$; $m - 4 = \pm\sqrt{14}$; $m - 4 = \sqrt{14}$ or $m - 4 = -\sqrt{14}$; $m = 4 + \sqrt{14}$ or $m = 4 - \sqrt{14}$; $m \doteq 4 + 3.74 \doteq 7.7$ or $m \doteq 4 - 3.74 \doteq .3$. $\{4 + \sqrt{14}, 4 - \sqrt{14}\}$; $\{7.7, .3\}$

6. $n^2 + 6n + 9 = -4 + 9$; $(n + 3)^2 = 5$; $n + 3 = \pm\sqrt{5}$; $n + 3 = \sqrt{5}$ or $n + 3 = -\sqrt{5}$; $n = -3 + \sqrt{5}$ or $n = -3 - \sqrt{5}$; $n \doteq -3 + 2.23 \doteq -.8$ or $n \doteq -3 - 2.23 \doteq -5.2$. $\{-3 + \sqrt{5}, -3 - \sqrt{5}\}$; $\{-.8, -5.2\}$

7. $a^2 + 7a + \frac{49}{4} = -5 + \frac{49}{4}$; $(a + \frac{7}{2})^2 = \frac{29}{4}$; $a + \frac{7}{2} = \pm\sqrt{\frac{29}{4}}$; $a + \frac{7}{2} = \frac{\sqrt{29}}{2}$ or $a + \frac{7}{2} = -\frac{\sqrt{29}}{2}$; $a = \frac{-7 + \sqrt{29}}{2}$ or $a = \frac{-7 - \sqrt{29}}{2}$; $a \doteq \frac{-7 + 5.39}{2} \doteq -.8$ or $a \doteq \frac{-7 - 5.39}{2} \doteq -6.2$; $\left\{\frac{-7 + \sqrt{29}}{2}, \frac{-7 - \sqrt{29}}{2}\right\} \cdot \{-.8, -6.2\}$

8. $b^2 - 5b + \frac{25}{4} = -3 + \frac{25}{4}$; $(b - \frac{5}{2})^2 = \frac{13}{4}$; $b - \frac{5}{2} = \pm\sqrt{\frac{13}{4}}$; $b - \frac{5}{2} = \frac{\sqrt{13}}{2}$ or $b - \frac{5}{2} = -\frac{\sqrt{13}}{2}$; $b = \frac{5 + \sqrt{13}}{2}$ or $b = \frac{5 - \sqrt{13}}{2}$; $b \doteq \frac{5 + 3.61}{2} \doteq 4.3$ or $b \doteq \frac{5 - 3.61}{2} \doteq .7$. $\left\{\frac{5 + \sqrt{13}}{2}, \frac{5 - \sqrt{13}}{2}\right\}$; $\{4.3, .7\}$

9. $m^2 - 3m + \frac{9}{4} = 0 + \frac{9}{4}$; $(m - \frac{3}{2})^2 = \frac{9}{4}$; $m - \frac{3}{2} = \pm\sqrt{\frac{9}{4}}$; $m - \frac{3}{2} = \frac{3}{2}$ or $m - \frac{3}{2} = -\frac{3}{2}$; $m = 3$ or $m = 0$. $\{3, 0\}$

10. $m^2 + 2m + 1 = 0 + 1$; $(m + 1)^2 = 1$; $m + 1 = \pm\sqrt{1}$; $m + 1 = 1$ or $m + 1 = -1$; $m = 0$ or $m = -2$. $\{0, -2\}$

11. $v^2 - 20v + 100 = -19 + 100$; $(v - 10)^2 = 81$; $v - 10 = \pm\sqrt{81}$; $v - 10 = 9$ or $v - 10 = -9$; $v = 19$ or $v = 1$. $\{19, 1\}$

12. $x^2 - 24x + 144 = -23 + 144$; $(x - 12)^2 = 121$; $x - 12 = \pm\sqrt{121}$; $x - 12 = 11$ or $x - 12 = -11$; $x = 23$ or $x = 1$. $\{23, 1\}$

13. $r^2 - 5r = 5$; $r^2 - 5r + \frac{25}{4} = 5 + \frac{25}{4}$; $(r - \frac{5}{2})^2 = \frac{45}{4}$; $r - \frac{5}{2} = \pm\sqrt{\frac{45}{4}}$; $r - \frac{5}{2} = \frac{3\sqrt{5}}{2}$ or $r - \frac{5}{2} = -\frac{3\sqrt{5}}{2}$; $r = \frac{5 + 3\sqrt{5}}{2}$ or $r = \frac{5 - 3\sqrt{5}}{2}$; $r \doteq \frac{5 + 3(2.24)}{2} \doteq 5.9$ or $r \doteq \frac{5 - 3(2.24)}{2} \doteq -.9$. $\left\{\frac{5 + 3\sqrt{5}}{2}, \frac{5 - 3\sqrt{5}}{2}\right\}$; $\{5.9, -.9\}$

14. $p^2 + 3p = 27$; $p^2 + 3p + \frac{9}{4} = 27 + \frac{9}{4}$; $(p + \frac{3}{2})^2 = \frac{117}{4}$; $p + \frac{3}{2} = \pm\sqrt{\frac{117}{4}}$; $p + \frac{3}{2} = \frac{3\sqrt{13}}{2}$ or $p + \frac{3}{2} = -\frac{3\sqrt{13}}{2}$; $p = \frac{-3 + 3\sqrt{13}}{2}$ or $p = \frac{-3 - 3\sqrt{13}}{2}$; $p \doteq \frac{-3 + 3(3.61)}{2} \doteq 3.9$ or $p \doteq \frac{-3 - 3(3.61)}{2} \doteq -6.9$. $\left\{\frac{-3 + 3\sqrt{13}}{2}, \frac{-3 - 3\sqrt{13}}{2}\right\}$; $\{3.9, -6.9\}$

15. $y^2 + 3y = -\frac{9}{4}$; $y^2 + 3y + \frac{9}{4} = -\frac{9}{4} + \frac{9}{4}$; $(y + \frac{3}{2})^2 = 0$; $y + \frac{3}{2} = 0$; $y = -\frac{3}{2}$. $\{-\frac{3}{2}\}$

16. $y^2 - \frac{2}{3}y = -\frac{1}{9}$; $y^2 - \frac{2}{3}y + \frac{1}{9} = -\frac{1}{9} + \frac{1}{9}$; $(y - \frac{1}{3})^2 = 0$; $y - \frac{1}{3} = 0$; $y = \frac{1}{3}$. $\{\frac{1}{3}\}$

17. $n^2 + \frac{7}{3}n = \frac{1}{3}$; $n^2 + \frac{7}{3}n + \frac{49}{36} = \frac{1}{3} + \frac{49}{36}$; $(n + \frac{7}{6})^2 = \frac{61}{36}$; $n + \frac{7}{6} = \pm\sqrt{\frac{61}{36}}$; $n + \frac{7}{6} = \frac{\sqrt{61}}{6}$ or $n + \frac{7}{6} = -\frac{\sqrt{61}}{6}$; $n = \frac{-7 + \sqrt{61}}{6}$ or $n = \frac{-7 - \sqrt{61}}{6}$; $n \doteq \frac{-7 + 7.81}{6} \doteq .1$ or $n \doteq \frac{-7 - 7.81}{6} \doteq -2.5$. $\left\{\frac{-7 + \sqrt{61}}{6}, \frac{-7 - \sqrt{61}}{6}\right\}$; $\{.1, -2.5\}$

18. $n^2 - \frac{8}{5}n = \frac{1}{5}$; $n^2 - \frac{8}{5}n + \frac{16}{25} = \frac{1}{5} + \frac{16}{25}$; $(n - \frac{4}{5})^2 = \frac{21}{25}$; $n - \frac{4}{5} = \pm\sqrt{\frac{21}{25}}$; $n - \frac{4}{5} = \frac{\sqrt{21}}{5}$ or $n - \frac{4}{5} = -\frac{\sqrt{21}}{5}$; $n = \frac{4 + \sqrt{21}}{5}$ or $n = \frac{4 - \sqrt{21}}{5}$; $n \doteq \frac{4 + 4.58}{5} \doteq 1.7$ or $n \doteq \frac{4 - 4.58}{5} \doteq -.1$. $\left\{\frac{4 + \sqrt{21}}{5}, \frac{4 - \sqrt{21}}{5}\right\}$; $\{1.7, -.1\}$

B **19.** $s^2 + \frac{2}{3}s + \frac{1}{9} = 0 + \frac{1}{9}$; $(s + \frac{1}{3})^2 = \frac{1}{9}$; $s + \frac{1}{3} = \pm\sqrt{\frac{1}{9}}$; $s + \frac{1}{3} = \frac{1}{3}$ or $s + \frac{1}{3} = -\frac{1}{3}$; $s = 0$ or $s = -\frac{2}{3}$. $\{0, -\frac{2}{3}\}$

20. $t^2 - \frac{3}{2}t + \frac{9}{16} = 0 + \frac{9}{16}$; $(t - \frac{3}{4})^2 = \frac{9}{16}$; $t - \frac{3}{4} = \pm\sqrt{\frac{9}{16}}$; $t - \frac{3}{4} = \frac{3}{4}$ or $t - \frac{3}{4} = -\frac{3}{4}$; $t = \frac{6}{4} = 1\frac{1}{2}$ or $t = 0$. $\{1\frac{1}{2}, 0\}$

21. $z^2 + \frac{5z}{2} + \frac{25}{16} = 25 + \frac{25}{16}$; $(z + \frac{5}{4})^2 = \frac{425}{16}$; $z + \frac{5}{4} = \pm\sqrt{\frac{425}{16}}$; $z + \frac{5}{4} = \frac{5\sqrt{17}}{4}$ or $z + \frac{5}{4} = -\frac{5\sqrt{17}}{4}$; $z = \frac{-5 + 5\sqrt{17}}{4}$ or $z = \frac{-5 - 5\sqrt{17}}{4}$; $z \doteq \frac{-5 + 5(4.12)}{4} \doteq 3.9$ or $z \doteq \frac{-5 - 5(4.12)}{4} \doteq -6.4$. $\left\{\frac{-5 + 5\sqrt{17}}{4}, \frac{-5 - 5\sqrt{17}}{4}\right\}$; $\{3.9, -6.4\}$

22. $w^2 + \frac{2w}{5} + \frac{1}{25} = 3 + \frac{1}{25}$; $(w + \frac{1}{5})^2 = \frac{76}{25}$; $w + \frac{1}{5} = \pm\sqrt{\frac{76}{25}}$; $w + \frac{1}{5} = \frac{2\sqrt{19}}{5}$ or $w + \frac{1}{5} = -\frac{2\sqrt{19}}{5}$; $w = \frac{-1 + 2\sqrt{19}}{5}$ or $w = \frac{-1 - 2\sqrt{19}}{5}$; $w \doteq \frac{-1 + 2(4.36)}{5} \doteq 1.5$ or $w \doteq \frac{-1 - 2(4.36)}{5} \doteq -1.9$. $\left\{\frac{-1 + 2\sqrt{19}}{5}, \frac{-1 - 2\sqrt{19}}{5}\right\}$; $\{1.5, -1.9\}$

23. $y - 2 + y = 2y(y - 2)$; $2y - 2 = 2y^2 - 4y$; $2y^2 - 6y = -2$; $y^2 - 3y = -1$; $y^2 - 3y + \frac{9}{4} = -1 + \frac{9}{4}$; $(y - \frac{3}{2})^2 = \frac{5}{4}$; $y - \frac{3}{2} = \pm\sqrt{\frac{5}{4}}$; $y - \frac{3}{2} = \frac{\sqrt{5}}{2}$ or $y - \frac{3}{2} = -\frac{\sqrt{5}}{2}$; $y = \frac{3 + \sqrt{5}}{2}$ or $y = \frac{3 - \sqrt{5}}{2}$; $y \doteq \frac{3 + 2.24}{2} \doteq 2.6$ or $y = \frac{3 - 2.24}{2} \doteq .4$. $\left\{\frac{3 + \sqrt{5}}{2}, \frac{3 - \sqrt{5}}{2}\right\}$; $\{2.6, .4\}$

24. $x + x + 1 = x(x + 1)$; $2x + 1 = x^2 + x$; $x^2 - x = 1$; $x^2 - x + \frac{1}{4} = 1 + \frac{1}{4}$; $(x - \frac{1}{2})^2 = \frac{5}{4}$; $x - \frac{1}{2} = \pm\sqrt{\frac{5}{4}}$; $x - \frac{1}{2} = \frac{\sqrt{5}}{2}$ or $x - \frac{1}{2} = -\frac{\sqrt{5}}{2}$; $x = \frac{1 + \sqrt{5}}{2}$ or $x = \frac{1 - \sqrt{5}}{2}$; $x \doteq \frac{1 + 2.24}{2} \doteq 1.6$ or $x \doteq \frac{1 - 2.24}{2} \doteq -.6$. $\left\{\frac{1 + \sqrt{5}}{2}, \frac{1 - \sqrt{5}}{2}\right\}$; $\{1.6, -.6\}$

25. $3(s - 1) + 1(s + 1) = 2(s^2 - 1)$; $3s - 3 + s + 1 = 2s^2 - 2$; $2s^2 - 4s = 0$; $s^2 - 2s = 0$; $s^2 - 2s + 1 = 0 + 1$; $(s - 1)^2 = 1$; $s - 1 = \pm\sqrt{1}$; $s - 1 = 1$ or $s - 1 = -1$; $s = 2$ or $s = 0$. $\{2, 0\}$

26. $2(t - 1) - 1(t + 1) = 3(t^2 - 1)$; $2t - 2 - t - 1 = 3t^2 - 3$; $3t^2 - t = 0$; $t^2 - \frac{1}{3}t = 0$; $t^2 - \frac{1}{3}t + \frac{1}{36} = 0 + \frac{1}{36}$; $(t - \frac{1}{6})^2 = \frac{1}{36}$; $t - \frac{1}{6} = \pm\sqrt{\frac{1}{36}}$; $t - \frac{1}{6} = \frac{1}{6}$ or $t - \frac{1}{6} = -\frac{1}{6}$; $t = \frac{1}{3}$ or $t = 0$. $\{\frac{1}{3}, 0\}$

27. $2m(m - 3) - m(m + 3) + 9 = 0$; $2m^2 - 6m - m^2 - 3m + 9 = 0$; $m^2 - 9m = -9$; $m^2 - 9m + \frac{81}{4} = -9 + \frac{81}{4}$; $(m - \frac{9}{2})^2 = \frac{45}{4}$; $m - \frac{9}{2} = \pm\sqrt{\frac{45}{4}}$; $m - \frac{9}{2} = \frac{3\sqrt{5}}{2}$ or $m - \frac{9}{2} = -\frac{3\sqrt{5}}{2}$; $m = \frac{9 + 3\sqrt{5}}{2}$ or $m = \frac{9 - 3\sqrt{5}}{2}$; $m \doteq \frac{9 + 3(2.24)}{2} \doteq 7.9$ or $m \doteq \frac{9 - 3(2.24)}{2} \doteq 1.1$. $\left\{\frac{9 + 3\sqrt{5}}{2}, \frac{9 - 3\sqrt{5}}{2}\right\}$; $\{7.9, 1.1\}$

28. $\frac{n}{n + 2} + \frac{2n}{n - 2} = \frac{3}{n^2 - 4}$; $n(n - 2) + 2n(n + 2) = 3$; $n^2 - 2n + 2n^2 + 4n = 3$; $3n^2 + 2n = 3$; $n^2 + \frac{2}{3}n = 1$; $n^2 + \frac{2}{3}n + \frac{1}{9} = 1 + \frac{1}{9}$; $(n + \frac{1}{3})^2 = \frac{10}{9}$; $n + \frac{1}{3} = \pm\sqrt{\frac{10}{9}}$; $n + \frac{1}{3} = \frac{\sqrt{10}}{3}$ or $n + \frac{1}{3} = -\frac{\sqrt{10}}{3}$; $n = \frac{-1 + \sqrt{10}}{3}$ or $n = \frac{-1 - \sqrt{10}}{3}$; $n \doteq \frac{-1 + 3.16}{3} \doteq .7$ or $n \doteq \frac{-1 - 3.16}{3} \doteq -1.4$. $\left\{\frac{-1 + \sqrt{10}}{3}, \frac{-1 - \sqrt{10}}{3}\right\}$; $\{.7, -1.4\}$

Pages 472–473 · **PROBLEMS**

A

1. $x(x+2) = 224$; $x^2 + 2x = 224$; $x^2 + 2x - 224 = 0$; $(x+16)(x-14) = 0$; $x + 16 = 0$ or $x - 14 = 0$; $x = -16$ (reject); $x = 14$ in, $x + 2 = 16$ in, *Ans.*

2. $w(w+10) = 119$; $w^2 + 10w = 119$; $w^2 + 10w - 119 = 0$; $(w+17)(w-7) = 0$; $w + 17 = 0$ or $w - 7 = 0$; $w = -17$ (reject); $w = 7$ ft, $w + 10 = 17$ ft, *Ans.*

3. Let x = length of first room; $x - 3$ = its width; $x(x-3)$ = its area. Also, $x + 2$ = width of second room; $2x$ = its length, $2x(x+2)$ = its area. $x(x-3) + 2x(x+2) = 690$; $x^2 - 3x + 2x^2 + 4x = 690$; $3x^2 + x - 690 = 0$; $(3x+46)(x-15) = 0$; $3x + 46 = 0$ or $x - 15 = 0$; $x = -\frac{46}{3}$ (reject); $x = 15$ ft, $x - 3 = 12$ ft, $x + 2 = 17$ ft, $2x = 30$ ft, *Ans.*

4. $s^2 + (s+5)^2 = 325$; $s^2 + s^2 + 10s + 25 = 325$; $2s^2 + 10s - 300 = 0$; $s^2 + 5s - 150 = 0$; $(s+15)(s-10) = 0$; $s + 15 = 0$ or $s - 10 = 0$; $s = -15$ (reject); $s = 10$ in, $s + 5 = 15$ in, *Ans.*

5. $c^2 = a^2 + b^2$; $c = 50$; a = distance of one bus; b = distance of other bus; $d = rt$; $t = 1$ hr; $a = r$; $b = r + 10$; $50^2 = r^2 + (r+10)^2$; $2500 = r^2 + r^2 + 20r + 100$; $2r^2 + 20r = 2400$; $r^2 + 10r - 1200 = 0$; $(r+40)(r-30) = 0$; $r + 40 = 0$ or $r - 30 = 0$; $r = -40$ (reject); $r = 30$ mph, $r + 10 = 40$ mph, *Ans.*

6. Let n = number contributing; then $\frac{300}{n}$ = amount each gave; $(n+5)\left(\frac{300}{n} - 2\right) = 300$; $\frac{(n+5)(300-2n)}{n} = 300$; $300n + 1500 - 2n^2 - 10n = 300n$; $1500 - 2n^2 - 10n = 0$; $n^2 + 5n - 750 = 0$; $(n+30)(n-25) = 0$; $n + 30 = 0$ or $n - 25 = 0$; $n = -30$ (reject); $n = 25$; $\frac{300}{25} = 12¢$, *Ans.*

7. Let n = original number; then $\frac{120}{n}$ = original share; $(n+2)\left(\frac{120}{n} - 2\right) = 120$; $(n+2)(120-2n) = 120n$; $120n + 240 - 2n^2 - 4n = 120n$; $240 - 2n^2 - 4n = 0$; $n^2 + 2n - 120 = 0$; $(n+12)(n-10) = 0$; $n + 12 = 0$ or $n - 10 = 0$; $n = -12$ (reject); $n = 10$; $\frac{120}{10} = 12$; $12 - 2 = \$10$, *Ans.*

8. Let s = rate in still air; $d = rt$; $\frac{600}{s-30} - \frac{600}{s+30} = \frac{1}{2}$; $1200(s+30) - 1200(s-30) = (s-30)(s+30)$; $1200s + 36{,}000 - 1200s + 36{,}000 = s^2 - 900$; $s^2 - 900 = 72{,}000$; $s^2 = 72{,}900$; $s = \sqrt{72{,}900} = 270$ mph, *Ans.*

9. Let n = number bought; then $\frac{4000}{n}$ = average cost; $(n-1)\left(\frac{4000}{n} + 200\right) = 4000$; $(n-1)(4000 + 200n) = 4000n$; $4000n - 4000 + 200n^2 - 200n = 4000n$; $n^2 - n - 20 = 0$; $(n-5)(n+4) = 0$; $n - 5 = 0$ or $n + 4 = 0$; $n = -4$ (reject); $n = 5$, *Ans.*

10. Let n = no. bought; $\frac{1040}{n}$ = average cost; $(n-4)\left(\frac{1040}{n} + 60\right) = 1040 + 220$; $(n-4)(1040 + 60n) = 1260n$; $1040n - 4160 + 60n^2 - 240n = 1260n$; $60n^2 - 460n - 4160 = 0$; $3n^2 - 23n - 208 = 0$; $(3n+16)(n-13) = 0$; $3n + 16 = 0$ or $n - 13 = 0$; $n = -\frac{16}{3}$ (reject); $n = 13$; $n - 4 = 9$, *Ans.*

Page 475 · **WRITTEN EXERCISES**

A

1. $a = 3$, $b = 5$, $c = 1$; $x = \frac{-5 \pm \sqrt{5^2 - 4(3)(1)}}{2(3)} = \frac{-5 \pm \sqrt{13}}{6}$; $x \doteq \frac{-5 + 3.61}{6} \doteq -.2$ or $x \doteq \frac{-5 - 3.61}{6} \doteq -1.4$. $\left\{\frac{-5+\sqrt{13}}{6}, \frac{-5-\sqrt{13}}{6}\right\}$; $\{-.2, -1.4\}$

2. $a = 4$, $b = 7$, $c = 2$; $x = \frac{-7 \pm \sqrt{7^2 - 4(4)(2)}}{2(4)} = \frac{-7 \pm \sqrt{17}}{8}$; $x \doteq \frac{-7 + 4.12}{8} \doteq -.4$ or $x \doteq \frac{-7 - 4.12}{8} \doteq -1.4$. $\left\{\frac{-7+\sqrt{17}}{8}, \frac{-7-\sqrt{17}}{8}\right\}$; $\{-.4, -1.4\}$

3. $a = 2$, $b = -8$, $c = 3$; $x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(3)}}{2(2)} = \frac{8 \pm \sqrt{40}}{4} = \frac{8 \pm 2\sqrt{10}}{4} = \frac{4 \pm \sqrt{10}}{2}$; $x \doteq \frac{4 + 3.16}{2} \doteq 3.6$ or $x \doteq \frac{4 - 3.16}{2} \doteq .4$. $\left\{\frac{4+\sqrt{10}}{2}, \frac{4-\sqrt{10}}{2}\right\}$; $\{3.6, .4\}$

4. $a = 4,\ b = -6,\ c = 1;\ x = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(4)(1)}}{2(4)} = \dfrac{6 \pm \sqrt{20}}{8} = \dfrac{6 \pm 2\sqrt{5}}{8} = \dfrac{3 \pm \sqrt{5}}{4}$;
$x \doteq \dfrac{3 + 2.24}{4} \doteq 1.3$ or $x \doteq \dfrac{3 - 2.24}{4} \doteq .2$. $\left\{\dfrac{3 + \sqrt{5}}{4}, \dfrac{3 - \sqrt{5}}{4}\right\}$; $\{1.3, .2\}$

5. $a = 2,\ b = -5,\ c = -12;\ x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-12)}}{2(2)} = \dfrac{5 \pm \sqrt{121}}{4} = \dfrac{5 \pm 11}{4}$;
$x = \frac{16}{4} = 4$ or $x = \dfrac{-6}{4} = -\dfrac{3}{2}$. $\{4, -\frac{3}{2}\}$

6. $a = 6,\ b = 1,\ c = -35;\ x = \dfrac{-1 \pm \sqrt{1^2 - 4(6)(-35)}}{2(6)} = \dfrac{-1 \pm \sqrt{841}}{12} = \dfrac{-1 \pm 29}{12}$; $x = \frac{28}{12} = \frac{7}{3}$
or $x = \dfrac{-30}{12} = -\dfrac{5}{2}$. $\{\frac{7}{3}, -\frac{5}{2}\}$

7. $x^2 + 4x - 3 = 0;\ a = 1,\ b = 4,\ c = -3;$
$x = \dfrac{-4 \pm \sqrt{4^2 - 4(1)(-3)}}{2(1)} = \dfrac{-4 \pm \sqrt{28}}{2} = \dfrac{-4 \pm 2\sqrt{7}}{2} = -2 \pm \sqrt{7};$
$x \doteq -2 + 2.65 \doteq .7$ or $x \doteq -2 - 2.65 \doteq -4.7$. $\{-2 + \sqrt{7}, -2 - \sqrt{7}\}$; $\{.7, -4.7\}$

8. $x^2 + 6x - 4 = 0;\ a = 1,\ b = 6,\ c = -4;$
$x = \dfrac{-6 \pm \sqrt{6^2 - 4(1)(-4)}}{2(1)} = \dfrac{-6 \pm \sqrt{52}}{2} = \dfrac{-6 \pm 2\sqrt{13}}{2} = -3 \pm \sqrt{13};$
$x \doteq -3 + 3.61 = .6$ or $x \doteq -3 - 3.61 \doteq -6.6$. $\{-3 + \sqrt{13}, -3 - \sqrt{13}\}$; $\{.6, -6.6\}$

9. $x^2 - 2x - 1 = 0;\ a = 1,\ b = -2,\ c = -1;$
$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)} = \dfrac{2 \pm \sqrt{8}}{2} = \dfrac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2};$
$x \doteq 1 + 1.41 \doteq 2.4$ or $x \doteq 1 - 1.41 \doteq -.4$. $\{1 + \sqrt{2}, 1 - \sqrt{2}\}$; $\{2.4, -.4\}$

10. $x^2 + x - 11 = 0;\ a = 1,\ b = 1,\ c = -11;$
$x = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(-11)}}{2(1)} = \dfrac{-1 \pm \sqrt{45}}{2} = \dfrac{-1 \pm 3\sqrt{5}}{2}$; $x \doteq \dfrac{-1 + 3(2.24)}{2} \doteq 2.9$ or
$x \doteq \dfrac{-1 - 3(2.24)}{2} \doteq -3.9$. $\left\{\dfrac{-1 + 3\sqrt{5}}{2}, \dfrac{-1 - 3\sqrt{5}}{2}\right\}$; $\{2.9, -3.9\}$

11. $20x^2 - 17x + 3 = 0;\ a = 20,\ b = -17,\ c = 3;$
$x = \dfrac{-(-17) \pm \sqrt{(-17)^2 - 4(20)(3)}}{2(20)} = \dfrac{17 \pm \sqrt{49}}{40} = \dfrac{17 \pm 7}{40}$; $x = \frac{24}{40} = \frac{3}{5}$ or $x = \frac{10}{40} = \frac{1}{4}$. $\{\frac{3}{5}, \frac{1}{4}\}$

12. $10x^2 - 17x + 3 = 0;\ a = 10,\ b = -17,\ c = 3;$
$x = \dfrac{-(-17) \pm \sqrt{(-17)^2 - 4(10)(3)}}{2(10)} = \dfrac{17 \pm \sqrt{169}}{20} = \dfrac{17 \pm 13}{20}$; $x = \frac{30}{20} = \frac{3}{2}$ or $x = \frac{4}{20} = \frac{1}{5}$. $\{\frac{3}{2}, \frac{1}{5}\}$

13. $a = 3,\ b = -1,\ c = 0;\ x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(0)}}{2(3)} = \dfrac{1 \pm \sqrt{1}}{6} = \dfrac{1 \pm 1}{6}$;
$x = \frac{2}{6} = \frac{1}{3}$ or $x = \frac{0}{6} = 0$. $\{\frac{1}{3}, 0\}$

14. $a = 5,\ b = 0,\ c = -17;\ x = \dfrac{-0 \pm \sqrt{0^2 - 4(5)(-17)}}{2(5)} = \dfrac{\pm\sqrt{340}}{10} = \dfrac{\pm 2\sqrt{85}}{10} = \dfrac{\pm\sqrt{85}}{5}$;
$x \doteq \dfrac{\pm 9.22}{5} \doteq \pm 1.8$. $\{\frac{1}{5}\sqrt{85}, -\frac{1}{5}\sqrt{85}\}$; $\{1.8, -1.8\}$

B **15.** $x^2 - 2x - 2 = 0;\ a = 1,\ b = -2,\ c = -2;$
$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-2)}}{2(1)} = \dfrac{2 \pm \sqrt{12}}{2} = \dfrac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3};$
factors are $(x - (1 + \sqrt{3}))(x - (1 - \sqrt{3})) = (x - 1 - \sqrt{3})(x - 1 + \sqrt{3})$

16. $x^2 - 2x - 4 = 0;\ a = 1,\ b = -2,\ c = -4;$
$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)} = \dfrac{2 \pm \sqrt{20}}{2} = \dfrac{2 \pm 2\sqrt{5}}{2} = 1 \pm \sqrt{5};$
factors are $(x - (1 + \sqrt{5}))(x - (1 - \sqrt{5})) = (x - 1 - \sqrt{5})(x - 1 + \sqrt{5})$

17. $y^2 + 6y + 3 = 0$; $a = 1, b = 6, c = 3$;
$y = \dfrac{-6 \pm \sqrt{6^2 - 4(1)(3)}}{2(1)} = \dfrac{-6 \pm \sqrt{24}}{2} = \dfrac{-6 \pm 2\sqrt{6}}{2} = -3 \pm \sqrt{6}$;
factors are $(y - (-3 + \sqrt{6}))(y - (-3 - \sqrt{6})) = (y + 3 - \sqrt{6})(y + 3 + \sqrt{6})$

18. $y^2 + 8y + 13 = 0$; $a = 1, b = 8, c = 13$;
$y = \dfrac{-8 \pm \sqrt{8^2 - 4(1)(13)}}{2(1)} = \dfrac{-8 \pm \sqrt{12}}{2} = \dfrac{-8 \pm 2\sqrt{3}}{2} = -4 \pm \sqrt{3}$;
factors are $(y - (-4 + \sqrt{3}))(y - (-4 - \sqrt{3})) = (y + 4 - \sqrt{3})(y + 4 + \sqrt{3})$

19. $a = 1, b = -2, c = 2$; $x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} = \dfrac{2 \pm \sqrt{-4}}{2}$;
negative nos. have no square roots in the set of real nos.

20. $a = 1, b = -2, c = 4$; $x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)} = \dfrac{2 \pm \sqrt{-12}}{2}$; see explanation in **19**.

21. $y = (y + 1)^2$; $y = y^2 + 2y + 1$; $y^2 + y + 1 = 0$; $a = 1, b = 1, c = 1$;
$y = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)} = \dfrac{-1 \pm \sqrt{-3}}{2}$; see explanation in **19**.

22. $4y = (y + 3)^2$; $4y = y^2 + 6y + 9$; $y^2 + 2y + 9 = 0$; $a = 1, b = 2, c = 9$;
$y = \dfrac{-2 \pm \sqrt{2^2 - 4(1)(9)}}{2(1)} = \dfrac{-2 \pm \sqrt{-32}}{2}$; see explanation in **19**.

Pages 475–476 · PROBLEMS

A

1. $3x^2 = 147$; $x^2 = 49$; $x = -7$ (reject); $x = 7$. Square 7 ft × 7 ft, rectangle, 7 ft × 14 ft, *Ans.*

2. $6(3x)(x) = 72$; $18x^2 = 72$; $x^2 = 4$; $x = -2$ (reject); $x = 2$ in, $3x = 6$ in, *Ans.*

3. Let s = length of side of table; $(s - 4)^2 = \frac{2}{3}s^2$; $s^2 - 8s + 16 = \frac{2}{3}s^2$; $3s^2 - 24s + 48 = 2s^2$;
$s^2 - 24s + 48 = 0$; $s = \dfrac{-(-24) \pm \sqrt{(-24)^2 - 4(1)(48)}}{2(1)} = \dfrac{24 \pm \sqrt{384}}{2} = \dfrac{24 \pm 8\sqrt{6}}{2} = 12 \pm 4\sqrt{6}$;
$s \doteq 12 - 4(2.45) \doteq 2.2$ (reject); $s \doteq 12 + 4(2.45) \doteq 21.8$ in square, *Ans.*

4. Let x = length of side of large tile; $100x^2 = 150(x - 1)^2$; $100x^2 = 150x^2 - 300x + 150$;
$x^2 - 6x + 3 = 0$; $x = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(3)}}{2(1)} = \dfrac{6 \pm \sqrt{24}}{2} = \dfrac{6 \pm 2\sqrt{6}}{2} = 3 \pm \sqrt{6}$;
$x \doteq 3 + 2.44 \doteq 5.4$ or $x \doteq 3 - 2.44 = .56$ (reject); $x - 1 = 4.4$ in, *Ans.*

5. $c^2 = a^2 + b^2$; $c^2 = (3x)^2 + (4x)^2 = 25x^2$; $c = 5x$; $p = a + b + c$; $2 = 3x + 4x + 5x$; $12x = 2$;
$x = \frac{1}{6}$; $3x = \frac{1}{2}$ ft; $4x = \frac{2}{3}$ ft; $5x = \frac{5}{6}$ ft. 6 in, 8 in, 10 in, *Ans.*

6. $c^2 = a^2 + b^2$; $c = 13$; a = distance of one group; b = distance of other group; $d = rt$; $t = 1$ hr;
$a = r$; $b = r + 7$; $13^2 = r^2 + (r + 7)^2$; $169 = 2r^2 + 14r + 49$; $r^2 + 7r - 60 = 0$;
$r = \dfrac{-7 \pm \sqrt{7^2 - 4(1)(-60)}}{2(1)} = \dfrac{-7 \pm \sqrt{289}}{2} = \dfrac{-7 \pm 17}{2}$; $r = \dfrac{-24}{2} = -12$ (reject);
$r = \frac{10}{2} = 5$ mph, $r + 7 = 12$ mph, *Ans.*

B

7. $\dfrac{400}{r} + 2 = \dfrac{400}{r - 10}$; $400(r - 10) + 2r(r - 10) = 400r$; $400r - 4000 + 2r^2 - 20r = 400r$;
$r^2 - 10r - 2000 = 0$; $(r - 50)(r + 40) = 0$; $r = -40$ (reject); $r = 50$ mph, *Ans.*

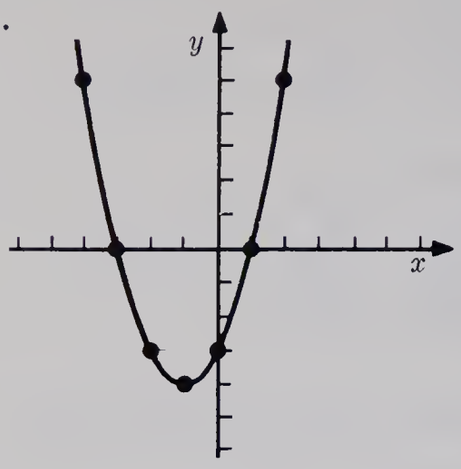
8. Let w = width of paper; $w + 11$ = length of paper; $(w - 16)(w + 11 - 16)(8) = 2800$;
$(w - 16)(w - 5) = 350$; $w^2 - 21w + 80 = 350$; $w^2 - 21w - 270 = 0$; $(w - 30)(w + 9) = 0$;
$w = -9$ in (reject); $w = 30$ in, $w + 11 = 41$ in, *Ans.*

9. a. $(1000)(25) + 4400 = (1000 - 4n)(25 + n)$; $29{,}400 = 25{,}000 - 100n + 1000n - 4n^2$;
$4n^2 - 900n + 4400 = 0$; $n^2 - 225n + 1100 = 0$; $(n - 5)(n - 220) = 0$; $n - 5 = 0$ or $n - 220 = 0$; $n = 5¢$ or $n = 220¢$, *Ans.*
b. Two answers are possible; one answer is practical.

10. a. $1000(16) + 900 = (1000 + n)(16 - n \cdot \frac{1}{100})$; $16{,}900 = 16{,}000 + 16n - 10n - \frac{1}{100}n^2$;
$\frac{1}{100}n^2 - 6n + 900 = 0$; $n^2 - 600n + 90{,}000 = 0$; $(n - 300)^2 = 0$; $n - 300 = 0$; $n = 300$, *Ans.*
b. One answer is possible.

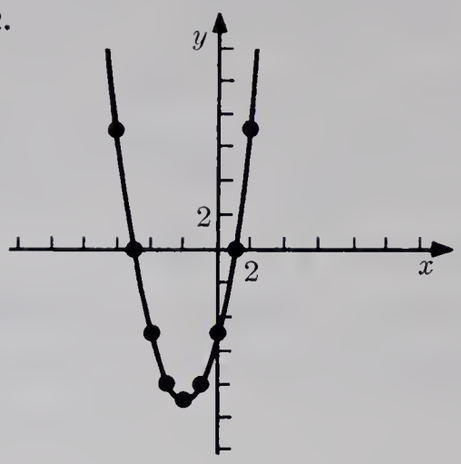
Page 478 · **WRITTEN EXERCISES**

A **1.**



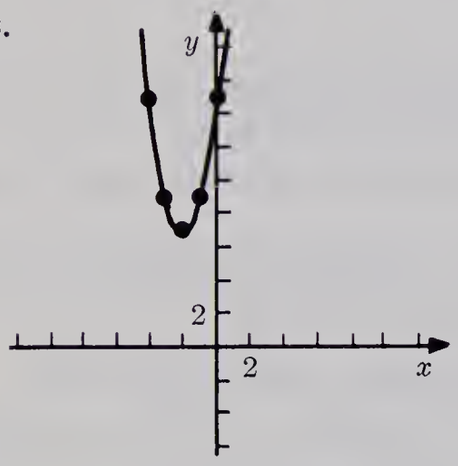
$b^2 - 4ac = 2^2 - 4(1)(-3) = 16$;
Two different real roots

2.



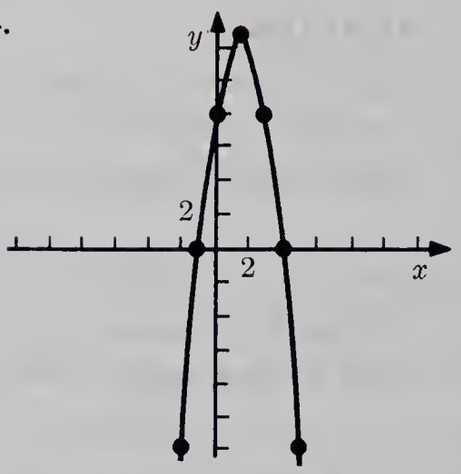
$b^2 - 4ac = 4^2 - 4(1)(-5) = 36$;
Two different real roots

3.



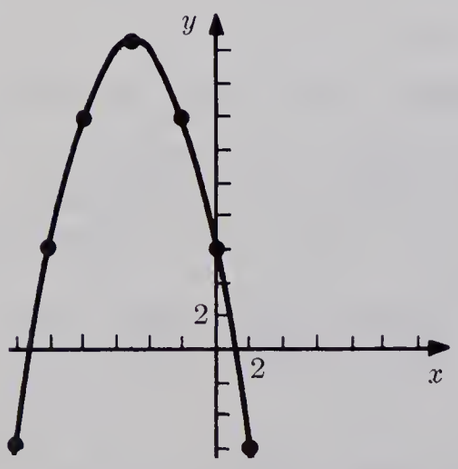
$b^2 - 4ac = 8^2 - 4(2)(15) = -56$;
No real roots

4.



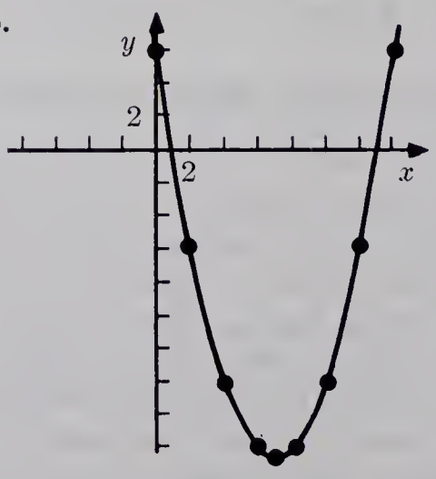
$b^2 - 4ac = 6^2 - 4(-2)(8) = 100$;
Two different real roots

5.



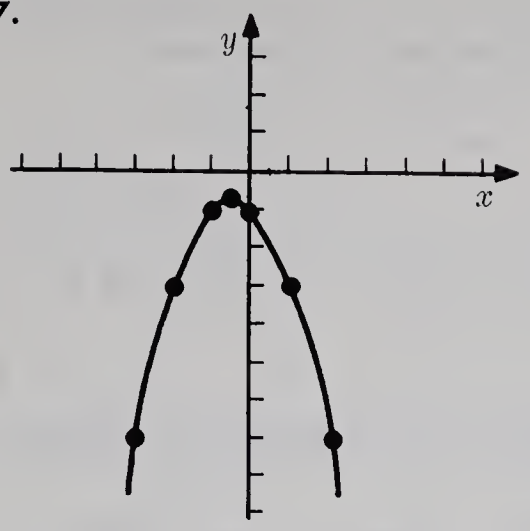
$b^2 - 4ac = (-5)^2 - 4(-\frac{1}{2})(6) = 37$;
Two different real roots

6.



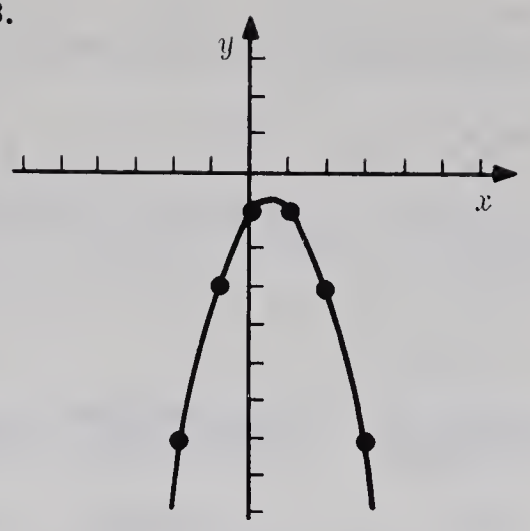
$b^2 - 4ac = (-7)^2 - 4(\frac{1}{2})(6) = 37$;
Two different real roots

7.



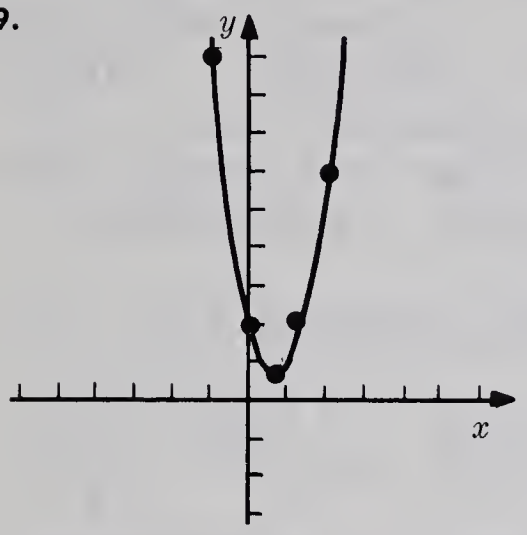
$b^2 - 4ac = (-1)^2 - 4(-1)(-1) = -3$;
No real roots

8.



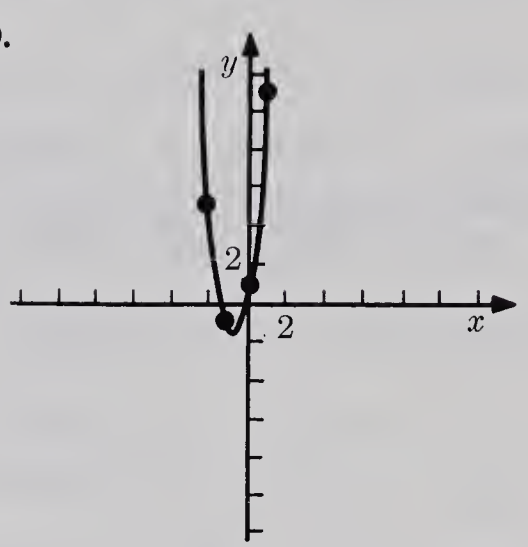
$b^2 - 4ac = (1)^2 - 4(-1)(-1) = -3$;
No real roots

9.



$b^2 - 4ac = (-4)^2 - 4(3)(2) = -8$;
No real roots

10.



$b^2 - 4ac = (6)^2 - 4(4)(1) = 20$;
Two different real roots

B **11.** $x^2 - 2x + 1 = 0$; $b^2 - 4ac = (-2)^2 - 4(1)(1) = 0$; factorable; $(x - 1)(x - 1)$

12. $x^2 + 4x - 4 = 0$; $b^2 - 4ac = 4^2 - 4(1)(-4) = 32$; $x = \dfrac{-4 \pm \sqrt{32}}{2}$; $x = -2 + 2\sqrt{2}$, $-2 - 2\sqrt{2}$ (roots); factors: $(x + 2 - 2\sqrt{2})(x + 2 + 2\sqrt{2})$

13. $-2x^2 + 4x - 1 = 0$; $b^2 - 4ac = 4^2 - 4(-2)(-1) = 8$; $x = = \dfrac{-4 \pm \sqrt{8}}{-4}$; $x = 1 - \dfrac{\sqrt{2}}{2}, 1 + \dfrac{\sqrt{2}}{2}$ (roots); factors: $-2\left(x - 1 + \dfrac{\sqrt{2}}{2}\right)\left(x - 1 - \dfrac{\sqrt{2}}{2}\right)$

14. $-3x^2 - 3x - 5 = 0$; $b^2 - 4ac = (-3)^2 - 4(-3)(-5) = -51$; not factorable over real nos.

15. $u^2 + \frac{1}{4}u - \frac{1}{8} = 0$; $b^2 - 4ac = (\frac{1}{4})^2 - 4(1)(-\frac{1}{8}) = \frac{9}{16}$; factorable; $(u + \frac{1}{2})(u - \frac{1}{4})$

16. $u^2 - \frac{1}{6}\ u + \frac{1}{6} = 0$; $b^2 - 4ac = (-\frac{1}{6})^2 - 4(1)(\frac{1}{6}) = -\frac{23}{36}$; not factorable over real nos.

17. $y(\frac{1}{8} - \frac{1}{8}y - \frac{3}{2}y^2)$; $-\frac{3}{2}y^2 - \frac{1}{8}y + \frac{1}{8} = 0$; $b^2 - 4ac = (-\frac{1}{8})^2 - 4(-\frac{3}{2})(\frac{1}{8}) = \frac{49}{64}$; factorable; $y(\frac{1}{4} - y)(\frac{1}{2} + \frac{3}{2}y)$

18. $\frac{1}{15}z(1 + 3z - 10z^2)$; $-10z^2 + 3z + 1 = 0$; $b^2 - 4ac = 3^2 - 4(-10)(1) = 49$; factorable; $\frac{1}{15}z(1 + 5z)(1 - 2z)$

Page 479 · **PROBLEMS**

A **1.** $h = vt + 16t^2$; (a) $288 = 48t + 16t^2$; $t^2 + 3t - 18 = 0$; $(t + 6)(t - 3) = 0$; $t + 6 = 0$ or $t - 3 = 0$; $t = -6$ (reject); $t = 3$ sec, *Ans.* (b) $288 = 112t + 16t^2$; $t^2 + 7t - 18 = 0$; $(t + 9)(t - 2) = 0$; $t + 9 = 0$ or $t - 2 = 0$; $t = -9$ (reject); $t = 2$ sec, *Ans.*

(c) $288 = 30t + 16t^2$; $8t^2 + 15t - 144 = 0$; $t = \dfrac{-15 \pm \sqrt{15^2 - 4(8)(-144)}}{2(8)} = \dfrac{-15 \pm \sqrt{4833}}{16} = \dfrac{-15 \pm 3\sqrt{537}}{16}$; $t \doteq \dfrac{-15 - 3(23.2)}{16} \doteq -5.3$ (reject); $t \doteq \dfrac{-15 + 3(23.2)}{16} \doteq 3.4$ sec, *Ans.*

2. $s^2 = 8rh - 4h^2$; $(36)^2 = 8(30)h - 4h^2$; $4h^2 - 240h + 1296 = 0$; $h^2 - 60h + 324 = 0$; $(h - 54)(h - 6) = 0$; $h - 54 = 0$ or $h - 6 = 0$; $h = 54$ (reject); $h = 6$ ft, *Ans.*

3. $l = \frac{p(p-1)}{2}$; $10 = \frac{p(p-1)}{2}$; $p^2 - p = 20$; $p^2 - p - 20 = 0$; $(p - 5)(p + 4) = 0$; $p - 5 = 0$ or $p + 4 = 0$; $p = -4$ (reject); $p = 5$, *Ans.*

4. $6 = \frac{p(p-1)}{2}$; $p^2 - p = 12$; $p^2 - p - 12 = 0$; $(p - 4)(p + 3) = 0$; $p - 4 = 0$ or $p + 3 = 0$; $p = -3$ (reject); $p = 4$, *Ans.*

5. $v^2 = r\left(\frac{64.4}{5280}\right)$; $v^2 = 4000\left(\frac{64.4}{5280}\right) \doteq 48.8$; $v = \pm\sqrt{48.8}$; $v \doteq -7.0$ (reject); $v \doteq 7.0$ mi/sec, *Ans.*

6. $v^2 = \frac{gR^2}{R+h}$; (a) $v^2 = \frac{(32.2)(4000)^2}{4000 + 500} = \frac{(32.2)(16{,}000{,}000)}{4500} \doteq 114{,}489$; $v = \pm\sqrt{114{,}489}$; $v \doteq -338.4$ (reject); $v \doteq 338.4$ mi/min, *Ans.* (b) $v^2 = \frac{(32.2)(4000)^2}{4000 + 236{,}000} = \frac{(32.2)(16{,}000{,}000)}{240{,}000} \doteq 2147$; $v = \pm\sqrt{2147}$; $v \doteq -46.3$ (reject); $v \doteq 46.3$ mi/min, *Ans.* (c) $v^2 = \frac{32.2(4000)^2}{4000 + 425} = \frac{(32.2)(16{,}000{,}000)}{4425} \doteq 116{,}429$; $v = \pm\sqrt{116{,}429}$; $v \doteq -341.2$ (reject); $v \doteq 341.2$ mi/min, *Ans.*

7. $d = .045v^2 + 1.1v$; $50 = .045v^2 + 1.1v$; $45v^2 + 1100v - 50{,}000 = 0$; $9v^2 + 220v - 10{,}000 = 0$; $v = \frac{-220 \pm \sqrt{(220)^2 - 4(9)(-10{,}000)}}{18} = \frac{-220 \pm \sqrt{408{,}400}}{18} = \frac{-220 \pm 20\sqrt{1021}}{18} = \frac{-110 \pm 10\sqrt{1021}}{9} \doteq \frac{-110 \pm 10(32.0)}{9}$; $v \doteq -47.8$ (reject); $v \doteq 23.3$ mph, *Ans.*

8. $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$; $(13)^2 = (7 - 2)^2 + (y_2 - 1)^2$; $169 = 25 + y_2^2 - 2y_2 + 1$; $y_2^2 - 2y_2 - 143 = 0$; $y_2 = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-143)}}{2(1)} = \frac{2 \pm \sqrt{576}}{2} = \frac{2 \pm 24}{2}$; $y_2 = \frac{26}{2} = 13$ or $y_2 = \frac{-22}{2} = -11$, *Ans.*

9. $d^2 = (4 - (-2))^2 + (-1 - 3)^2$; $d^2 = 36 + 16 = 52$; $d = \pm\sqrt{52}$; $d = -2\sqrt{13} \doteq -7.2$ (reject); $d = 2\sqrt{13} \doteq 2(3.61) \doteq 7.2$ units, *Ans.*

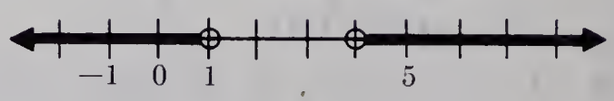
10. Points A $(-2, -2)$, B $(4, 0)$, C $(7, 1)$ are on a straight line if $\overline{AB} + \overline{BC} = \overline{AC}$.
For $\overline{AB}$: $d^2 = (-2 - 4)^2 + (-2 - 0)^2 = 36 + 4 = 40$; $d = \pm\sqrt{40}$; $d = 2\sqrt{10}$ or $d = -2\sqrt{10}$ (reject).
For $\overline{BC}$: $d^2 = (4 - 7)^2 + (0 - 1)^2 = 9 + 1 = 10$; $d = \pm\sqrt{10}$; $d = \sqrt{10}$; or $d = -\sqrt{10}$ (reject).
For $\overline{AC}$: $d^2 = (-2 - 7)^2 + (-2 - 1)^2 = 81 + 9 = 90$; $d = \pm\sqrt{90}$; $d = 3\sqrt{10}$ or $d = -3\sqrt{10}$ (reject); since $2\sqrt{10} + \sqrt{10} = 3\sqrt{10}$, the three points are on a straight line.

11. $d^2 = h(h + 8000)$; $d^2 = 4(4 + 8000) = 32{,}016$; $d = \pm\sqrt{32{,}016}$; $d = \pm 4\sqrt{2001} \doteq \pm 4(44.73) \doteq -178.9$ (reject); $\doteq 178.9$ mi, *Ans.*

12. $(446)^2 = h(h + 8000)$; $h^2 + 8000h - 198{,}916 = 0$; $h = \frac{-8000 \pm \sqrt{(8000)^2 - 4(1)(-198{,}916)}}{2(1)} = \frac{-8000 \pm \sqrt{64{,}795{,}664}}{2} \doteq \frac{-8000 \pm 8049.6}{2}$; $h \doteq \frac{-16049.6}{2} = -8024.8$ (reject); $h \doteq \frac{49.6}{2} = 24.8$ mi, *Ans.*

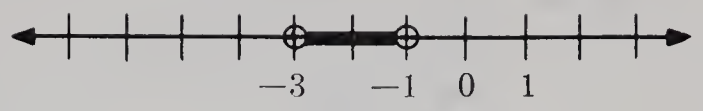
Page 482 · WRITTEN EXERCISES

A **1.** $(x - 4)(x - 1) > 0$;
$x - 4 > 0$ and $x - 1 > 0$ or $x - 4 < 0$ and $x - 1 < 0$
$x > 4$ and $x > 1$ or $x < 4$ and $x < 1$
$x > 4$ or $x < 1$

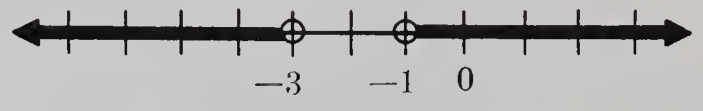


2. $(x - 4)(x - 1) < 0$;
$x - 4 > 0$ and $x - 1 < 0$ or $x - 4 < 0$ and $x - 1 > 0$
$x > 4$ and $x < 1$ or $x < 4$ and $x > 1$
$x \in \emptyset$ or $1 < x < 4$

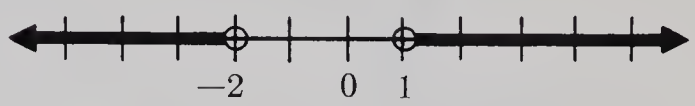
3. $(x+1)(x+3) < 0$;
$x+1 > 0$ and $x+3 < 0$ or $x+1 < 0$ and $x+3 > 0$
$x > -1$ and $x < -3$ or $x < -1$ and $x > -3$
$x \notin \emptyset$ or $-3 < x < -1$



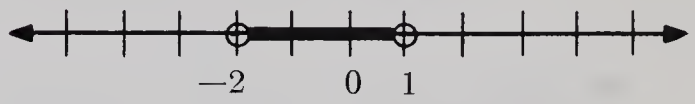
4. $(x+3)(x+1) > 0$;
$x+3 > 0$ and $x+1 > 0$ or $x+3 < 0$ and $x+1 < 0$
$x > -3$ and $x > -1$ or $x < -3$ and $x < -1$
$x > -1$ or $x < -3$



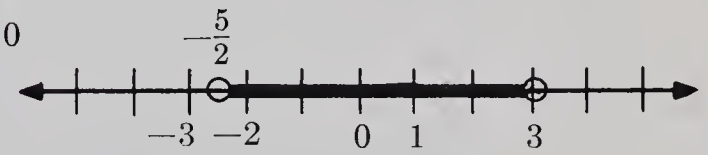
5. $y^2 + y - 2 > 0$; $(y+2)(y-1) > 0$;
$y+2 > 0$ and $y-1 > 0$ or $y+2 < 0$ and $y-1 < 0$
$y > -2$ and $y > 1$ or $y < -2$ and $y < 1$
$y > 1$ or $y < -2$



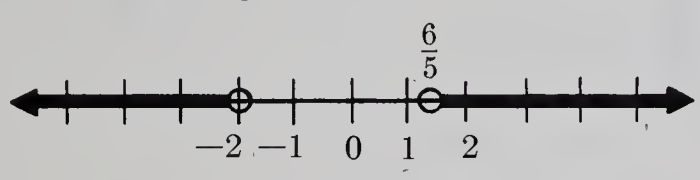
6. $y^2 + y - 2 < 0$; $(y+2)(y-1) < 0$;
$y+2 > 0$ and $y-1 < 0$ or $y+2 < 0$ and $y-1 > 0$
$y > -2$ and $y < 1$ or $y < -2$ and $y > 1$
$-2 < y < 1$ or $y \in \emptyset$



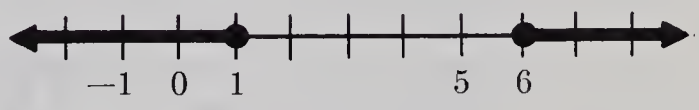
7. $2n^2 - n - 15 < 0$; $(2n+5)(n-3) < 0$;
$2n+5 > 0$ and $n-3 < 0$ or $2n+5 < 0$ and $n-3 > 0$
$n > -\frac{5}{2}$ and $n < 3$ or $n < -\frac{5}{2}$ and $n > 3$
$-\frac{5}{2} < n < 3$ or $n \in \emptyset$



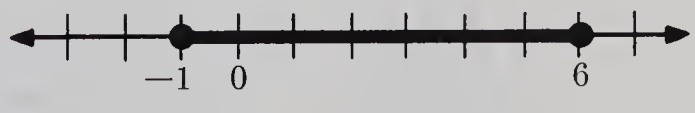
8. $5m^2 + 4m - 12 > 0$; $(5m-6)(m+2) > 0$;
$5m-6 > 0$ and $m+2 > 0$ or $5m-6 < 0$ and $m+2 < 0$
$m > \frac{6}{5}$ and $m > -2$ or $m < \frac{6}{5}$ and $m < -2$
$m > \frac{6}{5}$ or $m < -2$



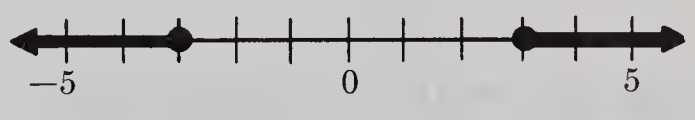
9. $r^2 - 7r + 6 \geq 0$; $(r-6)(r-1) \geq 0$;
$r-6 \geq 0$ and $r-1 \geq 0$ or $r-6 \leq 0$ and $r-1 \leq 0$
$r \geq 6$ and $r \geq 1$ or $r \leq 6$ and $r \leq 1$
$r \geq 6$ or $r \leq 1$



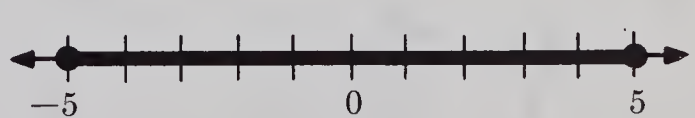
10. $s^2 - 5s - 6 \leq 0$; $(s-6)(s+1) \leq 0$;
$s-6 \geq 0$ and $s+1 \leq 0$ or $s-6 \leq 0$ and $s+1 \geq 0$
$s \geq 6$ and $s \leq -1$ or $s \leq 6$ and $s \geq -1$
$s \in \emptyset$ or $-1 \leq s \leq 6$



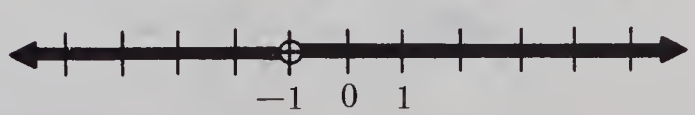
11. $x^2 - 9 \geq 0$; $(x+3)(x-3) \geq 0$;
$x+3 \geq 0$ and $x-3 \geq 0$ or $x+3 \leq 0$ and $x-3 \leq 0$
$x \geq -3$ and $x \geq 3$ or $x \leq -3$ and $x \leq 3$
$x \geq 3$ or $x \leq -3$



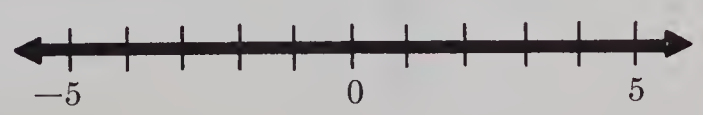
12. $x^2 - 25 \leq 0$; $(x+5)(x-5) \leq 0$;
$x+5 \geq 0$ and $x-5 \leq 0$ or $x+5 \leq 0$ and $x-5 \geq 0$
$x \geq -5$ and $x \leq 5$ or $x \leq -5$ and $x \geq 5$
$-5 \leq x \leq 5$ or $x \in \emptyset$



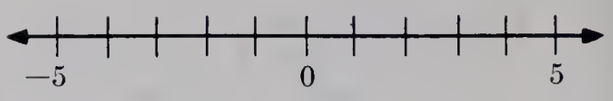
B **13.** $(x+1)(x+1) > 0$;
$x+1 > 0$ and $x+1 > 0$ or $x+1 < 0$ and $x+1 < 0$
$x > -1$ or $x < -1$



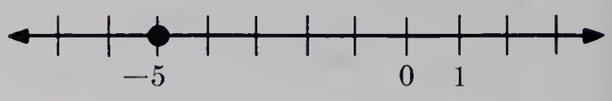
14. $(x-3)(x-3) \geq 0$;
$x-3 \geq 0$ and $x-3 \geq 0$ or $x-3 \leq 0$ and $x-3 \leq 0$
$x \geq 3$ or $x \leq 3$



15. $(x - 2)(x - 2) < 0$;
$(x - 2) > 0$ and $x - 2 < 0$
$x > 2$ and $x < 2$
$x \in \emptyset$

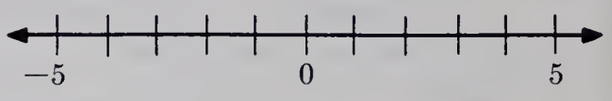


16. $(x + 5)(x + 5) \leq 0$;
$x + 5 \geq 0$ and $x + 5 \leq 0$
$x \geq -5$ and $x \leq -5$
$x = -5$



17. $(x)(x) > 0$;
$x > 0$ and $x > 0$ or $x < 0$ and $x < 0$
$x > 0$ or $x < 0$

18. $(x)(x) < 0$;
$x > 0$ and $x < 0$
$x \in \emptyset$



C **19.** $x^2 - 2x - 35 \geq 0$; $(x - 7)(x + 5) \geq 0$;
$x - 7 \geq 0$ and $x + 5 \geq 0$ or $x - 7 \leq 0$ and $x + 5 \leq 0$
$x \geq 7$ and $x \geq -5$ or $x \leq 7$ and $x \leq -5$
$x \geq 7$ or $x \leq -5$

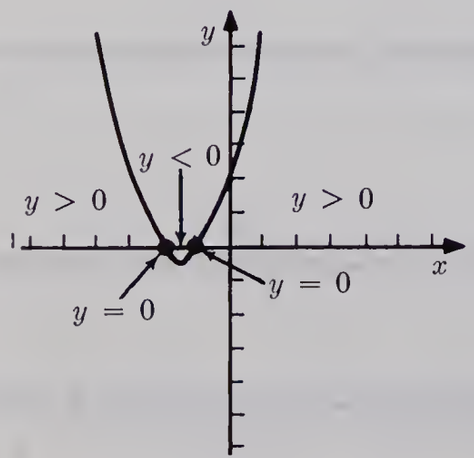
20. $x^2 - 12x + 35 \geq 0$; $(x - 7)(x - 5) \geq 0$;
$x - 7 \geq 0$ and $x - 5 \geq 0$ or $x - 7 \leq 0$ and $x - 5 \leq 0$
$x \geq 7$ and $x \geq 5$ or $x \leq 7$ and $x \leq 5$
$x \geq 7$ or $x \leq 5$

21. $x^2 + 5x \geq 0$; $x(x + 5) \geq 0$;
$x \geq 0$ and $x + 5 \geq 0$ or $x \leq 0$ and $x + 5 \leq 0$
$x \geq 0$ and $x \geq -5$ or $x \leq 0$ and $x \leq -5$
$x \geq 0$ or $x \leq -5$

22. $x^2 - x \geq 0$; $x(x - 1) \geq 0$;
$x \geq 0$ and $x - 1 \geq 0$ or $x \leq 0$ and $x - 1 \leq 0$
$x \geq 0$ and $x \geq 1$ or $x \leq 0$ and $x \leq 1$
$x \geq 1$ or $x \leq 0$

Page 484 · WRITTEN EXERCISES

A **1.**



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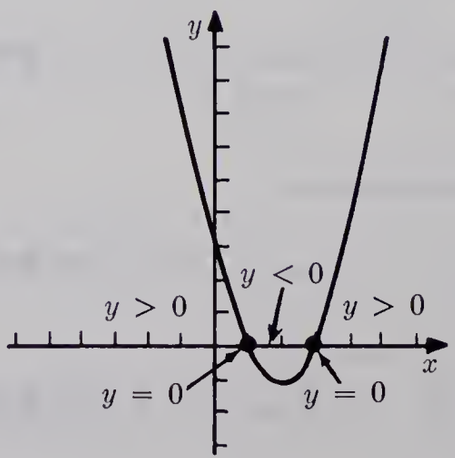

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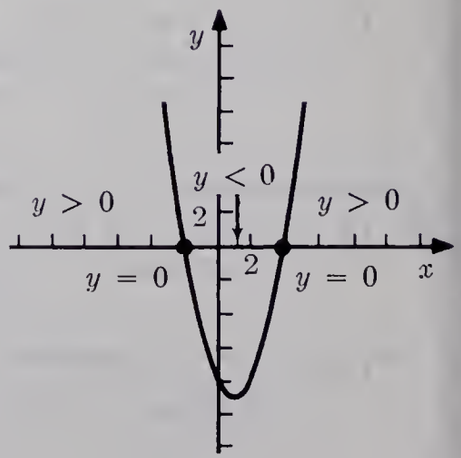

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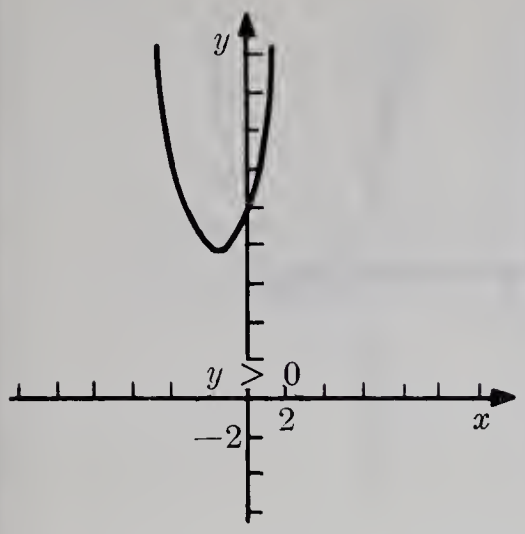

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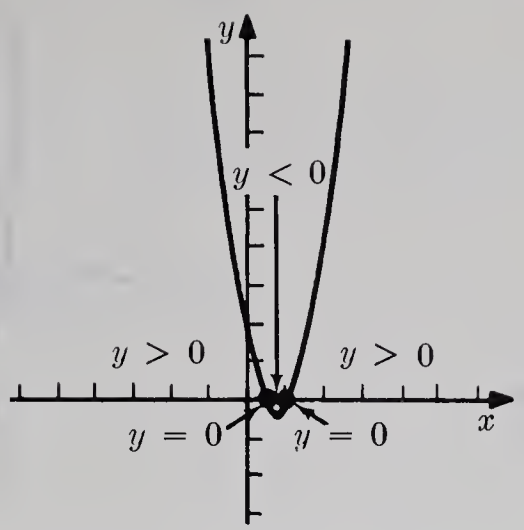

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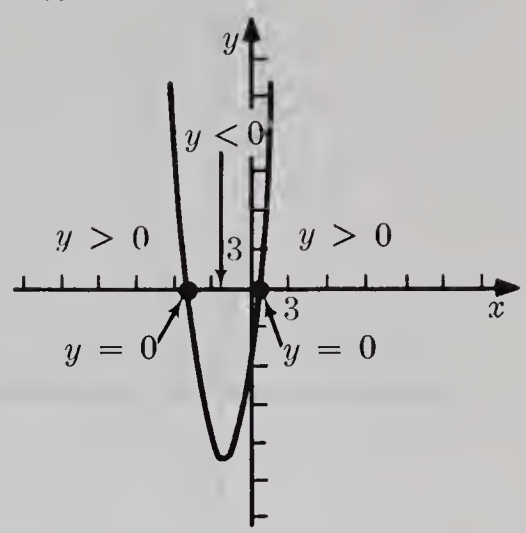

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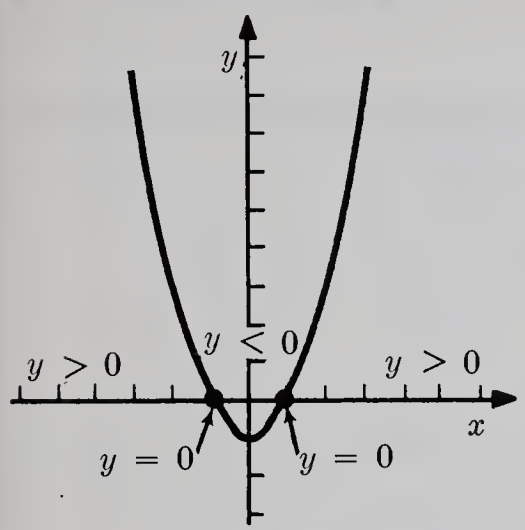

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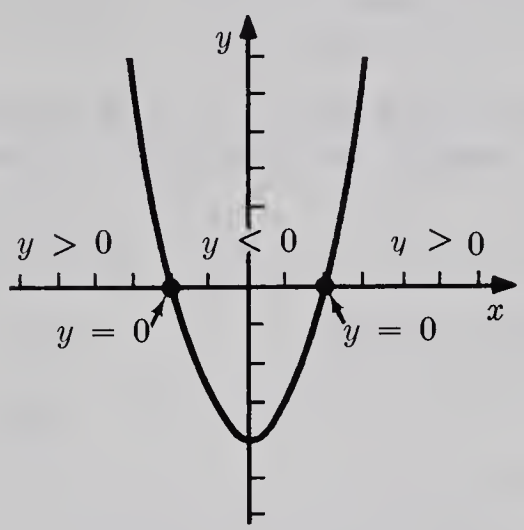

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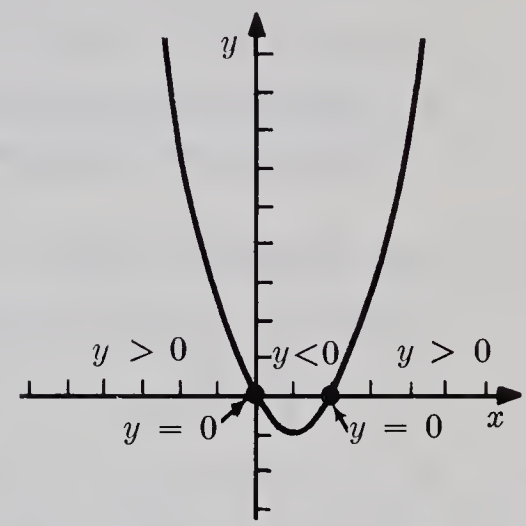

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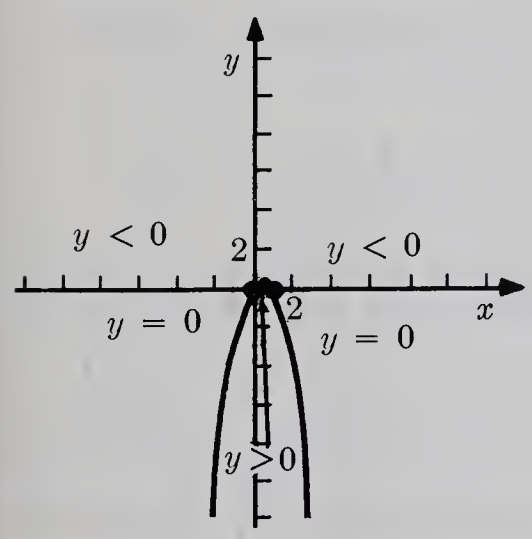

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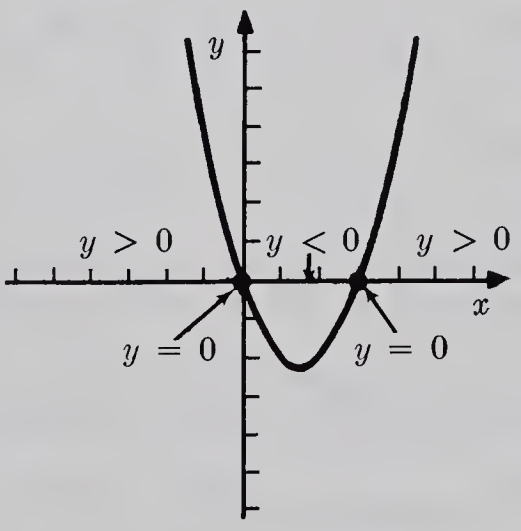

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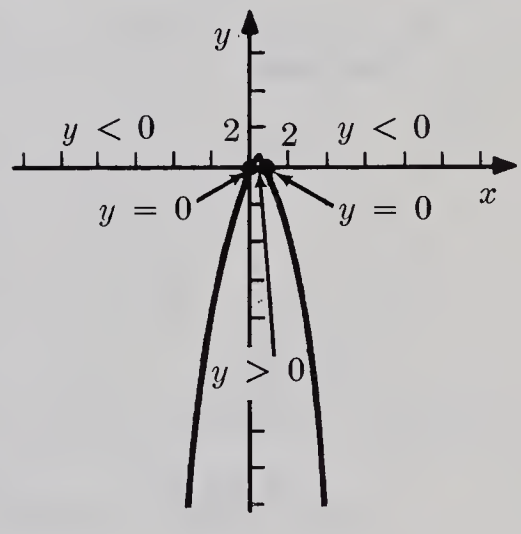
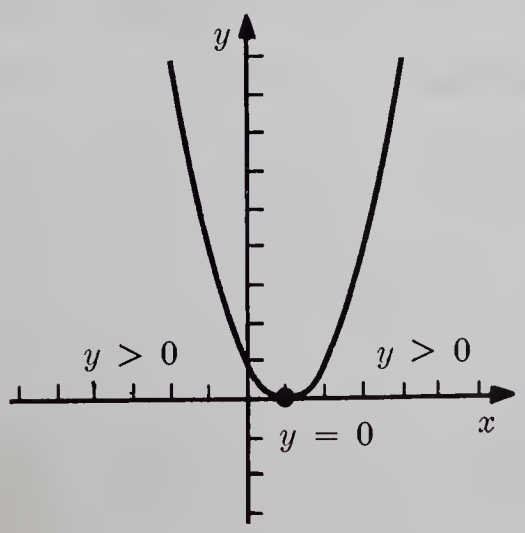


B **13.**



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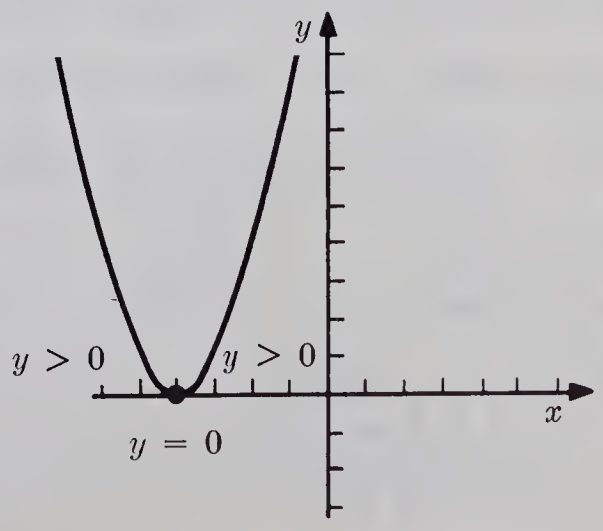

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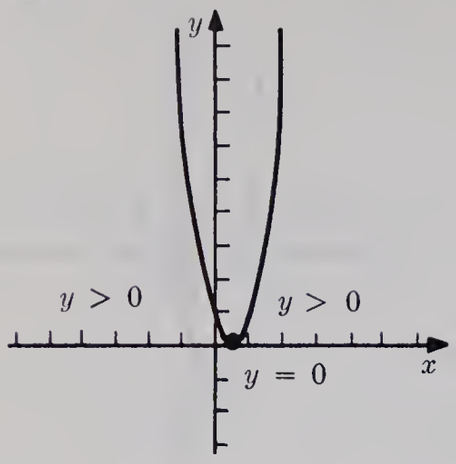

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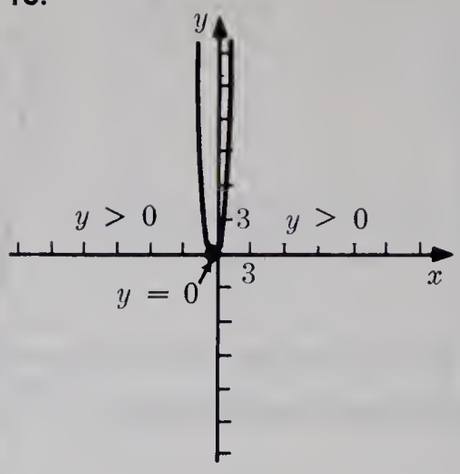


C **17.** $\sqrt{x^2 - 3x - 10}$ is a real no. if $x^2 - 3x - 10 \geq 0$. If $x^2 - 3x - 10 \geq 0$; $(x - 5)(x + 2) \geq 0$; $x - 5 \geq 0$ and $x + 2 \geq 0$; $x \geq 5$ and $x \geq -2$; $\therefore x \geq 5$; or $x - 5 \leq 0$ and $x + 2 \leq 0$; $x \leq 5$ and $x \leq -2$; $\therefore x \leq -2$. $\{x \geq 5 \text{ or } x \leq -2\}$

18. $\sqrt{x^2 - 3x}$ is a real no. if $x^2 - 3x \geq 0$, $x(x - 3) \geq 0$. Either $x \geq 0$ and $x \geq 3$ or $x \leq 0$ and $x \leq 3$; nos. satisfying these conditions are ≥ 3 or ≤ 0. $\{x \leq 0 \text{ or } x \geq 3\}$

Page 486 · **CHAPTER TEST**

1. $2y^2 = \frac{1}{8}$; $y^2 = \frac{1}{16}$; $y = \pm\frac{1}{4}$. $\{\frac{1}{4}, -\frac{1}{4}\}$

2. $(x - \frac{1}{2})^2 = \frac{9}{4}$; $x - \frac{1}{2} = \pm\sqrt{\frac{9}{4}}$; $x - \frac{1}{2} = \frac{3}{2}$ or $x - \frac{1}{2} = -\frac{3}{2}$; $x = 2$ or $x = -1$. $\{2, -1\}$

3. $(\frac{1}{2} - n)^2 = 0$; $\frac{1}{2} - n = 0$; $n = \frac{1}{2}$. $\{\frac{1}{2}\}$

4. $x^2 - 22x - 11 = 0$; $b = -22$, $c = -11$; $11 + 2\sqrt{33} + 11 - 2\sqrt{33} = 22 = -b$; $(11 + 2\sqrt{33})(11 - 2\sqrt{33}) = 121 - 132 = -11 = c$; correct

5. $x^2 + 30x + 29 = 0$; $b = 30$, $c = 29$; $1.1 - 31.1 = -30 = -b$; $(1.1)(-31.1) = -34.21 \neq c$; incorrect

6. $c = \left(\frac{1.4}{2}\right)^2 = .49$; $(x - .7)^2$

7. $x^2 - 3x + \frac{9}{4} = 9 + \frac{9}{4}$; $(x - \frac{3}{2})^2 = \frac{45}{4}$

8. $x^2 - 16x + 8 = 0$; $x^2 - 16x + 64 = -8 + 64$; $(x - 8)^2 = 56$; $x - 8 = \pm\sqrt{56}$; $x = 8 + 2\sqrt{14}$ or $x = 8 - 2\sqrt{14}$; $x \doteq 8 + 2(3.74) \doteq 15.5$ or $x \doteq 8 - 2(3.74) \doteq .5$. $\{8 + 2\sqrt{14}, 8 - 2\sqrt{14}\}$; $\{15.5, .5\}$

9. $a = 4, b = 12, c = 7$; $x = \dfrac{-12 \pm \sqrt{(12)^2 - 4(4)(7)}}{2(4)} = \dfrac{-12 \pm \sqrt{32}}{8} = \dfrac{-12 \pm 4\sqrt{2}}{8} = \dfrac{-3 \pm \sqrt{2}}{2}$;

$x \doteq \dfrac{-3 \pm 1.41}{2}$; $x \doteq \dfrac{-1.59}{2} \doteq -.8$ or $x \doteq \dfrac{-4.41}{2} \doteq -2.2$. $\left\{\dfrac{-3 + \sqrt{2}}{2}, \dfrac{-3 - \sqrt{2}}{2}\right\}$; $\{-.8, -2.2\}$

10. $(18 + 2x)(24 + 2x) = 720$; $432 + 36x + 48x + 4x^2 = 720$; $4x^2 + 84x - 288 = 0$; $x^2 + 21x - 72 = 0$; $(x + 24)(x - 3) = 0$; $x + 24 = 0$ or $x - 3 = 0$; $x = -24$ (reject); $x = 3$ in, *Ans.*

11. $b^2 - 4ac = (-3)^2 - 4(1)(-5) = 29$; two different real roots

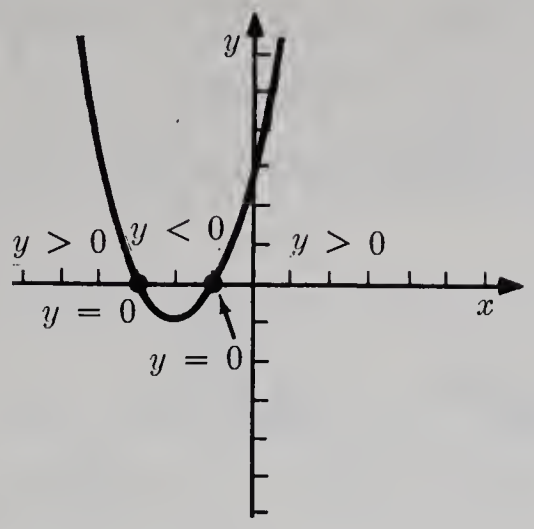
12. $b^2 - 4ac = 4^2 - 4(4)(1) = 0$; a double real root

13. $b^2 - 4ac = 1^2 - 4(1)(3) = -11$; no real roots

14. $b^2 - 4ac = (-3)^2 - 4(5)(7) = -131$; no real roots

15. $x^2 - 3x - 4 > 0$; $(x - 4)(x + 1) > 0$;
$x - 4 > 0$ and $x + 1 > 0$ or $x - 4 < 0$ and $x + 1 < 0$
$x > 4$ and $x > -1$ or $x < 4$ and $x < -1$
$x > 4$ or $x < -1$

16.



$x^2 + 4x + 3 \leq 0$ when $y \leq 0$,
i.e., when $-3 \leq x \leq -1$

Pages 486–489 · **CHAPTER REVIEW**

1. u, v; u, $-v$ **2.** $a = \pm 2$. $\{2, -2\}$ **3.** $b^2 = 64$; $b = \pm 8$. $\{8, -8\}$

4. $c + 1 = \pm 5$; $c = 4$ or $c = -6$. $\{4, -6\}$ **5.** opposite, coefficient, linear; constant

6. $x^2 - 4x - 4 = 0$; $b = -4$, $c = -4$; $2 - 2 = 0 \neq -b$; $2(-2) = -4 = c$; incorrect

7. $b = 26$, $c = 19$; $-13 + 5\sqrt{6} - 13 - 5\sqrt{6} = -26 = -b$;
$(-13 + 5\sqrt{6})(-13 - 5\sqrt{6}) = 169 - 150 = 19 = c$; correct

8. $k = \left(\dfrac{-20}{2}\right)^2 = 100$; $(s - 10)^2$ **9.** $k = (\frac{1}{2})^2 = \frac{1}{4}$; $(y + \frac{1}{2})^2$

10. $k = \left(\dfrac{.5}{2}\right)^2 = .0625$; $(z + .25)^2$

11. $x^2 + 2x + 1 = 23 + 1$; $(x + 1)^2 = 24$; $x + 1 = \pm\sqrt{24}$; $x + 1 = 2\sqrt{6}$ or $x + 1 = -2\sqrt{6}$;
$x = -1 + 2\sqrt{6}$ or $x = -1 - 2\sqrt{6}$; $x \doteq -1 + 2(2.45) = 3.9$ or $x \doteq -1 - 2(2.45) = -5.9$.
$\{-1 + 2\sqrt{6}, -1 - 2\sqrt{6}\}$; $\{3.9, -5.9\}$

12. $x^2 - 6x = 12$; $x^2 - 6x + 9 = 12 + 9$; $(x - 3)^2 = 21$; $x - 3 = \pm\sqrt{21}$; $x = 3 + \sqrt{21}$ or
$x = 3 - \sqrt{21}$; $x \doteq 3 + 4.58 \doteq 7.6$ or $x \doteq 3 - 4.58 \doteq -1.6$. $\{3 + \sqrt{21}, 3 - \sqrt{21}\}$; $\{7.6, -1.6\}$

13. $y^2 + 5y + \frac{25}{4} = 3 + \frac{25}{4}$; $(y + \frac{5}{2})^2 = \frac{37}{4}$; $y + \frac{5}{2} = \pm\sqrt{\frac{37}{4}}$; $y + \frac{5}{2} = \dfrac{\sqrt{37}}{2}$ or $y + \frac{5}{2} = -\dfrac{\sqrt{37}}{2}$;

$y = \dfrac{-5 + \sqrt{37}}{2}$ or $y = \dfrac{-5 - \sqrt{37}}{2}$; $y \doteq \dfrac{-5 + 6.08}{2} \doteq .5$ or $y \doteq \dfrac{-5 - 6.08}{2} \doteq -5.5$.

$\left\{\dfrac{-5 + \sqrt{37}}{2}, \dfrac{-5 - \sqrt{37}}{2}\right\}$; $\{.5, -5.5\}$

14. $(y - 3)^2 = 20$; $y - 3 = \pm\sqrt{20}$; $y - 3 = 2\sqrt{5}$ or $y - 3 = -2\sqrt{5}$; $y = 3 + 2\sqrt{5}$ or
$y = 3 - 2\sqrt{5}$; $y \doteq 3 + 2(2.24) \doteq 7.5$ or $y \doteq 3 - 2(2.24) \doteq -1.5$. $\{3 + 2\sqrt{5}, 3 - 2\sqrt{5}\}$;
$\{7.5, -1.5\}$

15. Let s = rate in still water; $\dfrac{18}{s + 3} + \dfrac{18}{s - 3} = 8$; $18(s - 3) + 18(s + 3) = 8(s^2 - 9)$;

$18s - 54 + 18s + 54 = 8s^2 - 72$; $8s^2 - 36s - 72 = 0$; $2s^2 - 9s - 18 = 0$; $(2s + 3)(s - 6) = 0$;
$2s + 3 = 0$ or $s - 6 = 0$; $s = -\frac{3}{2}$ (reject); $s = 6$ mph, *Ans.*

16. Let r = rate of car; $r + 10$ = rate of train; $\dfrac{400}{r} - \dfrac{400}{r + 10} = 2$; $400(r + 10) - 400r = 2r(r + 10)$;

$400r + 4000 - 400r = 2r^2 + 20r$; $2r^2 + 20r - 4000 = 0$; $r^2 + 10r - 2000 = 0$;
$(r + 50)(r - 40) = 0$; $r + 50 = 0$ or $r - 40 = 0$; $r = -50$ (reject); $r = 40$ mph,
$r + 10 = 50$ mph, *Ans.*

17. 0 **18.** real **19.** $\dfrac{-b + \sqrt{b^2 - 4ac}}{2a}, \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$

20. $a = 2$, $b = 11$, $c = 3$; $x = \dfrac{-11 \pm \sqrt{(11)^2 - 4(2)(3)}}{2(2)} = \dfrac{-11 \pm \sqrt{97}}{4}$; $x \doteq \dfrac{-11 \pm 9.85}{4}$;

$x \doteq \dfrac{-1.15}{4} \doteq -.3$ or $x \doteq \dfrac{-20.85}{4} \doteq -5.2$. $\left\{\dfrac{-11 + \sqrt{97}}{4}, \dfrac{-11 - \sqrt{97}}{4}\right\}$; $\{-.3, -5.2\}$

21. $a = 1,\ b = -18,\ c = 4;\ x = \dfrac{-(-18) \pm \sqrt{(-18)^2 - 4(1)(4)}}{2(1)} = \dfrac{18 \pm \sqrt{308}}{2} = \dfrac{18 \pm 2\sqrt{77}}{2} = 9 \pm \sqrt{77};\ x \doteq 9 + 8.78 \doteq 17.8$ or $x \doteq 9 - 8.78 \doteq .2.$ $\{9 + \sqrt{77}, 9 - \sqrt{77}\};$ $\{17.8, .2\}$

22. $a = 3,\ b = -14,\ c = -5;\ x = \dfrac{-(-14) \pm \sqrt{(-14)^2 - 4(3)(-5)}}{2(3)} = \dfrac{14 \pm \sqrt{256}}{6} = \dfrac{14 \pm 16}{6};$

$x = \frac{30}{6} = 5$ or $x = \dfrac{-2}{6} = -\dfrac{1}{3}.$ $\{5, -\frac{1}{3}\}$

23. $5x^2 + 8x - 3 = 0;\ a = 5,\ b = 8,\ c = -3;$

$x = \dfrac{-8 \pm \sqrt{8^2 - 4(5)(-3)}}{2(5)} = \dfrac{-8 \pm \sqrt{124}}{10} = \dfrac{-8 \pm 2\sqrt{31}}{10} = \dfrac{-4 \pm \sqrt{31}}{5};$

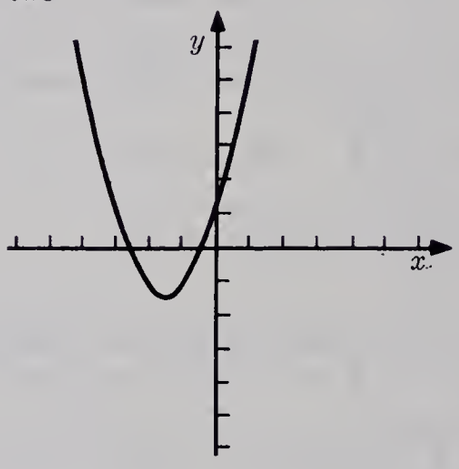
$x \doteq \dfrac{-4 \pm 5.57}{5};\ x \doteq \dfrac{1.57}{5} \doteq .3$ or $x \doteq \dfrac{-9.57}{5} \doteq -1.9.$ $\left\{\dfrac{-4 + \sqrt{31}}{5}, \dfrac{-4 - \sqrt{31}}{5}\right\};$ $\{.3, -1.9\}$

24. $200(35) + 1100 = (200 - 2n)(35 + 1 \cdot n);\ 8100 = 7000 - 70n + 200n - 2n^2;$
$2n^2 - 130n + 1100 = 0;\ n^2 - 65n + 550 = 0;\ (n - 55)(n - 10) = 0;\ n - 55 = 0$ or
$n - 10 = 0;\ n = 55$ (reject, not reasonable); $n = \$10,$ *Ans.*

25. $(x - 2)(x - 2)(1) = 16;\ (x - 2)^2 = 16;\ x - 2 = \pm\sqrt{16};\ x - 2 = 4$ or $x - 2 = -4;\ x = 6$ or $x = -2$ (reject); 6 inches square, *Ans.*

26. $y = ax^2 + bx + c$

27. two



28. a. $b^2 - 4ac = (-7)^2 - 4(2)(2) = 33$; two different real roots
b. $2x^2 - 8x + 8 = 0;\ b^2 - 4ac = (-8)^2 - 4(2)(8) = 0$; a double real root
c. $b^2 - 4ac = (3)^2 - 4(1)(2) = 1$; two different real roots
d. $x^2 + 3x + 3 = 0;\ b^2 - 4ac = (3)^2 - 4(1)(3) = -3$; no real roots

29. $32 = 48t - 16t^2;\ t^2 - 3t + 2 = 0;\ (t - 2)(t - 1) = 0;\ t - 2 = 0$ or $t - 1 = 0;$
$t = 2$ (reject); $t = 1$ sec, *Ans.*

30. $(22)^2 = 8(20)h - 4h^2;\ 4h^2 - 160h + 484 = 0;\ h^2 - 40h + 121 = 0;$

$h = \dfrac{-(-40) \pm \sqrt{(-40)^2 - 4(1)(121)}}{2(1)} = \dfrac{40 \pm \sqrt{1116}}{2} = \dfrac{40 \pm 6\sqrt{31}}{2} = 20 \pm 3\sqrt{31};$

$h \doteq 20 + 3(5.57) \doteq 36.7$ (reject); $h \doteq 20 - 3(5.57) \doteq 3.3$ ft, *Ans.*

31. $630 = \frac{1}{2}t^2 - \frac{1}{2}t;\quad t^2 - t - 1260 = 0;\quad (t + 35)(t - 36) = 0;\quad t + 35 = 0$ or $t - 36 = 0;$
$t = -35$ (reject); $t = 36,$ *Ans.*

32. $>,\ <$ **33.** $\leq,\ \geq$ **34.** no

35. $(x + 2)(x + 1) > 0;$
$x + 2 > 0$ and $x + 1 > 0$ or $x + 2 < 0$ and $x + 1 < 0$
$x > -2$ and $x > -1$ or $x < -2$ and $x < -1$
$x > -1$ or $x < -2.$
{real numbers greater than -1 or less than -2}

36. $x^2 - x - 12 \leq 0;\ (x - 4)(x + 3) \leq 0;$
$x - 4 \geq 0$ and $x + 3 \leq 0$ or $x - 4 \leq 0$ and $x + 3 \geq 0$
$x \geq 4$ and $x \leq -3$ or $x \leq 4$ and $x \geq -3$
$x \in \emptyset$ or $-3 \leq x \leq 4.$
{real numbers greater than or equal to -3 and less than or equal to 4}

37. $(x + 5)(x - 5) \geq 0$;

$x + 5 \geq 0$ and $x - 5 \geq 0$ or $x + 5 \leq 0$ and $x - 5 \leq 0$

$x \geq -5$ and $x \geq 5$ or $x \leq -5$ and $x \leq 5$

$x \geq 5$ or $x \leq -5$.

{real numbers greater than or equal to 5 or less than or equal to -5}

38. $(x + 1)(x - 1) < 0$;

$x + 1 > 0$ and $x - 1 < 0$ or $x + 1 < 0$ and $x - 1 > 0$

$x > -1$ and $x < 1$ or $x < -1$ and $x > 1$

$-1 < x < 1$ or $x \in \emptyset$.

{real numbers greater than -1 and less than 1}

39. $>$, *Ans.* $x^2 - x - 2 > 0$; $(x - 2)(x + 1) > 0$;

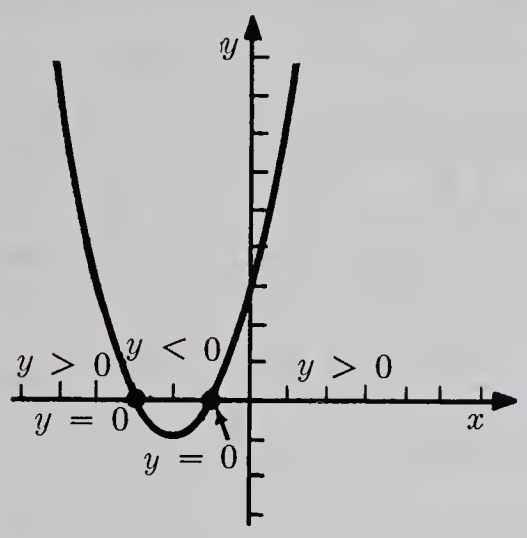
$x - 2 > 0$ and $x + 1 > 0$ or $x - 2 < 0$ and $x + 1 < 0$

$x > 2$ and $x > -1$ or $x < 2$ and $x < -1$

$x > 2$ or $x < -1$.

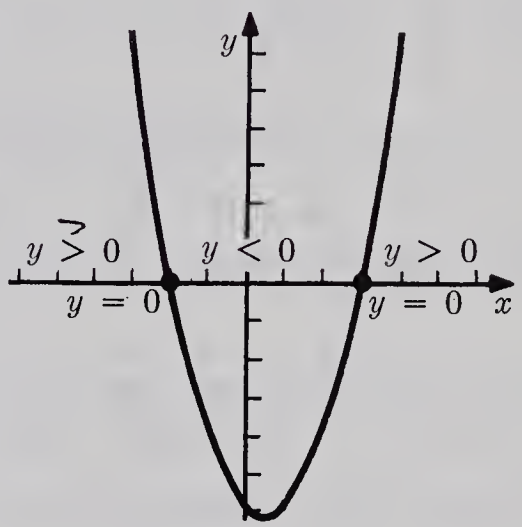
40. $(x + 1)(x - 2) < 0$, $x^2 - x < 2$. $<$, $<$, *Ans.*

41.



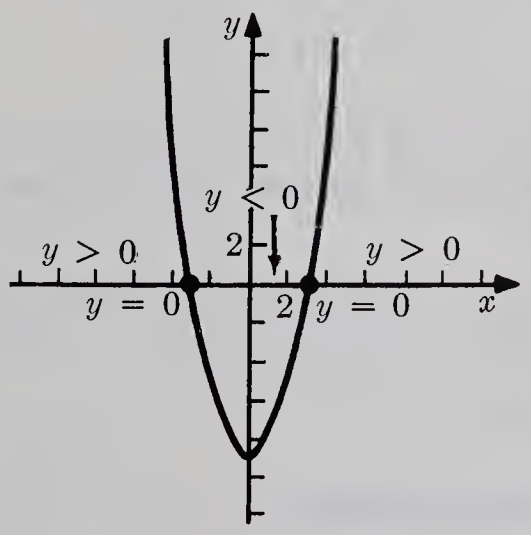
$x^2 + 4x + 3 \geq 0$ when $y \geq 0$,
i.e., when $x \geq -1$ or $x \leq -3$

42.



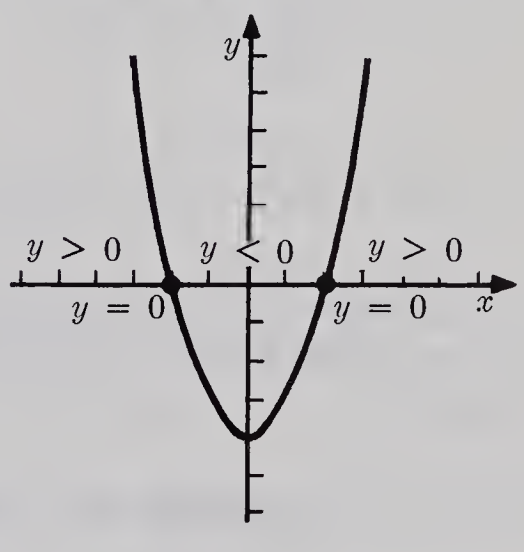
$x^2 < x + 6$ or $x^2 - x - 6 < 0$
when $y < 0$, i.e., when $-2 < x < 3$

43.



$x^2 - 9 > 0$ when $y > 0$, i.e.,
when $x > 3$ or $x < -3$

44.



$x^2 - 4 \leq 0$ when $y \leq 0$, i.e.,
when $-2 \leq x \leq 2$

Page 490 · EXTRA FOR EXPERTS

1. a. $n = 1$: $(1)^{156} - 2(1)^{41} + 1 = 1 - 2 + 1 = 0$

b. $x = -1$: $(-1)^{71} - 3(-1)^{28} + 4 = -1 - 3 + 4 = 0$

c. $x = -2$: $(-2)^3 - (-2)^2 - (-2) + 10 = -8 - 4 + 2 + 10 = 0$

d. $t = 3$: $3(3)^4 - 16(3)^3 + 19(3)^2 + 5(3) + 3 = 243 - 432 + 171 + 15 + 3 = 0$

2. a. $x = 1$: $(1)^3 - 7(1) + 6 = 1 - 7 + 6 = 0$; $\dfrac{x^3 - 7x + 6}{x - 1} = x^2 + x - 6 = (x + 3)(x - 2)$; factors are $(x - 1)(x + 3)(x - 2)$

b. $y = 1$: $2(1)^3 + 3(1)^2 - 2(1) - 3 = 2 + 3 - 2 - 3 = 0$;
$\dfrac{2y^3 + 3y^2 - 2y - 3}{y - 1} = 2y^2 + 5y + 3 = (2y + 3)(y + 1)$; factors are $(y - 1)(2y + 3)(y + 1)$

c. not factorable

d. $m = -2$: $2(-2)^4 + 2(-2)^3 - 17(-2)^2 - 23(-2) + 6 = 32 - 16 - 68 + 46 + 6 = 0$;
$\dfrac{2m^4 + 2m^3 - 17m^2 - 23m + 6}{m + 2} = 2m^3 - 2m^2 - 13m + 3$;
$m = 3$: $2(3)^3 - 2(3)^2 - 13(3) + 3 = 54 - 18 - 39 + 3 = 0$;
$\dfrac{2m^3 - 2m^2 - 13m + 3}{m - 3} = 2m^2 + 4m - 1$; factors are $(m + 2)(m - 3)(2m^2 + 4m - 1)$

3. a. $x^3 - 4x^2 - x + 4 = 0$; $x = 1$: $1 - 4 - 1 + 4 = 0$; $\dfrac{x^3 - 4x^2 - x + 4}{x - 1} = x^2 - 3x - 4$:
$(x - 1)(x^2 - 3x - 4) = 0$; $(x - 1)(x - 4)(x + 1) = 0$; $x - 1 = 0$ or $x - 4 = 0$ or $x + 1 = 0$; $x = 1$ or $x = 4$ or $x = -1$. $\{1, 4, -1\}$.

b. $3z^3 - 5z^2 - 3z + 2 = 0$; $z = 2$: $3(2)^3 - 5(2)^2 - 3(2) + 2 = 24 - 20 - 6 + 2 = 0$;
$\dfrac{3z^3 - 5z^2 - 3z + 2}{z - 2} = 3z^2 + z - 1$; $(z - 2)(3z^2 + z - 1) = 0$; $z - 2 = 0$ or $3z^2 + z - 1 = 0$;
$z = 2$ or $z = \dfrac{-1 \pm \sqrt{(1)^2 - 4(3)(-1)}}{2(3)} = \dfrac{-1 \pm \sqrt{13}}{6}$. $\left\{2, \dfrac{-1 + \sqrt{13}}{6}, \dfrac{-1 - \sqrt{13}}{6}\right\}$

c. $2z^3 - 3z^2 - 14z + 15 = 0$; $z = 1$: $2(1)^3 - 3(1)^2 - 14(1) + 15 = 2 - 3 - 14 + 15 = 0$;
$\dfrac{2z^3 - 3z^2 - 14z + 15}{z - 1} = 2z^2 - z - 15 = (2z + 5)(z - 3)$; $(z - 1)(2z + 5)(z - 3) = 0$;
$z - 1 = 0$ or $2z + 5 = 0$ or $z - 3 = 0$; $z = 1$ or $z = -\frac{5}{2}$ or $z = 3$. $\{1, -\frac{5}{2}, 3\}$

d. $y^4 - 7y^3 + 5y^2 + 40y - 48 = 0$;
$y = 4$: $(4)^4 - 7(4)^3 + 5(4)^2 + 40(4) - 48 = 256 - 448 + 80 + 160 - 48 = 0$;
$\dfrac{y^4 - 7y^3 + 5y^2 + 40y - 48}{y - 4} = y^3 - 3y^2 - 7y + 12$;
$y = 4$: $(4)^3 - 3(4)^2 - 7(4) + 12 = 64 - 48 - 28 + 12 = 0$; $\dfrac{y^3 - 3y^2 - 7y + 12}{y - 4} = y^2 + y - 3$;
$(y - 4)(y - 4)(y^2 + y - 3) = 0$; $y - 4 = 0$ or $y - 4 = 0$ or $y^2 + y - 3 = 0$; $y = 4$ or $y = \dfrac{-1 \pm \sqrt{(1)^2 - 4(1)(-3)}}{2(1)} = \dfrac{-1 \pm \sqrt{13}}{2}$. $\left\{4, \dfrac{-1 + \sqrt{13}}{2}, \dfrac{-1 - \sqrt{13}}{2}\right\}$

4. a. $x = -1$: $2(-1)^3 + k(-1) - 4 = -2 - k - 4 = -6 - k$; if $-6 - k = 0$, then $k = -6$

b. $x = 2$: $(2)^3 + k(2)^2 - 14(2) + 2k = 8 + 4k - 28 + 2k = 6k - 20$; if $6k - 20 = 0$, then $k = \frac{10}{3}$

5. a. $x = y$: $(y)^n - y^n = 0$

b. $x = -y$: $(-y)^n + y^n$; if n is an odd integer, then $(-y)^n = -y^n$, and $-y^n + y^n = 0$; if n is an even integer, then $(-y)^n = y^n$, and $y^n + y^n \neq 0$.

CHAPTER 14. Geometry and Trigonometry

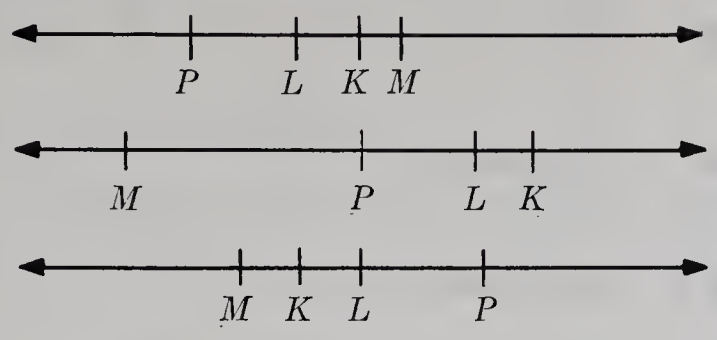
Pages 494–495 · **WRITTEN EXERCISES**

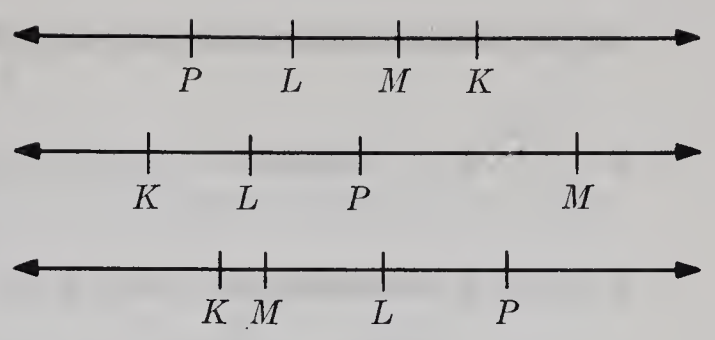
A

1. Axiom I **2.** Axiom II **3.** Axiom IV **4.** Axiom II
5. Axiom II **6.** Axiom IV **7.** Axiom IV or Axiom III **8.** R
9. S **10.** S, W, or T **11.** 6 **12.** 3
13. 5 **14.** 6 **15.** $4 + 2 = 6$ **16.** $6 + 3 = 9$

17.

18.





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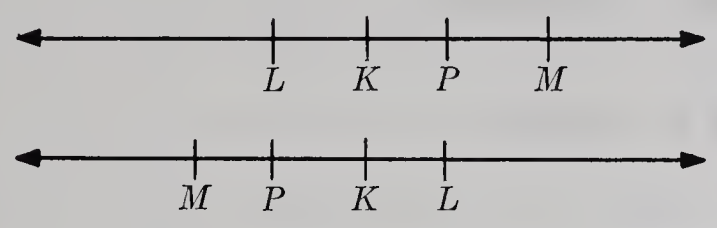

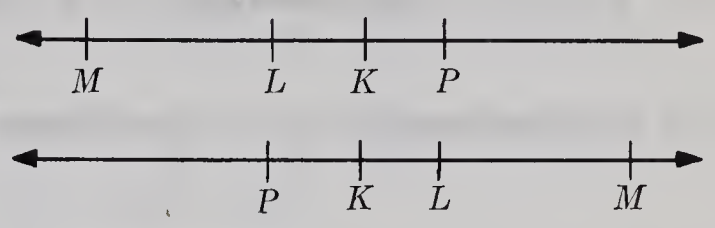

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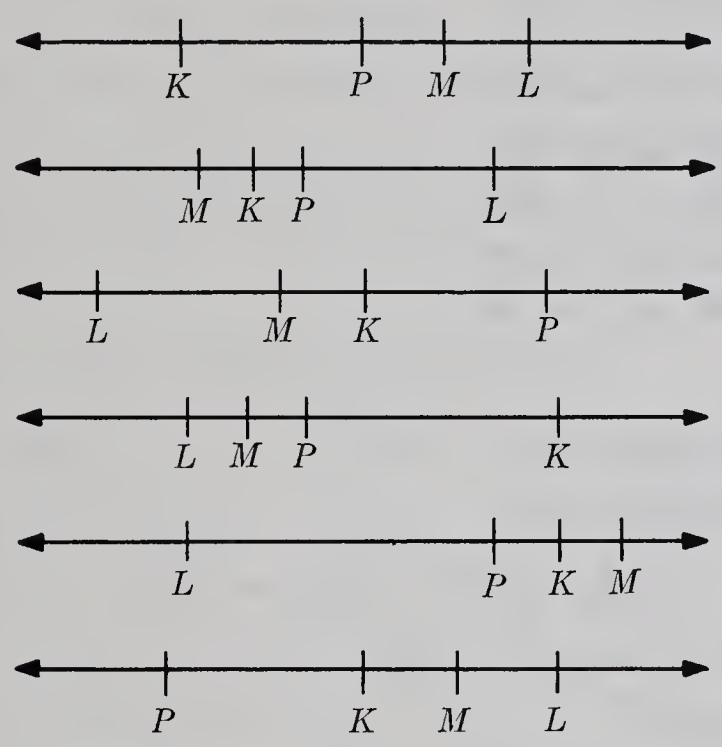
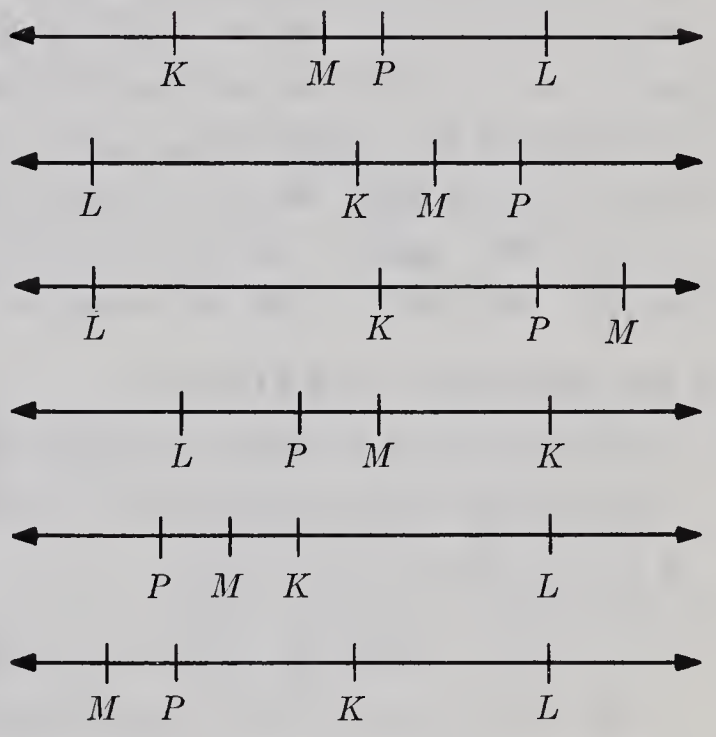




Pages 497–498 · **WRITTEN EXERCISES**

A **1.** The sum of the measures of the angles of a triangle is 180°

2. Subst. principle

3. Given

4. Addition axiom for equality

5. Subtraction axiom for equality

6. The sum of the measures of the three angles of a triangle cannot exceed 180°

7. The three vertices of a triangle determine a plane (Axiom IV); then use Axiom VI taking two vertices at a time.

8. Either both are right angles, or both are equal acute angles, or one is an acute and the other is a right angle.

9.

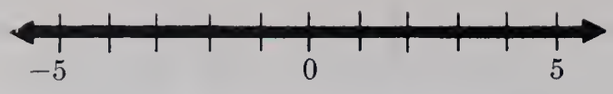
A ray

10.

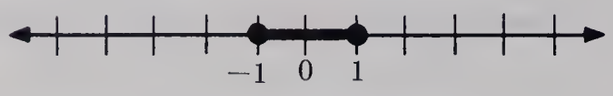
A point

11.

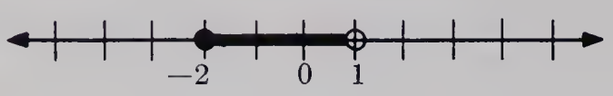
None of these

12. 

A line

13. 

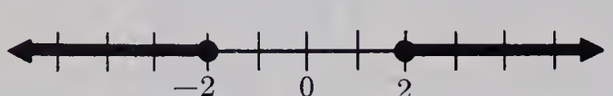
A line segment

14. 

None of these

15.

A line segment

16. 

Two rays

17. $40 + 60 + m\angle C = 180$; $100 + m\angle C = 180$; $m\angle C = 80°$

18. $70 + 50 + m\angle C = 180$; $120 + m\angle C = 180$; $m\angle C = 60°$

19. $45 + 45 + m\angle C = 180$; $90 + m\angle C = 180$; $m\angle C = 90°$

20. $60 + 30 + m\angle C = 180$; $90 + m\angle C = 180$; $m\angle C = 90°$

21. $90 + 35 + m\angle C = 180$; $125 + m\angle C = 180$; $m\angle C = 55°$

22. $24 + 90 + m\angle C = 180$; $114 + m\angle C = 180$; $m\angle C = 66°$

23. $110 + 50 + m\angle C = 180$; $160 + m\angle C = 180$; $m\angle C = 20°$

24. $130 + 10 + m\angle C = 180$; $140 + m\angle C = 180$; $m\angle C = 40°$

Page 500 · **WRITTEN EXERCISES**

A **1.** $20 + 65 + m\angle L = 180$; $85 + m\angle L = 180$; $m\angle L = 95°$

2. $40 + 55 + m\angle L = 180$; $95 + m\angle L = 180$; $m\angle L = 85°$

3. $\frac{3}{5} = \frac{a'}{15}$; $5a' = 45$; $a' = 9$ ft

4. $\frac{4}{2} = \frac{30}{b'}$; $4b' = 60$; $b' = 15$ ft

5. $c = AB$, $c' = AB'$; $\frac{.30}{.39} = \frac{.50}{c'}$; $.3c' = .195$; $3c' = 1.95$; $c' = .65$;
$BB' = AB' - AB = .65 - .39 = .26$ M, *Ans.*

6. $c = AB$, $c' = AB' = AB + BB' = 4.2 + 6.0 = 10.2$ M; $\frac{3.5}{4.2} = \frac{b'}{10.2}$; $4.2b' = 35.7$;
$42b' = 357$; $b' = 8.5$ M, *Ans.*

7. $\frac{15}{20} = \frac{x}{12}$; $20x = 180$; $x = 9$ in; $\frac{15}{20} = \frac{y}{16}$; $20y = 240$; $y = 12$ in. 9 in, 12 in, *Ans.*

8. $\frac{12}{8} = \frac{x}{10}$; $8x = 120$; $x = 15$ M; $\frac{12}{8} = \frac{y}{15}$; $8y = 180$; $y = 22\frac{1}{2}$ M. 15 M, $22\frac{1}{2}$ M, *Ans.*

9. $\frac{CF}{AC} = \frac{5}{12}$

10. $\frac{DE}{AD} = \frac{5}{12}$

11. $\frac{JH}{AJ} = \frac{5}{12}$

12. $\frac{CF}{AF} = \frac{BG}{AG} = \frac{DE}{AE}$

13. $\frac{AB}{AG} = \frac{AC}{AF} = \frac{AD}{AE}$

14. $AD = 12 + 8 + 8.8 = 28.8$ in; $\frac{DE}{28.8} = \frac{5}{12}$; $12(DE) = 144$; $DE = 12$ in, *Ans.*

Pages 500–501 · **PROBLEMS**

A **1.** $\frac{6}{8} = \frac{h}{40}$; $8h = 240$; $h = 30$ ft

2. $\frac{h}{50} = \frac{6}{7}$; $7h = 300$; $h = 42\frac{6}{7}$ ft

3. Since 1 yd = 3 ft, $\frac{h}{50} = \frac{3}{5}$; $5h = 150$; $h = 30$ ft

4. $\frac{5}{4} = \frac{h}{14}$; $4h = 70$; $h = 17\frac{1}{2}$ ft

5. $\frac{16}{14} = \frac{w}{27}$; $14w = 432$; $w = 30\frac{6}{7}$ ft

6. $\frac{l}{28} = \frac{17}{14}$; $14l = 476$; $l = 34$ ft

Pages 503–504 · **PROBLEMS**

A **1.** $A = 37°$; $b = 40$ ft; $\tan 37° = \frac{a}{b}$; $.7536 \doteq \frac{a}{40}$; $a \doteq 40(.7536) = 30.144 \doteq 30$ ft, *Ans.*

2. $A = 40°$; $b = 50$ ft; $\tan 40° = \frac{a}{b}$; $.8391 \doteq \frac{a}{50}$; $a \doteq 50(.8391) = 41.955 \doteq 42$ ft, *Ans.*

3. $A = 40°$; $b = 120$ ft; $\tan 40° = \frac{a}{b}$; $.8391 \doteq \frac{a}{120}$; $a \doteq 120(.8391) = 100.692 \doteq 101$ ft, *Ans.*

4. $A = 60°$; $b = 50$ ft; $\tan 60° = \frac{a}{b}$; $1.7321 \doteq \frac{a}{50}$; $a \doteq 50(1.7321) = 86.605 \doteq 87$ ft, *Ans.*

5. $A = 25°$; $a = 150$ ft; $\tan 25° = \frac{a}{b}$; $.4663 \doteq \frac{150}{b}$; $.4663b \doteq 150$; $b \doteq \frac{150}{.4663} \doteq 321.6 \doteq 322$ ft, *Ans.*

6. $A = 14°$; $a = 1200$ ft; $\tan 14° = \frac{a}{b}$; $.2493 \doteq \frac{1200}{b}$; $.2493b \doteq 1200$;

$b \doteq \frac{1200}{.2493} \doteq 4813.4 \doteq 4813$ ft, *Ans.*

7. $a = 70$ ft; $b = 100$ ft; $\tan A = \frac{70}{100} = .7$; $A \doteq 35°$, *Ans.*

8. $a = 6$ ft; $b = 10$ ft; $\tan A = \frac{6}{10} = .6$; $A \doteq 31°$, *Ans.*

9. $A = 28°$; $a = 40$ ft; $\tan 28° = \frac{a}{b}$; $.5317 \doteq \frac{40}{b}$; $.5317b \doteq 40$; $b \doteq \frac{40}{.5317} \doteq 75.2 \doteq 75$ ft, *Ans.*

10. $A = 32°$; $a = 200$ ft; $\tan 32° = \frac{a}{b}$; $.6249 \doteq \frac{200}{b}$; $.6249b \doteq 200$; $b \doteq \frac{200}{.6249} \doteq 320$ ft, *Ans.*

Pages 505–506 · **WRITTEN EXERCISES**

A **1.** $\frac{12}{15} = \frac{4}{5}$ **2.** $\frac{12}{15} = \frac{4}{5}$ **3.** $\frac{a}{c} = \frac{12}{15} = \frac{4}{5}$ **4.** $\frac{8}{10} = \frac{4}{5}$ **5.** $\frac{8}{10} = \frac{4}{5}$

6. $\frac{a'}{c'} = \frac{8}{10} = \frac{4}{5}$ **7.** $\frac{6}{10} = \frac{3}{5}$ **8.** $\frac{6}{10} = \frac{3}{5}$ **9.** $\frac{b'}{c'} = \frac{6}{10} = \frac{3}{5}$ **10.** $\frac{9}{15} = \frac{3}{5}$

11. $\frac{9}{15} = \frac{3}{5}$ **12.** $\frac{b}{c} = \frac{9}{15} = \frac{3}{5}$ **13.** $\frac{b}{c} = \frac{9}{15} = \frac{3}{5}$ **14.** $\frac{b'}{c'} = \frac{6}{10} = \frac{3}{5}$ **15.** $\frac{a'}{c'} = \frac{8}{10} = \frac{4}{5}$

16. $\frac{a}{c} = \frac{12}{15} = \frac{4}{5}$ **17.** $\frac{a}{b} = \frac{12}{9} = \frac{4}{3}$ **18.** $\frac{b}{a} = \frac{9}{12} = \frac{3}{4}$

Page 506 · **PROBLEMS**

A **1.** Let a = height of hill in ft; $c = 1250$ ft; $A = 18°$; $\sin 18° = \frac{a}{c}$; $.3090 \doteq \frac{a}{1250}$;

$a \doteq 1250(.3090) = 386.25 \doteq 386.3$ ft, *Ans.*

2. Let a = height of tower in ft; $c = 800$ ft; $A = 27°$; $\sin 27° = \frac{a}{c}$; $.4540 \doteq \frac{a}{800}$;

$a \doteq 800(.4540) \doteq 363.2$ ft, *Ans.*

3. Let b = distance from building in ft; $c = 40$ ft; $A = 75°$; $\cos 75° = \frac{b}{c}$; $.2588 \doteq \frac{b}{40}$;

$b \doteq 40(.2588) = 10.352 \doteq 10.4$ ft, *Ans.*

4. Let a = distance up wall in ft; $B = 15°$; $c = 30$ ft; $\cos 15° = \frac{a}{c}$; $.9659 \doteq \frac{a}{30}$;

$a \doteq 30(.9659) = 28.977 \doteq 29.0$ ft, *Ans.*

5. Let c = length of diagonal of rectangle in inches; $a = 25$ in; $A = 42°$; $\sin 42° = \frac{a}{c}$; $.6691 \doteq \frac{25}{c}$;

$.6691c \doteq 25$; $c \doteq \frac{25}{.6691} \doteq 37.36 \doteq 37.4$ in, *Ans.*

6. Let c = length of ladder in ft; $b = 25$ ft; $A = 75°$; $\sin 75° = \frac{b}{c}$; $.9659 \doteq \frac{25}{c}$; $.9659c \doteq 25$;

$c \doteq \frac{25}{.9659} \doteq 25.88 \doteq 25.9$ ft, *Ans.*

7. Let c = distance along road in ft; $a = 40$ ft; $A = 9°$; $\sin 9° = \frac{a}{c}$; $.1564 \doteq \frac{40}{c}$; $.1564c \doteq 40$; $c \doteq \frac{40}{.1564} \doteq 255.75 \doteq 255.8$ ft, *Ans.*

8. Let b = horizontal (ground) distance in ft; $a = 600$ ft; $A = 10°$; $\tan 10° = \frac{a}{b}$; $.1763 \doteq \frac{600}{b}$; $.1763b \doteq 600$; $b \doteq \frac{600}{.1763} \doteq 3403.28 \doteq 3403.3$ ft; *or* let c = air distance in ft; $a = 600$ ft; $A = 10°$; $\sin 10° = \frac{a}{c}$; $.1736 \doteq \frac{600}{c}$; $.1736c \doteq 600$; $c \doteq \frac{600}{.1736} \doteq 3456.22 \doteq 3456.2$ ft. 3403.3 ft along ground or 3456.2 ft through air, *Ans.*

9. Let c = length of wire in ft; $b = 18$ ft; $A = 40°$; $\cos 40° = \frac{b}{c}$; $.7660 \doteq \frac{18}{c}$; $.7660c \doteq 18$; $c \doteq \frac{18}{.7660} \doteq 23.49 \doteq 23.5$ ft, *Ans.*

10. Let c = length of string in ft; $b = 200$ ft; $A = 42°$; $\cos 42° = \frac{b}{c}$; $.7431 \doteq \frac{200}{c}$; $.7431c \doteq 200$; $c \doteq \frac{200}{.7431} \doteq 269.14 \doteq 269.1$ ft, *Ans.*

Page 507 · WRITTEN EXERCISES

A **1.** .0872, .9962, .0875 **2.** .0349, .9994, .0349 **3.** .9511, .3090, 3.0777 **4.** .4540, .8910, .5095
5. .5878, .8090, .7265 **6.** .8660, .5000, 1.7321 **7.** .7431, .6691, 1.1106 **8.** .8572, .5150, 1.6643
9. .9998, .0175, 57.2900 **10.** .0175, .9998, .0175 **11.** .8387, .5446, 1.5399 **12.** .7880, .6157, 1.2799
13. 33° **14.** 57° **15.** 29° **16.** 68° **17.** 78° **18.** 50° **19.** 30° **20.** 41° **21.** 53°

Pages 507–508 · PROBLEMS

A **1.** (a) Let b = distance along AE in mi; $c = 180$ mi; $\angle BAE = 90° - 70° = 20°$; $\cos 20° = \frac{b}{c}$; $.9397 \doteq \frac{b}{180}$; $b \doteq 180(.9397) = 169.146 \doteq 169.1$ mi, *Ans.*

(b) Let b = distance along AN in mi; $c = 180$ mi; $\angle BAN = 70°$; $\cos 70° = \frac{b}{c}$; $.3420 \doteq \frac{b}{180}$; $b \doteq 180(.3420) = 61.56 \doteq 61.6$ mi, *Ans.* (*Note:* If a = distance traveled in northerly direction, $\sin 20° = \frac{a}{180}$ would give same result.)

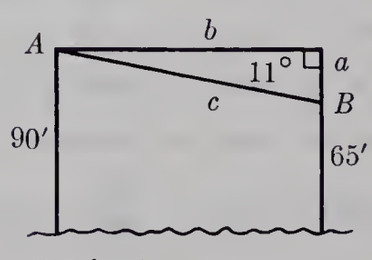
2. (a) $\sin 32° = \frac{x}{20}$; $.5299 \doteq \frac{x}{20}$; $x \doteq 20(.5299) = 10.598 \doteq 10.6$ ft, *Ans.*

(b) Let $y = \frac{1}{2}$ width of house in ft; $\cos 32° = \frac{y}{20}$; $.8480 \doteq \frac{y}{20}$; $y \doteq 20(.8480) = 16.96$; $2y \doteq 2(16.96) \doteq 33.9$ ft, *Ans.*

3. (a) Let b = ground distance in ft covered; $c = 2500$ ft; $A = 16°$; $\cos 16° = \frac{b}{c}$; $.9613 \doteq \frac{b}{2500}$; $b \doteq 2500(.9613) = 2403.25 \doteq 2403.3$ ft, *Ans.*

(b) Let a = change in altitude in ft; $\sin 16° = \frac{a}{c}$; $.2756 \doteq \frac{a}{2500}$; $a \doteq 2500(.2756) \doteq 689.0$ ft, *Ans.*

4. Let b = distance in ft between towers; $\angle CAB = 11°$; $BC = a = 90 - 65 = 25$ ft; $\tan 11° = \frac{a}{b}$; $.1944 \doteq \frac{25}{b}$; $.1944b \doteq 25$; $b \doteq \frac{25}{.1944} \doteq 128.6$ ft, *Ans.*



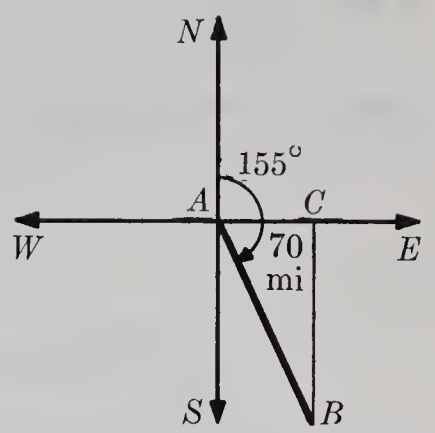
5. $\tan A = \frac{BC}{AC}$; $\tan 50° = \frac{BC}{100}$; $1.1918 \doteq \frac{BC}{100}$; $BC \doteq 119.18 \doteq 119.2$ ft, *Ans.*

6. $\tan A = \frac{BC}{AC}$; $\tan 63° = \frac{BC}{600}$; $1.9626 \doteq \frac{BC}{600}$; $BC \doteq 600(1.9626) = 1177.56 \doteq 1177.6$ ft, *Ans.*

7. $c = 12{,}500$; $a = 1500$; $\sin A = \dfrac{1500}{12{,}500} = .1200$; $A \doteq 7°$, *Ans.*

8. (a) Let c = distance in mi sailed along AB; $b = 70$ mi;
$\angle CAB = 155° - 90° = 65°$; $\cos 65° = \dfrac{b}{c}$; $.4226 \doteq \dfrac{70}{c}$;
$.4226c \doteq 70$; $c \doteq \dfrac{70}{.4226} \doteq 165.64 \doteq 165.6$ mi, *Ans.*

(b) Let a = distance south of port; $\tan 65° = \dfrac{a}{b}$; $2.1445 \doteq \dfrac{a}{70}$;
$a \doteq 70(2.1445) = 150.115 \doteq 150.1$ mi, *Ans.*

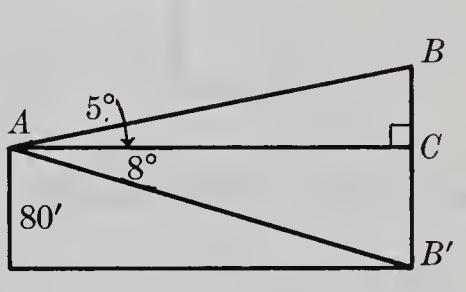


9. The triangle is a right triangle since $(8)^2 + (15)^2 = (17)^2$; $64 + 225 = 289$; $289 = 289$. Thus $\angle C = 90°$; $\tan A = \dfrac{a}{c} = \dfrac{8}{15}$; $\tan A \doteq .5333$; $\angle A \doteq 28°$; $\angle B \doteq 90° - 28° = 52°$.
28°, 52°, 90°, *Ans.*

10. Let c = length in ft of railway; $a = 210$ ft; $\angle A = 14°$; $\sin 14° = \dfrac{210}{c}$; $.2419 \doteq \dfrac{210}{c}$;
$.2419c \doteq 210$; $c \doteq \dfrac{210}{.2419} \doteq 868.13 \doteq 868.1$ ft, *Ans.*

B **11.** For pole P, $A = 20°$, $b = 50$ yd; $\tan 20° = \dfrac{a}{b}$; $.3640 \doteq \dfrac{a}{50}$; $a \doteq 50(.3640) = 18.200$;
for pole P', $A' = 15°$, $b' = 50$ yd; $\tan 15° = \dfrac{a'}{b'}$; $.2679 \doteq \dfrac{a'}{50}$; $a' \doteq 50(.2679) = 13.395$;
$a - a' \doteq 18.200 - 13.395 = 4.805 \doteq 4.8$ yd, *Ans.*

12. To find AC: $\tan 8° = \dfrac{B'C}{AC}$; $.1405 \doteq \dfrac{80}{AC}$; $.1405(AC) \doteq 80$;
$AC \doteq \dfrac{80}{.1405} \doteq 569.39 \doteq 569.4$ ft; to find BC: $\tan 5° = \dfrac{BC}{AC}$;
$.0875 \doteq \dfrac{BC}{569.4}$; $BC \doteq (.0875)(569.4) = 49.8225 \doteq 49.8$ ft;
$B'C + BC \doteq 80 + 49.8 = 129.8$;
height of lighthouse $\doteq 129.8$ ft, *Ans.*



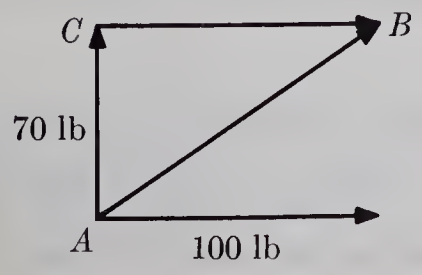
13. $\sin B = \dfrac{b}{c}$; $\cos B = \dfrac{a}{c}$; $\dfrac{\sin B}{\cos B} = \dfrac{\frac{b}{c}}{\frac{a}{c}} = \dfrac{\frac{b}{c}(c)}{\frac{a}{c}(c)} = \dfrac{b}{a} = \tan B$

14. $\sin A = \dfrac{a}{c}$; $\cos A = \dfrac{b}{c}$; $\left(\dfrac{a}{c}\right)^2 + \left(\dfrac{b}{c}\right)^2 = \dfrac{a^2}{c^2} + \dfrac{b^2}{c^2} = \dfrac{a^2 + b^2}{c^2} = \dfrac{c^2}{c^2}$ (since $c^2 = a^2 + b^2$ in a right triangle with right angle at C) $= 1$

Page 511 · **WRITTEN EXERCISES**

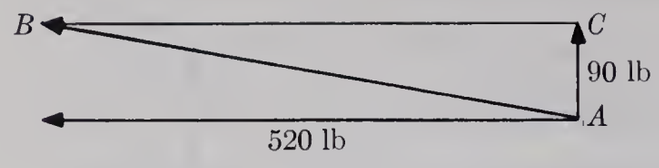
A **1.** No **2.** No **3.** Yes **4.** Yes **5.** Yes **6.** Yes
7. Yes **8.** No **9.** $-\overrightarrow{MN}$ *or* $\overrightarrow{NM}$ **10.** $-\overrightarrow{KL}$ *or* $\overrightarrow{LK}$
11. $-\overrightarrow{NK}$ *or* $\overrightarrow{KN}$ **12.** $-\overrightarrow{LM}$ *or* $\overrightarrow{ML}$ **13.** $\overrightarrow{VS}$ **14.** $\overrightarrow{SV}$
15. $\overrightarrow{WR}$ **16.** $\overrightarrow{TW}$ **17.** $\overrightarrow{ST}$ **18.** $\overrightarrow{WS}$

B **19.** (a)



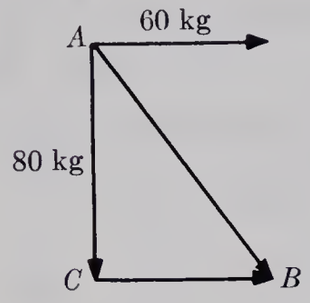
(b) $\tan \angle BAC = \dfrac{CB}{AC} = \dfrac{100}{70} \doteq 1.4286$;
$\angle BAC \doteq 55°$; $90° - 55° \doteq 35°$;
$(AB)^2 = (AC)^2 + (CB)^2 = (70)^2 + (100)^2 = 14{,}900$; $AB = \sqrt{14{,}900} = 10\sqrt{149} \doteq 10(12.2) = 122$; resultant is force of approx. 122 lb at an angle of 35°, *Ans.*

20. (a)



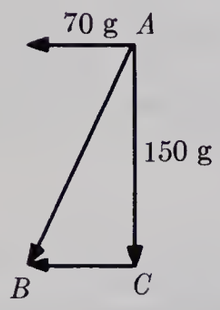
(b) $\tan \angle CAB = \frac{CB}{AC} = \frac{520}{90} \doteq 5.7778$;
$\angle CAB \doteq 80°$; $90° + 80° = 170°$;
$(AB)^2 = (AC)^2 + (CB)^2 = (90)^2 + (520)^2 = 278{,}500$; $AB = \sqrt{278{,}500} = 10\sqrt{2785} \doteq 10(52.8) = 528$; resultant is force of approx. 528 lb at an angle of 170°, *Ans.*

21. (a)



(b) $\tan \angle BAC = \frac{CB}{AC} = \frac{60}{80} = .7500$;
$\angle BAC \doteq 37°$; $270° + 37° = 307°$;
$(AB)^2 = (AC)^2 + (CB)^2 = (80)^2 + (60)^2 = 10{,}000$; $AB = \sqrt{10{,}000} = 100$; resultant is force of 100 kg at an angle of 307°, *Ans.*

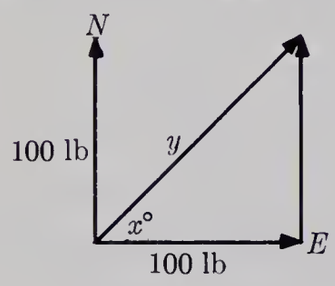
22. (a)



(b) $\tan \angle BAC = \frac{CB}{AC} = \frac{70}{150} \doteq .4667$;
$\angle BAC \doteq 25°$; $270° - 25° = 245°$;
$(AB)^2 = (AC)^2 + (CB)^2 = (150)^2 + (70)^2 = 27{,}400$; $AB = \sqrt{27{,}400} = 10\sqrt{274} \doteq 10(16.6) = 166$; resultant is force of approx. 166 g at angle of 245°, *Ans.*

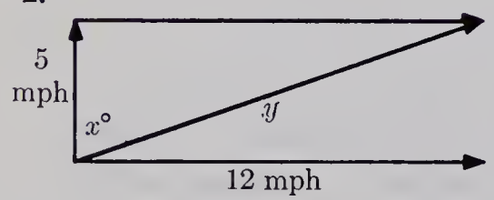
Pages 511–512 · **PROBLEMS**

A **1.**



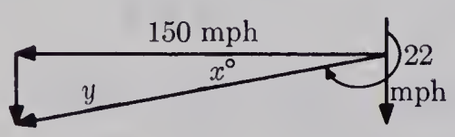
$\tan x° = \frac{100}{100} = 1.0$; $x = 45°$;
$\sin 45° = \frac{100}{y}$; $.7071 \doteq \frac{100}{y}$;
$y \doteq \frac{100}{.7071} \doteq 141.4 \doteq 141$;
force of approx. 141 lb at an angle of 45°, *Ans.*

2.



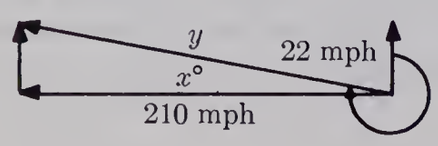
$\tan x° = \frac{12}{5} = 2.4$; $x \doteq 67°$;
$\cos 67° = \frac{5}{y}$; $.3907 \doteq \frac{5}{y}$;
$y \doteq \frac{5}{.3907} \doteq 12.8 \doteq 13$;
rate of (approx.) 13 mph at a bearing of 67°, *Ans.*

3.



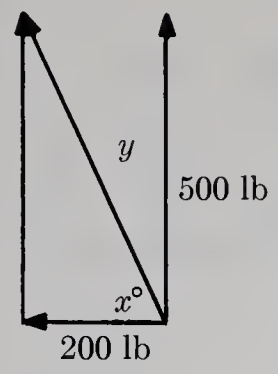
$\tan x° = \frac{22}{150} = .14$; $x \doteq 8°$; $270° - 8° \doteq 262°$;
$\cos 8° = \frac{150}{y}$; $.9903 \doteq \frac{150}{y}$; $y \doteq \frac{150}{.9903} \doteq 151.4 \doteq 151$;
rate of approx. 151 mph at a bearing of 262° *or* at an angle of 188°, *Ans.*

4.



$\tan x° = \frac{22}{210} \doteq .1047$; $x \doteq 6°$; $270° + 6° \doteq 276°$;
$\cos 6° = \frac{210}{y}$; $.9945 \doteq \frac{210}{y}$; $y \doteq \frac{210}{.9945} \doteq 211.2 \doteq 211$;
rate of approx. 211 mph at bearing of 276°, *Ans.*

5.



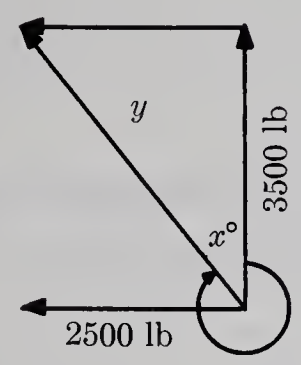
$\tan x° = \frac{500}{200} = 2.5$; $x \doteq 68°$;

$\cos 68° = \dfrac{200}{y}$; $.3746 \doteq \dfrac{200}{y}$;

$y \doteq \dfrac{200}{.3746} \doteq 533.9 \doteq 534$;

force of approx. 534 lb at an angle of 68° (or 112°) with the ground, *Ans.*

6.



$\tan x° = \frac{2500}{3500} \doteq .7142$; $x \doteq 36°$; $360° - 36° = 324°$;

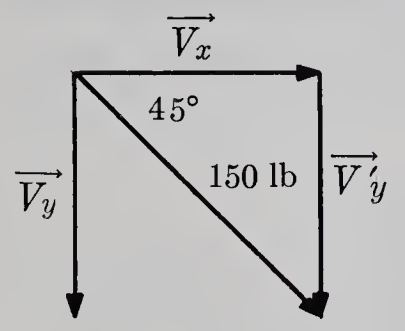
$\cos 36° = \dfrac{3500}{y}$; $.8090 \doteq \dfrac{3500}{y}$;

$y \doteq \dfrac{3500}{.8090} \doteq 4326.3 \doteq 4326$;

force of approx. 4326 lb at a bearing of 324°, *Ans.*

Page 513 · WRITTEN EXERCISES

A **1.**



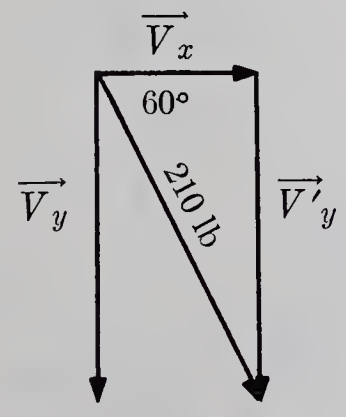
$\sin 45° = \dfrac{\overrightarrow{V'y}}{150}$; $.7071 \doteq \dfrac{\overrightarrow{V'y}}{150}$;

$\overrightarrow{V'y} \doteq 150(.7071) = 106.065 \doteq 106$;

$\cos 45° = \dfrac{\overrightarrow{Vx}}{150}$; $.7071 \doteq \dfrac{\overrightarrow{Vx}}{150}$;

$\overrightarrow{V_x} \doteq 150(.7071) \doteq 106$. Force of 106 lb horizontally and 106 lb vertically downward, *Ans.*

2.



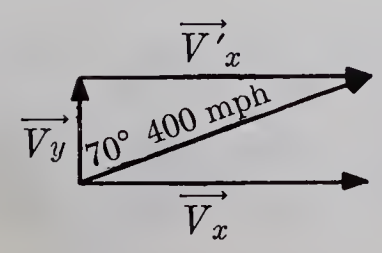
$\sin 60° = \dfrac{\overrightarrow{V'y}}{210}$; $.8660 \doteq \dfrac{\overrightarrow{V'y}}{210}$;

$\overrightarrow{V'y} \doteq 210(.8660) = 181.86 \doteq 182$;

$\cos 60° = \dfrac{\overrightarrow{Vx}}{210}$; $.5000 \doteq \dfrac{\overrightarrow{Vx}}{210}$; $\overrightarrow{Vx} \doteq 210(.5000) = 105$.

Force of 105 lb horizontally and 182 lb vertically downward, *Ans.*

3.



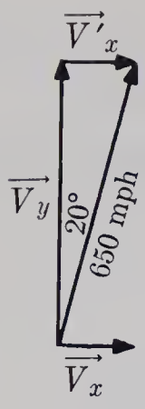
$\sin 70° = \dfrac{\overrightarrow{V'x}}{400}$; $.9397 \doteq \dfrac{\overrightarrow{V'x}}{400}$; $\overrightarrow{V'x} \doteq 400(.9397) =$

$375.88 \doteq 376$; $\cos 70° = \dfrac{\overrightarrow{Vy}}{400}$; $.3420 \doteq \dfrac{\overrightarrow{Vy}}{400}$;

$\overrightarrow{Vy} \doteq 400(.3420) = 136.8 \doteq 137$.

Velocity of 376 mph east and 137 mph north, *Ans.*

4.



$\sin 20° = \dfrac{\overrightarrow{V'x}}{650}$; $.3420 \doteq \dfrac{\overrightarrow{V'x}}{650}$;

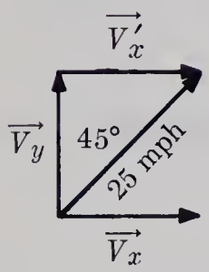
$\overrightarrow{V'x} \doteq 650(.3420) = 222.3 \doteq 222$;

$\cos 20° = \dfrac{\overrightarrow{Vy}}{650}$; $.9397 \doteq \dfrac{\overrightarrow{Vy}}{650}$;

$\overrightarrow{Vy} \doteq 650(.9397) = 610.805 \doteq 611$.

Velocity of 222 mph east and 611 mph north, *Ans.*

5.



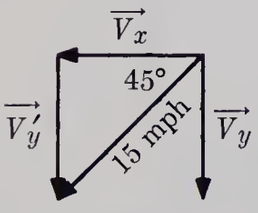
$\cos 45° = \dfrac{\overrightarrow{Vy}}{25}$; $.7071 \doteq \dfrac{\overrightarrow{Vy}}{25}$;

$\overrightarrow{Vy} \doteq 25(.7071) = 17.6775 \doteq 18$;

$\sin 45° = \dfrac{\overrightarrow{V'x}}{25}$; $.7071 \doteq \dfrac{\overrightarrow{V'x}}{25}$; $\overrightarrow{V'x} \doteq 25(.7071) \doteq 18$.

Velocity of 18 mph east and 18 mph north, *Ans.*

6.



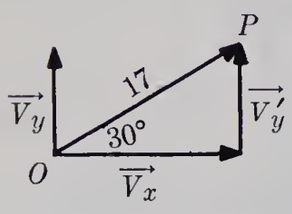
$\sin 45° = \dfrac{\overrightarrow{V'y}}{15}$; $.7071 \doteq \dfrac{\overrightarrow{V'y}}{15}$;

$\overrightarrow{V'y} \doteq 15(.7071) = 10.6065 \doteq 11$;

$\cos 45° = \dfrac{\overrightarrow{Vx}}{15}$; $.7071 \doteq \dfrac{\overrightarrow{Vx}}{15}$; $\overrightarrow{Vx} \doteq 15(.7071) \doteq 11$.

Velocity of 11 mph west and 11 mph south, *Ans.*

B **7.**



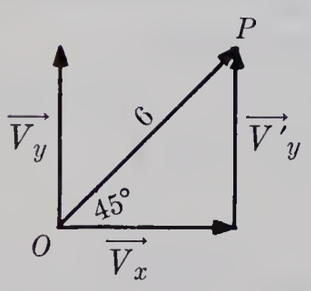
$\cos 30° = \dfrac{\overrightarrow{Vx}}{17}$; $.8660 \doteq \dfrac{\overrightarrow{Vx}}{17}$;

$\overrightarrow{Vx} \doteq 17(.8660) = 14.722 \doteq 14.7$;

$\sin 30° = \dfrac{\overrightarrow{V'y}}{17}$; $.5000 \doteq \dfrac{\overrightarrow{V'y}}{17}$; $\overrightarrow{V'y} \doteq 17(.5000) = 8.5$.

$P(14.7, 8.5)$, *Ans.*

8.



$\cos 45° = \dfrac{\overrightarrow{Vx}}{6}$; $.7071 \doteq \dfrac{\overrightarrow{Vx}}{6}$;

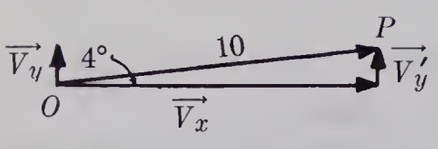
$\overrightarrow{Vx} \doteq 6(.7071) = 4.2426 \doteq 4.2$;

$\sin 45° = \dfrac{\overrightarrow{V'y}}{6}$; $.7071 \doteq \dfrac{\overrightarrow{V'y}}{6}$;

$\overrightarrow{V'y} \doteq 6(.7071) = 4.2426 \doteq 4.2$.

$P(4.2, 4.2)$, *Ans.*

9.

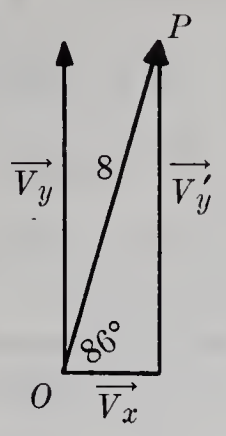


$\cos 4° = \dfrac{\overrightarrow{Vx}}{10}$; $.9976 \doteq \dfrac{\overrightarrow{Vx}}{10}$; $\overrightarrow{Vx} \doteq 10(.9976) = 9.976 \doteq 10.0$;

$\sin 4° = \dfrac{\overrightarrow{V'y}}{10}$; $.0698 \doteq \dfrac{\overrightarrow{V'y}}{10}$; $\overrightarrow{V'y} \doteq 10(.0698) = .698 \doteq .7$.

$P(10.0, .7)$, *Ans.*

10.



$\cos 86° = \frac{\overrightarrow{Vx}}{8}$; $.0698 \doteq \frac{\overrightarrow{Vx}}{8}$;
$\overrightarrow{Vx} \doteq 8(.0698) = .5584 \doteq .6$;
$\sin 86° = \frac{\overrightarrow{V'y}}{8}$; $.9976 \doteq \frac{\overrightarrow{V'y}}{8}$;
$\overrightarrow{V'y} \doteq 8(.9976) = 7.9808 \doteq 8.0$.
$P(.6, 8.0)$, *Ans.*

11. $X_R = 4 + 5 = 9$; $Y_R = 3 + 6 = 9$; (9, 9) **12.** $X_R = 7 + 9 = 16$; $Y_R = 8 + 4 = 12$; (16, 12)

13. $X_R = 10 + 3 = 13$; $Y_R = -7 + 5 = -2$; (13, −2)

14. $X_R = -3 + 4 = 1$; $Y_R = 1 + 8 = 9$; (1, 9)

Page 514 · **PROBLEMS**

A **1.** $\cos 37° = \frac{x}{45}$; $.7986 \doteq \frac{x}{45}$;
$x \doteq 35.937 \doteq 36$ lb, *Ans.*

2. $\cos 55° = \frac{x}{30}$; $.5736 \doteq \frac{x}{30}$;
$x \doteq 17.208 \doteq 17$ lb, *Ans.*

3. $\cos 35° = \frac{x}{15}$; $.8192 \doteq \frac{x}{15}$;
$x \doteq 12.288 \doteq 12$ lb, *Ans.*

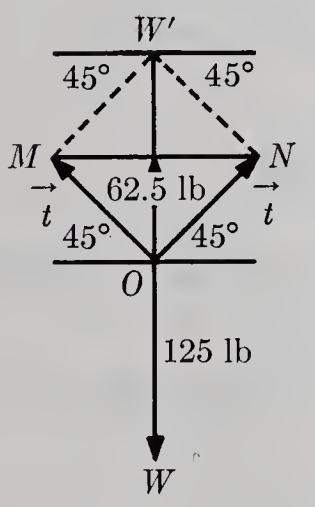
4. $\sin 60° = \frac{x}{150}$; $.8660 \doteq \frac{x}{150}$;
$x \doteq 150(.8660) = 129.900 \doteq 130$ lb, *Ans.*

5. $\cos 20° = \frac{x}{1200}$; $.9397 \doteq \frac{x}{1200}$;
$x \doteq 1127.64 \doteq 1128$ lb, *Ans.*

6. $\cos 52° = \frac{x}{500}$; $.6157 \doteq \frac{x}{500}$; $x = 307.85 \doteq 308$ lb, *Ans.*

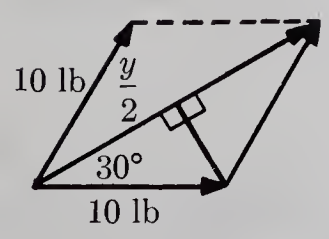
7. Let $\vec{w}$ = horizontal force of wind, and let $\vec{r}$ = resultant force (pull on rope).
$\tan 35° = \frac{3000}{\vec{w}}$; $.7002 \doteq \frac{3000}{\vec{w}}$; $\vec{w} \doteq \frac{3000}{.7002} \doteq 4284.4 \doteq 4284$ lb;
$\sin 35° = \frac{3000}{\vec{r}}$; $.5736 \doteq \frac{3000}{\vec{r}}$; $\vec{r} \doteq \frac{3000}{.5736} \doteq 5230.1 \doteq 5230$ lb. 4284 lb, 5230 lb, *Ans.*

B **8.**



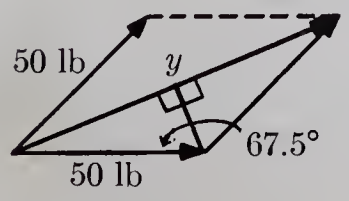
From point O, draw $\overrightarrow{OW}$ representing 125 lb, and lines representing ropes at angles of 45° to the horizontal. Draw OW' equal to $\overrightarrow{OW}$, and lines through W' at angles of 45° from the horizontal. Connect M and N. Let $\vec{t}$ = tension in rope.
$\sin 45° = \frac{62.5}{\vec{t}}$; $.7071 \doteq \frac{62.5}{\vec{t}}$;
$\vec{t} \doteq \frac{62.5}{.7071} \doteq 88.3 \doteq 88$ lb, *Ans.*

9.



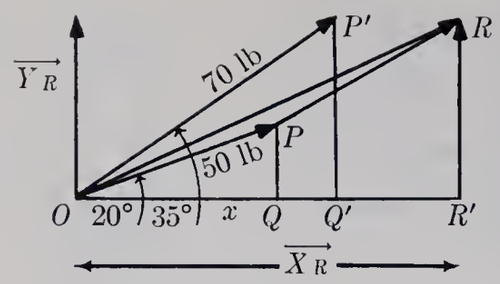
Let $\vec{y}$ represent the resultant force.
Then $\cos 30° = \frac{\frac{\vec{y}}{2}}{10}$; $.8660 = \frac{\vec{y}}{20}$;
$\vec{y} = 20(.8660) = 17.32 \doteq 17$ lb, *Ans.*

10.



Let $\vec{y}$ represent the resultant force. Then $\sin 67.5° = \frac{\frac{\vec{y}}{2}}{50}$;
since $67.5° \doteq 68°$, we have $\sin 68° = \frac{\vec{y}}{100}$; $100(.9272) = \vec{y}$;
$\vec{y} = 92.72 \doteq 93$ lb, *Ans.*

11.



$\sin 20° = \frac{PQ}{50}$; $.3420 \doteq \frac{PQ}{50}$; $PQ \doteq 17.1 \doteq 17$;

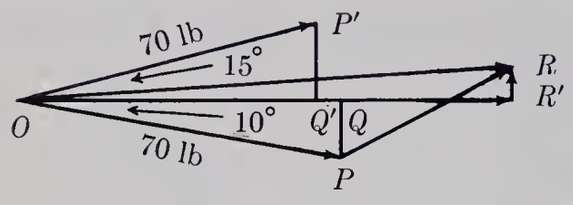
$\cos 20° = \frac{OQ}{50}$; $.9397 \doteq \frac{OQ}{50}$; $OQ \doteq 46.985 \doteq 47$;

$\sin 35° = \frac{P'Q'}{70}$; $.5736 \doteq \frac{P'Q'}{70}$; $P'Q' \doteq 40.152 \doteq 40$;

$\cos 35° = \frac{OQ'}{70}$; $.8192 \doteq \frac{OQ'}{70}$; $OQ' \doteq 57.344 \doteq 57$.

From $P(47, 17)$ and $P'(57, 40)$, we have $R(47 + 57, 17 + 40) = R(104, 57)$. Force of approx. 104 lb horizontally and 57 lb vertically, *Ans.*

12.



$\sin 10° = \frac{PQ}{70}$; $.1736 \doteq \frac{PQ}{70}$; $PQ \doteq 12.152 \doteq 12$;

$\cos 10° = \frac{OQ}{70}$; $.9848 \doteq \frac{OQ}{70}$; $OQ \doteq 68.936 \doteq 69$;

$\sin 15° = \frac{P'Q'}{70}$; $.2588 \doteq \frac{P'Q'}{70}$; $P'Q' \doteq 18.116 \doteq 18$;

$\cos 15° = \frac{OQ'}{70}$; $.9659 \doteq \frac{OQ'}{70}$; $OQ' \doteq 67.613 \doteq 68$;

From $P(69, -12)$ and $P'(68, 18)$, we have $R(69 + 68, -12 + 18) = R(137, 6)$. Force of approx. 137 lb horizontally and 6 lb vertically, *Ans.*

Pages 515–516 · CHAPTER TEST

1. False; if line l_1 contains points P, Q, R and if line l_2 contains points Q, R, S, then by Axiom II, $l_1 = l_2$.

2. True if and only if point D coincides with point Q.

3. $m\angle C + 25° + 40° = 180°$; $m\angle C + 65° = 180°$; $m\angle C = 115°$

4. A ray **5.** A line segment **6.** A point **7.** None of these

8. $\frac{8}{17} = \frac{40}{TM}$; $8(TM) = 680$; $TM = 85$; $LM = 85 - 17 = 68$

9. Let x = distance in ft of boat from cliff; $\tan 58° = \frac{x}{150}$; $1.60 \doteq \frac{x}{150}$; $x \doteq 150(1.60) = 240$ ft, *Ans.*

10. Let y = vertical change in mi; $\sin 10° = \frac{y}{2}$; $.174 \doteq \frac{y}{2}$; $y \doteq 2(.174) = .348$; $y \doteq .3$.

Let x = horizontal change in mi; $\cos 10° = \frac{x}{2}$; $.985 \doteq \frac{x}{2}$; $x \doteq 2(.985) = 1.97 \doteq 2.0$.

Vertical, approx. .3 mi; horizontal, approx. 2.0 mi, *Ans.*

11. Let x = height in ft of building and flagpole; $\tan 41° = \frac{x}{100}$; $.8693 \doteq \frac{x}{100}$;

$x \doteq 100(.8693) = 86.93 \doteq 87$; $x - 75 \doteq 12$ ft, *Ans.*

12. Let r = magnitude of resultant; $r^2 = (90)^2 + (135)^2 = 26{,}325$; $r = \sqrt{26{,}325} = \sqrt{5^2 \cdot 9^2 \cdot 13} = 45\sqrt{13} \doteq 45(3.606) \doteq 162.2 \doteq 162$ lb, *Ans.*

(*Note:* Ex. could also be done using $\tan x° = \frac{135}{90}$; $x \doteq 56°$; $\sin 56° = \frac{135}{r}$, yielding $r \doteq 161$ lb)

13. $\tan x° = \frac{135}{90} = 1.5$; $x \doteq 56°$

14. $\sin 15° = \frac{x}{145}$; $.2588 \doteq \frac{x}{145}$; $x \doteq 37.526 \doteq 38$; $\cos 15° = \frac{y}{145}$; $.9659 \doteq \frac{y}{145}$; $y \doteq 140.0555 \doteq 140$.

Force of approx. 38 lb horizontally and 140 lb vertically, *Ans.*

Pages 516–518 · CHAPTER REVIEW

1. two different, three **2.** C, Q, P **3.** $x \geq 5$ **4.** 180° **5.** equal

6. $\frac{x}{10} = \frac{52}{13}$; $13x = 520$; $x = 40$; $\frac{y}{12} = \frac{52}{13}$; $13y = 624$; $y = 48$. 40, 48, *Ans.*

7. Yes; each has angles with measure 35°, 55°, 90°, respectively.

8. $\frac{a'}{20} = \frac{4}{5}$; $5a' = 80$; $a' = 16$

9. $\frac{a'}{21} = \frac{2}{3}$; $3a' = 42$; $a' = 14$

10. $\frac{a}{b} = \frac{28}{70}$; $a:b = 2:5$

11. opposite, adjacent

12. a, b

13. b, a

14. Let $x + 5$ = no. of ft in height of flagpole; $\tan 45° = \frac{x}{50}$; $1 = \frac{x}{50}$; $x = 50$; $x + 5 = 55$ ft, *Ans.*

15. $\sin 28° = \frac{a}{1.3}$; $.47 \doteq \frac{a}{1.3}$; $a \doteq (1.3)(.47) = .611 \doteq .6$ in, *Ans.*

16. $\sin 35° = \frac{a}{DQ}$; $.57 \doteq \frac{.6}{DQ}$; $.57(DQ) \doteq .6$; $DQ \doteq \frac{.6}{.57} \doteq 1.1$ in, *Ans.*

17. Let $x + y = PQ$; $\cos 28° = \frac{x}{1.3}$; $.88 \doteq \frac{x}{1.3}$; $x \doteq (1.3)(.88) = 1.144 \doteq 1.1$ in; $\cos 35° \doteq \frac{y}{1.1}$; $.82 \doteq \frac{y}{1.1}$; $y \doteq (1.1)(.82) = .902 \doteq .9$; $PQ \doteq 1.1 + .9 \doteq 2.0$ in, *Ans.*

18. a, c **19.** $\sin 15° = .2588$; $\sin 55° = .8192$; $\therefore \sin 55°$ is greater

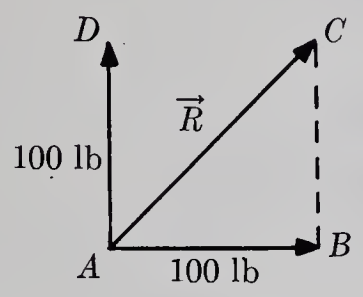
20. .8192 **21.** .1736 **22.** .2309 **23.** 12° **24.** 72° **25.** 48° **26.** 56°

27. Let x = height in ft that wire reaches up pole; $\sin 55° = \frac{x}{25}$; $.8192 \doteq \frac{x}{25}$; $x \doteq 25(.8192) = 20.48 \doteq 20$ ft, *Ans.*

28. Let c = length in ft of ladder; $\cos 50° = \frac{40}{c}$; $.6428 = \frac{40}{c}$; $.6428c = 40$; $c = \frac{40}{.6428} \doteq 62$ ft, *Ans.*

29. $\tan A = \frac{7}{11} \doteq .6363$; $A \doteq 32°$, *Ans.*

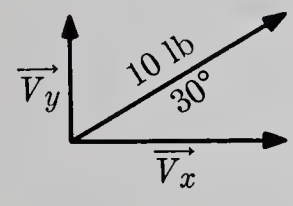
30.



31. $R^2 = (100)^2 + (100)^2 = 20{,}000$; $R = \sqrt{20{,}000} = 100\sqrt{2} \doteq 100(1.414) \doteq 141$ lb; $\tan x° = \frac{100}{100} = 1.0$; $x = 45°$. 141 lb; 45°, *Ans.*

32. Let $x°$ = bearing; $\tan x° = \frac{2}{5} = .4$; $x \doteq 22°$; $r^2 = (2)^2 + (5)^2$; $r^2 = 4 + 25 = 29$; $r = \sqrt{29} \doteq 5.4$. Rate of approx. 5.4 mph at a bearing of 22°, *Ans.*

33.



34. $\cos 55° = \frac{x}{200}$; $.5736 \doteq \frac{x}{200}$; $x \doteq 114.72 \doteq 115$; $\sin 55° = \frac{y}{200}$; $.8192 = \frac{y}{200}$; $y \doteq 163.84 \doteq 164$. Force of approx. 115 lb horizontally and 164 lb vertically, *Ans.*

Page 522 · EXTRA FOR EXPERTS

1. a. $\sqrt{64} \cdot \sqrt{-1} = 8i$; b. $\sqrt{27} \cdot \sqrt{-1} = 3i\sqrt{3}$; c. $\sqrt{\frac{81}{25}} \cdot \sqrt{-1} = \frac{9}{5}i$; d. $\sqrt{.25} \cdot \sqrt{-1} = .5i$

2. a. $-5 - 2i$; $5 - 2i$ b. $-6 + 7i$; $6 + 7i$

c. $3 + \sqrt{-1}$ *or* $3 + i$; $-3 + \sqrt{-1}$ *or* $-3 + i$ d. $-7 - 3i$; $7 - 3i$

3. a. $r + s = (3 + 7i) + (i) = 3 + 8i$; $r - s = (3 + 7i) - (i) = 3 + 6i$;

$r \cdot s = (3 + 7i)(i) = 3i + 7(-1) = -7 + 3i$; $r \div s = \frac{3 + 7i}{i} = \frac{3 + 7i}{i} \cdot \frac{i}{i} = \frac{-7 + 3i}{-1} = 7 - 3i$

b. $r + s = (15 + i) + (1 - i) = 16$ *or* $16 + 0i$; $r - s = (15 + i) - (1 - i) = 14 + 2i$;

$r \cdot s = (15 + i)(1 - i) = 15 - (-1) - 15i + i = 16 - 14i$;

$r \div s = \frac{15 + i}{1 - i} = \frac{15 + i}{1 - i} \cdot \frac{1 + i}{1 + i} = \frac{14 + 16i}{2} = 7 + 8i$

c. $r + s = (-i) + (1 - 2i) = 1 - 3i$; $r - s = (-i) - (1 - 2i) = -1 + i$;

$r \cdot s = (-i)(1 - 2i) = -i + 2(-1) = -2 - i$;

$r \div s = \frac{-i}{1 - 2i} = \frac{-i}{1 - 2i} \cdot \frac{1 + 2i}{1 + 2i} = \frac{2 - i}{5} = \frac{2}{5} - \frac{1}{5}i$

d. $r + s = (i) + (-3 + 2i) = -3 + 3i$; $r - s = (i) - (-3 + 2i) = 3 - i$;

$r \cdot s = (i)(-3 + 2i) = -3i + 2(-1) = -2 - 3i$;

$r \div s = \frac{i}{-3 + 2i} = \frac{i}{-3 + 2i} \cdot \frac{-3 - 2i}{-3 - 2i} = \frac{2 - 3i}{13} = \frac{2}{13} - \frac{3}{13}i$

e. $r + s = (9 - 3i) + (7 + 6i) = 16 + 3i$; $r - s = (9 - 3i) - (7 + 6i) = 2 - 9i$;

$r \cdot s = (9 - 3i)(7 + 6i) = 63 - 18(-1) + 54i - 21i = 81 + 33i$;

$r \div s = \dfrac{9 - 3i}{7 + 6i} = \dfrac{9 - 3i}{7 + 6i} \cdot \dfrac{7 - 6i}{7 - 6i} = \dfrac{45 - 75i}{85} = \dfrac{45}{85} - \dfrac{75}{85}i = \dfrac{9}{17} - \dfrac{15}{17}i$

f. $r + s = (-3 - 5i) + (6 + i) = 3 - 4i$; $r - s = (-3 - 5i) - (6 + i) = -9 - 6i$;
$r \cdot s = (-3 - 5i)(6 + i) = -18 - 5(-1) - 3i - 30i = -13 - 33i$;
$r \div s = \dfrac{-3 - 5i}{6 + i} = \dfrac{-3 - 5i}{6 + i} \cdot \dfrac{6 - i}{6 - i} = \dfrac{-23 - 27i}{37} = -\dfrac{23}{37} - \dfrac{27}{37}i$

4. a. $i^2 \cdot i = -1 \cdot i = -i$; b. $i^2 \cdot i^2 = (-1)(-1) = 1$;
c. $(-3 + 7i)(-3 + 7i) = 9 + 49(-1) - 21i - 21i = -40 - 42i$;
d. $x^2 - y^2(-1) + xyi - xyi = x^2 + y^2$; e. $4 + 4i$;
f. $\dfrac{3 + 3i}{8 + 6i} \cdot \dfrac{8 - 6i}{8 - 6i} = \dfrac{42 + 6i}{100}$; $\dfrac{7 + 4i}{2 - 3i} \cdot \dfrac{2 + 3i}{2 + 3i} = \dfrac{2 + 29i}{13}$;
$\dfrac{42 + 6i}{100} \cdot \dfrac{13}{13} - \dfrac{2 + 29i}{13} \cdot \dfrac{100}{100} = \dfrac{(546 + 78i) - (200 + 2900i)}{1300} = \dfrac{346 - 2822i}{1300} = \dfrac{173}{650} - \dfrac{1411}{650}i$
g. $(-i)[-3 + 2(-1) + i - 6i] = -i(-5 - 5i) = 5i + 5(-1) = -5 + 5i$
h. $\dfrac{3 - 9i}{7 + 2i} \cdot \dfrac{7 - 2i}{7 - 2i} = \dfrac{3 - 69i}{53}$; $\dfrac{8 + 3i}{3 - 4i} \cdot \dfrac{3 + 4i}{3 + 4i} = \dfrac{12 + 41i}{25}$; $\dfrac{3 - 9i}{7 + 2i} \cdot \dfrac{8 + 3i}{3 - 4i} =$
$\dfrac{3 - 69i}{53} \cdot \dfrac{12 + 41i}{25} = \dfrac{(3 - 69i)(12 + 41i)}{(53)(25)} = \dfrac{36 - 2829(-1) + 123i - 828i}{1325} =$
$\dfrac{2865 - 705i}{1325} = \dfrac{573}{265} - \dfrac{141}{265}i$

5. a. $x^2 = -25$; $x = \pm\sqrt{-25} = \pm\sqrt{25} \cdot \sqrt{-1} = \pm 5i$. $\{5i, -5i\}$
b. $y^2 = -75$; $y = \pm\sqrt{-75} = \pm\sqrt{25 \cdot 3} \cdot \sqrt{-1} = \pm 5i\sqrt{3}$. $\{5i\sqrt{3}, -5i\sqrt{3}\}$
c. $x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} = \dfrac{2 \pm \sqrt{-4}}{2} = \dfrac{2 \pm 2i}{2} = 1 \pm i$. $\{1 + i, 1 - i\}$
d. $z = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)} = \dfrac{4 \pm \sqrt{-36}}{2} = \dfrac{4 \pm 6i}{2} = 2 \pm 3i$. $\{2 + 3i, 2 - 3i\}$
e. $t = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(34)}}{2(1)} = \dfrac{6 \pm \sqrt{-100}}{2} = \dfrac{6 \pm 10i}{2} = 3 \pm 5i$. $\{3 + 5i, 3 - 5i\}$
f. $t = \dfrac{-(6) \pm \sqrt{6^2 - 4(1)(34)}}{2(1)} = \dfrac{-6 \pm \sqrt{-100}}{2} = \dfrac{-6 \pm 10i}{2} = -3 \pm 5i$. $\{-3 + 5i, -3 - 5i\}$

CHAPTER 15. Comprehensive Review and Tests

Pages 525–526 · **PROPERTIES OF NUMBERS: STRUCTURE**

1. Distrib. prop. for mult. with respect to subt. $\left(\text{since } \dfrac{4a - 6}{2} = \tfrac{1}{2}(4a - 6)\right)$ *or* distrib. prop. for div. with respect to subt. $\left(\text{since } \dfrac{4a - 6}{2} = [4a - 6] \div 2\right)$
2. Comm. prop. for add. **3.** Meaning of subt. **4.** Prop. of opposites **5.** Mult. prop. of equality
6. Theorem: a product is zero if, and only if, at least one of the factors is zero.
7. Distrib. prop. for mult. with respect to subt. **8.** Defn. of an exponent
9. Subt. prop. of equality **10.** Meaning of div.
11. Order prop. of nos. **12.** Closure prop. for add. **13.** Defn. of subset
14. Defn. of opposite of zero; no other no. has this property
15. Defn. of multiple **16.** Trans. prop. of inequality **17.** Defn. of equiv. equations
18. F; {0, 2, 4, 6, . . .} is closed under addition but {0, 2, 4, 6} is not closed under addition.
19. T **20.** F; graph shown is for $-2 < x < -1$
21. F; it is an assumption **22.** T **23.** F; a could be >0 and $b < 0$
24. F; $\sin A = \frac{3}{5}$ **25.** (25, *E*) **26.** (26, *B*) *or* (26, *C*)
27. (27, *D*) **28.** (28, *A*) **29.** (29, *F*) **30.** (30, *C*)

Pages 527–528 · ALGEBRAIC REPRESENTATION

1. Average $= \dfrac{a+n+p}{3}$, *Ans.*

2. $A = s^2$, $p = 4s$; $s^2 = 9n^2$; $s = 3n$; $p = 12n$, *Ans.*

3. Let $c =$ length of diagonal; $c^2 = a^2 + b^2$; $c^2 = n^2 + 5^2$; $c = \sqrt{n^2+25}$, *Ans.*

4. 12 in = 1 ft; f ft $-$ (t ft 3 in) = $(12f - 12t - 3)$ in, *Ans.*

5. Cost per article $= \dfrac{c}{n}$; $25 \div \dfrac{c}{n} = \dfrac{25n}{c}$, *Ans.*

6. Let $x =$ cost of 1 pencil; then $c = nt + px$; $px = c - nt$; $x = \dfrac{c-nt}{p}$, *Ans.*

7. (a) $m + .20m = 1.2m$ dollars; (b) $m - .10m = .9m$ dollars

8. $12s \le n < t$ **9.** $49\pi < A < 64\pi$ **10.** $3(3)(-1)^2 = 3 \cdot 3 \cdot 1 = 9$

11. $\dfrac{2(3)(-2)}{\frac{1}{2} - (-1)} = \dfrac{-12}{\frac{3}{2}} = \dfrac{-12 \cdot 2}{\frac{3}{2} \cdot 2} = \dfrac{-24}{3} = -8$ **12.** $2(3^2) - 4(\frac{1}{2}) = 2(9) - 2 = 18 - 2 = 16$

13. $5(3)(-1) - 2(\frac{1}{2})^3(-2)^2 + 1 = -15 - 2(\frac{1}{8})(4) + 1 = -15 - 1 + 1 = -15$

14. $\sqrt{4(-2-0)^2 - 6(3)(-1)(\frac{1}{2})} = \sqrt{4(4) + (3)(3)} = \sqrt{16+9} = \sqrt{25} = 5$

15. $3 \times (-2) \div [2(-1)] \div \frac{1}{2} \times (-2) = -6 \div (-2) \div \frac{1}{2} \times (-2) = 3 \div \frac{1}{2} \times (-2) = 6 \times (-2) = -12$

16. 59. **17.** 5.8 **18.** .1 **19.** 1. **20.** 1.0 **21.** .4 **22.** 7.549 $\doteq$ 7.5 (7.6 from table)

23. $\sqrt{\frac{71}{10}} = \sqrt{\frac{71}{10} \cdot \frac{10}{10}} = \frac{1}{10}\sqrt{71} \cdot \sqrt{10} \doteq .1(8.426)(3.162) \doteq .84(3.2) \doteq 2.7$

24. 3 ft : 2 yd = 1 yd : 2 yd = 1 : 2, *Ans.* **25.** 3000 lb : $4\frac{1}{2}$ tons = $\frac{3}{2}$ tons : $\frac{9}{2}$ tons = 1 : 3, *Ans.*

26. 2 weeks : 26 days = 14 days : 26 days = 7 : 13, *Ans.*

27. $(1 \text{ in})^3 : (3 \text{ in})^3 = 1^3 : 3^3 = 1 : 27$, *Ans.*

28. Area of rectangle less areas of two triangles: $(2)(6) - \frac{1}{2}(1)(2) - \frac{1}{2}(1)(1) = 12 - 1 - \frac{1}{2} =$ 10.5 sq ft = 1512.0 sq in, *Ans.*

29. $\dfrac{EF}{12} = \dfrac{7}{6}$; $6(EF) = 84$; $EF = 14$, *Ans.*

30. $\sin 40° = \dfrac{BC}{12}$; $.643 \doteq \dfrac{BC}{12}$; $BC \doteq 12(.643) = 7.716 \doteq 7.7$; $\cos 40° = \dfrac{AC}{12}$; $.766 \doteq \dfrac{AC}{12}$; $AC \doteq 12(.766) = 9.192 \doteq 9.2$. BC, 7.7; AC, 9.2, *Ans.*

Pages 528–529 · FUNDAMENTAL OPERATIONS AND FACTORING

1. $5a + 10 - 3a - 12 = 2a - 2 = 2(a-1)$ **2.** $2m^2n(3m^3n) - 2m^2n(5n^2) = 6m^5n^2 - 10m^2n^3$

3. $6x^2 - x - 2$

4.
$$\begin{array}{r} a^2 + 3ab + 9b^2 \\ a \quad - 3b \\ \hline a^3 + 3a^2b + 9ab^2 \\ \quad - 3a^2b - 9ab^2 - 27b^3 \\ \hline a^3 \qquad\qquad - 27b^3 \end{array}$$

5. $2a - b$

6. $1 - 8a + 16a^2$

7.
$$\begin{array}{r} x^2 - 2x + 4 \\ x+2 \overline{)\, x^3 + 0 \quad + 0 \quad + 8} \\ \underline{x^3 + 2x^2} \\ \quad -2x^2 \\ \quad \underline{-2x^2 - 4x} \\ \qquad\quad 4x + 8 \\ \qquad\quad \underline{4x + 8} \\ \qquad\quad\quad 0 \end{array}$$

Q: $x^2 - 2x + 4$

8. $\dfrac{18ax^3}{-3ax} - \dfrac{6a^2x^2}{-3ax} + \dfrac{3a^3x}{-3ax} = -6x^2 + 2ax - a^2$

9.
$$\begin{array}{r|l} t+3 & 5t^2 - 2t + 11 \\ \hline & 5t^3 + 13t^2 + 5t + 7 \\ & 5t^3 + 15t^2 \\ \hline & -2t^2 + 5t \\ & -2t^2 - 6t \\ \hline & 11t + 7 \\ & 11t + 33 \\ \hline & -26 \end{array} \quad Q: 5t^2 - 2t + 11 + \frac{-26}{t+3}$$

10. $8 - 8a - 2[a + 2b - 6b + 3] = 8 - 8a - 2a - 4b + 12b - 6 = -10a + 8b + 2$

11. $3[14c + 2 - 3c] - 5[3c - 6] - 3 = 42c + 6 - 9c - 15c + 30 - 3 = 18c + 33$

12. $6a^2 + 2ac - 3ab - bc$

13. $\dfrac{2(b^2 - 9)(3)(5)}{2(3)(b)(b-3)} = \dfrac{2(b-3)(3)(b+3)(5)}{2(b-3)(3)(b)} = \dfrac{5(b+3)}{b}$

14. $\dfrac{2a^3x^5}{15b^4} \cdot \dfrac{9b^4y}{8a^6x^6} = \dfrac{2a^3x^5 \cdot 3 \cdot 3b^4 \cdot y}{5 \cdot 3b^4 \cdot 2a^3x^5 \cdot 4a^3x} = \dfrac{2a^3x^5 \cdot 3b^4 \cdot 3 \cdot y}{2a^3x^5 \cdot 3b^4 \cdot 5 \cdot 4a^3x} = \dfrac{3y}{20a^3x}$

15. $\dfrac{(x+1)(x-1)}{(x-1)(x-1)} = \dfrac{x+1}{x-1}$

16. $\dfrac{2(a+4)}{(a+4)(a-7)} = \dfrac{2}{a-7}$

17. $\dfrac{(3c+1)(3c-1)}{(3c+1)} \cdot \dfrac{6c^2+2c}{6c-2} = \dfrac{(3c-1)(2)(c)(3c+1)}{1 \cdot 2(3c-1)} = \dfrac{(3c-1)(2)(c)(3c+1)}{(3c-1)(2)(1)} = \dfrac{c(3c+1)}{1} = 3c^2 + c$

18. $3x^2(1 - 7x)$

19. $(1 + 3a)(1 - 3a)$

20. $2\pi R(2R - H)$

21. $(y - 4)(y + 3)$

22. $(x - 5)^2$

23. $P(1 + RT)$

24. $(9x + 1)(4x - 1)$

25. $(d + .2)(d - .2)$

26. $\frac{1}{2}h(b_1 + b_2)$

27. $2(h^2 + 9)(h + 3)(h - 3)$

28. $(n+3)(n-3)(n+4)(n-4)$

29. $3r(6r - 5)(r + 6)$

30. $\dfrac{6a}{6(a+1)} + \dfrac{5}{6(a+1)} = \dfrac{6a+5}{6(a+1)}$

31. $\dfrac{h^2}{(h+2)(h-2)} - \dfrac{h(h-2)}{(h+2)(h-2)} = \dfrac{h^2 - h^2 + 2h}{h^2 - 4} = \dfrac{2h}{h^2 - 4}$

32. $\dfrac{y(2x+3y)}{y(x^2y)} - \dfrac{x(2x-4y)}{x(xy^2)} = \dfrac{2xy + 3y^2 - 2x^2 + 4xy}{x^2y^2} = \dfrac{3y^2 + 6xy - 2x^2}{x^2y^2}$

33. $\dfrac{3r(2r+3s)}{3r(4s)} - \dfrac{2s(7r+5s)}{2s(6r)} = \dfrac{6r^2 + 9rs - 14rs - 10s^2}{12rs} = \dfrac{6r^2 - 5rs - 10s^2}{12rs}$

34. $\dfrac{2x+3}{(x+1)(x-1)} + \dfrac{x}{(-1)(x-1)} = \dfrac{2x+3+(-1)(x)(x+1)}{(x+1)(x-1)} = \dfrac{2x+3-x^2-x}{(x+1)(x-1)} = \dfrac{3+x-x^2}{x^2-1}$

35. $\dfrac{t-1}{t-1} - \dfrac{t+1}{t-1} = \dfrac{t-1-t-1}{t-1} = \dfrac{-2}{t-1}$ *or* $\dfrac{2}{1-t}$

36. $\dfrac{(a-b)}{(a+b)(a-b)} - \dfrac{1}{(a+b)(a-b)} + \dfrac{a+b}{(a+b)(a-b)} = \dfrac{a-b-1+a+b}{(a+b)(a-b)} = \dfrac{2a-1}{a^2-b^2}$

37. $\dfrac{\left(1+\frac{n}{4}\right)4}{\left(\frac{n}{2}+2\right)4} = \dfrac{4+n}{2n+8} = \dfrac{4+n}{2(n+4)} = \dfrac{1}{2}$

38. $\dfrac{(c^2 - \frac{1}{4})4c}{\left(3+\frac{3}{2c}\right)4c} = \dfrac{4c^3 - c}{12c+6} = \dfrac{c(4c^2-1)}{6(2c+1)} = \dfrac{c(2c-1)(2c+1)}{6(2c+1)} = \dfrac{c(2c-1)}{6}$

39. $\dfrac{\left(1-\frac{r-1}{r+1}\right)[(r+1)(r-1)]}{\left(1-\frac{r+1}{r-1}\right)[(r+1)(r-1)]} = \dfrac{(r+1)(r-1) - (r-1)^2}{(r+1)(r-1) - (r+1)^2} = \dfrac{r^2 - 1 - r^2 + 2r - 1}{r^2 - 1 - r^2 - 2r - 1} = \dfrac{2r-2}{-2r-2} = \dfrac{2(r-1)}{-2(r+1)} = \dfrac{1-r}{1+r}$

Page 529 · **RADICALS**

1. $\sqrt{4 \cdot 7} = 2\sqrt{7}$

2. $\frac{2}{3}\sqrt{36a^2 \cdot 5} = \frac{2}{3} \cdot 6a\sqrt{5} = 4a\sqrt{5}$

3. $\frac{1}{2} \cdot 2b^8 = b^8$

4. $6\sqrt{\frac{1}{3} \cdot \frac{3}{3}} = \frac{6}{3}\sqrt{3} = 2\sqrt{3}$

5. $2\sqrt{\frac{7}{8} \cdot \frac{2}{2}} = 2\sqrt{\frac{14}{16}} = \frac{2}{4}\sqrt{14} = \frac{1}{2}\sqrt{14}$

6. $\sqrt{\frac{4}{3} \cdot \frac{3}{3}} = \frac{2}{3}\sqrt{3}$

7. $3\sqrt{4(a^2 + b^2)} = 3 \cdot 2\sqrt{a^2 + b^2} = 6\sqrt{a^2 + b^2}$

8. $\frac{3}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3}{2}\sqrt{2}$

9. $\frac{\sqrt{2}}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{2 \cdot 3} = \frac{\sqrt{6}}{6}$

10. $\frac{20}{2\sqrt{2}} = \frac{10}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{10}{2}\sqrt{2} = 5\sqrt{2}$

11. $\frac{3(\sqrt{2} + 2)}{3\sqrt{2}} = \frac{\sqrt{2} + 2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2 + 2\sqrt{2}}{2} = \frac{2(1 + \sqrt{2})}{2} = 1 + \sqrt{2}$

12. $\frac{a}{\sqrt{2a}} \cdot \frac{\sqrt{2a}}{\sqrt{2a}} = \frac{a\sqrt{2a}}{2a} = \frac{\sqrt{2a}}{2}$

13. $\frac{1(\sqrt{2} - 1)}{(\sqrt{2} + 1)(\sqrt{2} - 1)} = \frac{\sqrt{2} - 1}{(\sqrt{2})^2 - 1^2} = \frac{\sqrt{2} - 1}{2 - 1} = \frac{\sqrt{2} - 1}{1} = \sqrt{2} - 1$

14. $\frac{6(3 + \sqrt{5})}{(3 - \sqrt{5})(3 + \sqrt{5})} = \frac{6(3 + \sqrt{5})}{3^2 - (\sqrt{5})^2} = \frac{6(3 + \sqrt{5})}{9 - 5} = \frac{6(3 + \sqrt{5})}{4} = \frac{2 \cdot 3(3 + \sqrt{5})}{2 \cdot 2} = \frac{3(3 + \sqrt{5})}{2}$

15. $\sqrt{16 \cdot 2} - \sqrt{4 \cdot 2} = 4\sqrt{2} - 2\sqrt{2} = 2\sqrt{2}$

16. $3\sqrt{25 \cdot 2} - 4\sqrt{49 \cdot 2} = 15\sqrt{2} - 28\sqrt{2} = -13\sqrt{2}$

17. $4\sqrt{9 \cdot 3} + 3 - 2\sqrt{16 \cdot 3} = 12\sqrt{3} + 3 - 8\sqrt{3} = 3 + 4\sqrt{3}$

18. $\frac{1}{2}\sqrt{4 \cdot 3} - 3\sqrt{25 \cdot 3} + \sqrt{9 \cdot 2} = \sqrt{3} - 15\sqrt{3} + 3\sqrt{2} = 3\sqrt{2} - 14\sqrt{3}$

19. $\frac{3}{2}\sqrt{\frac{1}{6} \cdot \frac{6}{6}} - 3\sqrt{4 \cdot 6} + 4\sqrt{\frac{3}{2} \cdot \frac{2}{2}} = \frac{3}{12}\sqrt{6} - 6\sqrt{6} + \frac{4}{2}\sqrt{6} = -\frac{15\sqrt{6}}{4}$

20. $10\sqrt{\frac{4}{5} \cdot \frac{5}{5}} - \frac{4}{3}\sqrt{9 \cdot 5} = 10 \cdot \frac{2}{5}\sqrt{5} - \frac{4}{3} \cdot 3\sqrt{5} = 4\sqrt{5} - 4\sqrt{5} = 0$

21. $\sqrt{18} - 2\sqrt{3} = 3\sqrt{2} - 2\sqrt{3}$

22. $12\sqrt{12} - 8\sqrt{18} = 12 \cdot 2\sqrt{3} - 8 \cdot 3\sqrt{2} = 24\sqrt{3} - 24\sqrt{2}$

23. $9\sqrt{4} - \sqrt{9} = 18 - 3 = 15$

24. $12\sqrt{30 \cdot \frac{4}{5}} = 12\sqrt{6 \cdot 4} = 24\sqrt{6}$

25. 5.4

26. $\sqrt{25 \cdot 17} = 5\sqrt{17} \doteq 5(4.123) = 20.615 \doteq 20.6$

27.
$$\begin{aligned} \text{Let } a^2 &\doteq 37.8 \\ a &= 6.1 \\ 37.8 \div 6.1 &\doteq 6.196 \\ 2)&\overline{12.296} \\ a &= 6.148 \\ 37.8 \div 6.148 &\doteq 6.1483 \\ \sqrt{37.8} &\doteq 6.1 \end{aligned}$$

28. $\sqrt{\frac{824}{100}} = \frac{\sqrt{824}}{10} = \frac{\sqrt{4 \cdot 206}}{10} = \frac{\sqrt{206}}{5} \doteq \frac{14.35}{5} = 2.87 \doteq 2.9$

To find $\sqrt{206}$:

$$\begin{aligned} \text{Let } a^2 &\doteq 206 \\ a &= 14 \\ 206 \div 14 &\doteq 14.71 \\ 2)&\overline{28.71} \\ a &= 14.355 \\ 206 \div 14.355 &\doteq 14.350 \\ \sqrt{206} &\doteq 14.35 \end{aligned}$$

29. $(2\sqrt{3})^3 - 2(2\sqrt{3})^2 - 12(2\sqrt{3}) + 24 = 8\sqrt{27} - 8\sqrt{9} - 24\sqrt{3} + 24 = 24\sqrt{3} - 24 - 24\sqrt{3} + 24 = 0$

30. $(\sqrt{6}+1)(\sqrt{6}-1)\left(\dfrac{\sqrt{6}}{\sqrt{6}+1}-\dfrac{\sqrt{6}}{\sqrt{6}-1}\right)=(\sqrt{6}+1)(\sqrt{6}-1)(k\sqrt{6})$;
$(\sqrt{6})^2-\sqrt{6}-(\sqrt{6})^2-\sqrt{6}=((\sqrt{6})^2-1^2)(k\sqrt{6})$; $6-\sqrt{6}-6-\sqrt{6}=5k\sqrt{6}$;
$-2\sqrt{6}=5k\sqrt{6}$; $k=\dfrac{-2\sqrt{6}}{5\sqrt{6}}$; $k=-\frac{2}{5}$. $\{-\frac{2}{5}\}$

31. $100N = 139.39\overline{39}$
$N = 1.39\overline{39}$
$99N = 138.0000$; $N=\frac{138}{99}=\frac{46}{33}$

32. $10N = 5.7\overline{7}$
$N = .5\overline{7}$
$9N = 5.20$; $N=\dfrac{5.2}{9}=\frac{52}{90}=\frac{26}{45}$

Pages 530–531 · **EQUATIONS**

1. $-2a=-18$; $a=9$. $\{9\}$

2. $2c=6c-3$; $-4c=-3$; $c=\frac{3}{4}$. $\{\frac{3}{4}\}$

3. L.C.D. $=12$; $4(2f)+3(3f)+2(17)=0$; $8f+9f+34=0$; $17f=-34$; $f=-2$. $\{-2\}$

4. L.C.D. $=6$; $3g-2(g+2)=6$; $3g-2g-4=6$; $g=10$. $\{10\}$

5. L.C.D. $=6p$; $3-1=3p$; $3p=2$; $p=\frac{2}{3}$. $\{\frac{2}{3}\}$

6. $8d+4-4d+2=0$; $4d=-6$; $d=-\frac{3}{2}$. $\{-\frac{3}{2}\}$

7. L.C.D. $=3(2t+3)$; $6+6(2t+3)=10(2t+3)$; $6+12t+18=20t+30$; $24+12t=20t+30$; $-8t=6$; $t=-\frac{3}{4}$. $\{-\frac{3}{4}\}$

8. $2[6k+3-k+3]=5-6k+15$; $12k+6-2k+6=20-6k$; $10k+12=20-6k$; $16k=8$; $k=\frac{1}{2}$. $\{\frac{1}{2}\}$

9. $\dfrac{1-v}{3(2v-1)}-\dfrac{2}{3}=\dfrac{1}{2v-1}$; L.C.D. $=3(2v-1)$; $1-v-2(2v-1)=3$; $1-v-4v+2=3$; $-5v=0$; $v=0$. $\{0\}$

10. L.C.D. $=(w-5)(w+5)$; $10+w(w+5)=(30+w)(w-5)$; $10+w^2+5w=30w-150+w^2-5w$; $-20w=-160$; $w=8$. $\{8\}$

11. $\dfrac{2y}{2y-1}-\dfrac{-5}{2y-1}=3$; $2y+5=3(2y-1)$; $2y+5=6y-3$; $-4y=-8$; $y=2$. $\{2\}$

12. L.C.D. $=(x+3)(3-x)$; $6(3-x)+2(x+3)=0$; $18-6x+2x+6=0$; $-4x=-24$; $x=6$. $\{6\}$

13. $\dfrac{-1}{c^2-1}+\dfrac{1}{c-1}=0$; $-1+c+1=0$; $c=0$. $\{0\}$

14. $4.8x-9.6-2.7x-3=0$; $2.1x=12.6$; $21x=126$; $x=6$. $\{6\}$

15. $ax=3b$; $x=\dfrac{3b}{a}$. $\left\{\dfrac{3b}{a}\right\}$

16. $mx+nx=m^2-n^2$; $x(m+n)=(m+n)(m-n)$; $x=m-n$. $\{m-n\}$

17. $2mx-2nx=mn$; $x(2m-2n)=mn$; $x=\dfrac{mn}{2m-2n}$. $\left\{\dfrac{mn}{2m-2n}\right\}$

18. $x^2-16x+64=0$; $(x-8)^2=0$; $x-8=0$; $x=8$. $\{8\}$

19. $x^2=25$; $x=\pm\sqrt{25}=\pm5$. $\{5,-5\}$

20. $h(h-3)=0$; $h=0$ or $h-3=0$; $h=0$ or $h=3$. $\{0,3\}$

21. $t=\dfrac{-(-9)\pm\sqrt{(-9)^2-4(3)(1)}}{2(3)}$; $t=\dfrac{9\pm\sqrt{69}}{6}\doteq\dfrac{9\pm8.307}{6}$;
$t\doteq\dfrac{17.307}{6}\doteq2.88\doteq2.9$ or $t\doteq\dfrac{.693}{6}\doteq.11\doteq.1$. $\{2.9,.1\}$

22. $\sqrt{2y}=8$; $2y=64$; $y=32$. $\{32\}$

23. $9x^2-1=9x^2$; $-1=0$. $\emptyset$

24. $s+2=s+2$. $\{\text{real nos.}\}$

25. $\sqrt{11x+20}=2-x$; $11x+20=4-4x+x^2$; $x^2-15x-16=0$; $(x-16)(x+1)=0$; $x-16=0$ or $x+1=0$; $x=16$ or $x=-1$
Check: $\sqrt{11(16)+20}\stackrel{?}{=}2-16$; $14\neq-14$
$\sqrt{11(-1)+20}\stackrel{?}{=}2-(-1)$; $3=3$. $\{-1\}$

26. $3x - 3y = 3$
$3x - 2y = 0$
$\underline{\qquad\qquad}$
$-y = 3;\ y = -3;\ x - (-3) = 1;\ x + 3 = 1;\ x = -2.\ \{(-2, -3)\}$

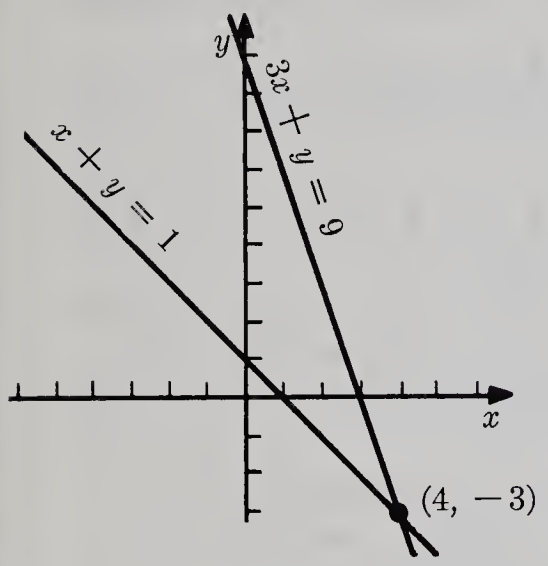
27. $8x - 4y = -24$
$6x + 4y = 3$
$\underline{\qquad\qquad}$
$14x = -21;\ x = -\frac{3}{2};\ 2(-\frac{3}{2}) - y = -6;\ -3 - y = -6;\ y = 3.\ \{(-\frac{3}{2}, 3)\}$

28. $10t + u = 8t + 8u + 7;\ 2t - 7u = 7;\ 2t - 7(t - 6) = 7;\ 2t - 7t + 42 = 7;\ -5t = -35;$
$t = 7;\ u = 7 - 6 = 1.\ \{(7, 1)\}$

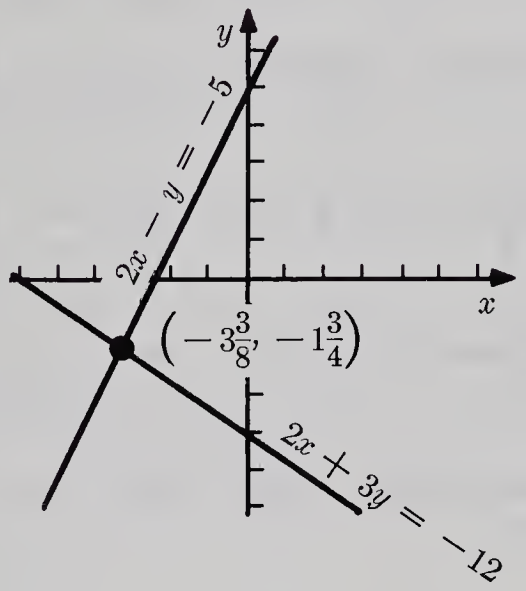
29. $m + 6n = -30$
$m - 20n = 152$
$\underline{\qquad\qquad}$
$26n = -182;\ n = -7;\ \frac{m}{2} + 3(-7) = -15;\ \frac{m}{2} = 6;\ m = 12.\ \{(12, -7)\}$

30. $6r + 4s = 190$
$6r + 15s = 465$
$\underline{\qquad\qquad}$
$-11s = -275;\ s = 25;\ .3r + .2(25) = 9.5;\ .3r = 4.5;\ r = 15.\ \{(15, 25)\}$

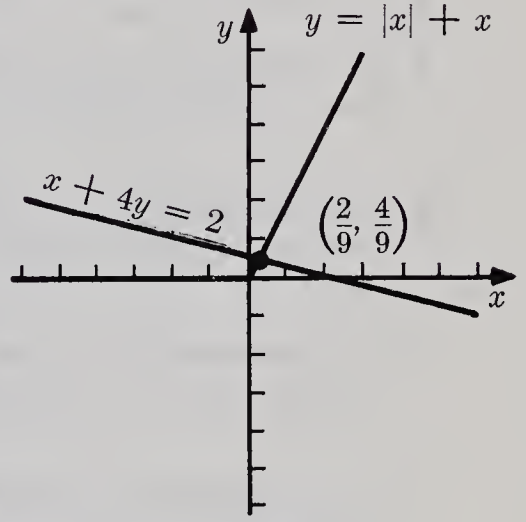
31.



32.



33.



34. $k(4) - 3(-3) = 1;\ 4k + 9 = 1;\ 4k = -8;\ k = -2,$ *Ans.*

35. $2(-2) + 3k(3) = k;\ -4 + 9k = k;\ 8k = 4;\ k = \frac{1}{2},$ *Ans.*

36. $2x - 3(0) = 6;\ 2x = 6;\ x = 3,$ *Ans.*

37. $4(0) - 3y = 12;\ 3y = -12;\ y = -4,$ *Ans.*

38. $y = -4x + b;\ \frac{1}{2} = -4(5) + b;\ \frac{1}{2} = -20 + b;\ b = 20\frac{1}{2};\ y = -4x + 20\frac{1}{2}$ *or* $2y = -8x + 41;$
$8x + 2y - 41 = 0,$ *Ans.*

39. $y = \frac{3}{2}x + b;\ -6 = \frac{3}{2}(-1) + b;\ -6 = -\frac{3}{2} + b;\ b = -\frac{9}{2};\ y = \frac{3}{2}x - \frac{9}{2}$ *or* $2y = 3x - 9;$
$3x - 2y - 9 = 0,$ *Ans.*

40. $-y = -3x + 2;\ y = 3x - 2;\ y = 3x + b;\ 4 = 3(2) + b;\ 4 = 6 + b;\ b = -2;\ y = 3x - 2;$
$3x - y - 2 = 0,$ *Ans.*

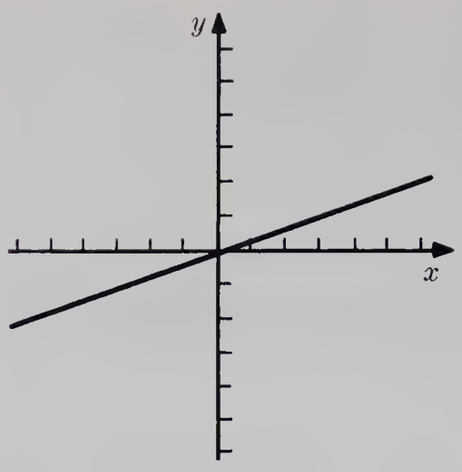
41. $3y = -2x + 3;\ y = -\frac{2}{3}x + 1;\ y = -\frac{2}{3}x + b;\ y = -\frac{2}{3}x + 3$ *or* $3y = -2x + 9;\ 2x + 3y - 9 = 0,$ *Ans.*

42. $m = \dfrac{-2 - 10}{1 - (-1)} = \dfrac{-12}{2} = -6;\ y = -6x + b;\ 10 = -6(-1) + b;\ 10 = 6 + b;\ b = 4;$
$y = -6x + 4;\ 6x + y - 4 = 0,$ *Ans.*

43. $m = \dfrac{6 - (-9)}{1 - (-1)} = \dfrac{6 + 9}{1 + 1} = \dfrac{15}{2};\ y = \frac{15}{2}x + b;\ 6 = \frac{15}{2}(1) + b;\ b = -\frac{3}{2};$
$y = \frac{15}{2}x - \frac{3}{2}$ *or* $2y = 15x - 3;\ 15x - 2y - 3 = 0,$ *Ans.*

44. $ky = -6x + k;\ y = -\dfrac{6}{k}x + 1;\ -\dfrac{6}{k} = 2;\ -6 = 2k;\ k = -3,$ *Ans.*

45.



Pages 531–532 · **FUNCTIONS AND VARIATION**

1. $s = \frac{v^2}{2g}$

2. $9R = 4(F - 32)$; $9R = 4F - 128$; $4F = 9R + 128$; $F = \frac{9R + 128}{4}$ *or* $F = \frac{9}{4}R + 32$

3. $v^2 = 2gh$; $h = \frac{v^2}{2g}$

4. $2A = h(B + b)$; $B + b = \frac{2A}{h}$; $B = \frac{2A}{h} - b$

5. As x increases by 1, y increases by 1; y is 1 less than x; so $y = x - 1$

or

$m = \frac{2}{2} = \frac{1}{1}$; $m = 1$
$0 = 1(1) + b$; $b = -1$
$y = x - 1$

6. As x increases by 1, y increases by 3; y is 2 more than $3x$; so $y = 3x + 2$

or

$m = \frac{6}{2} = \frac{3}{1}$; $m = 3$
$5 = 3(1) + b$; $b = 2$
$y = 3x + 2$

7. As x increases by 1, y increases by 2; y is 1 less than $2x$; so $y = 2x - 1$

or

$m = \frac{2}{1} = \frac{4}{2} = \frac{2}{1}$; $m = 2$
$1 = 2(1) + b$; $b = -1$
$y = 2x - 1$

8. As n increases by 1, c increases by 5; c is 3 less than $5n$; so $c = 5n - 3$

or

Let $x = n$, and let $y = c$
$m = \frac{5}{1} = \frac{10}{2} = \frac{5}{1}$; $m = 5$
$2 = 5(1) + b$; $b = -3$
$y = 5x - 3$; $c = 5n - 3$

9. As t increases by 1, s increases by $\frac{1}{2}$; s is 2 less than $\frac{1}{2}t$; so $s = \frac{1}{2}t - 2$ *or* $2s = t - 4$

or

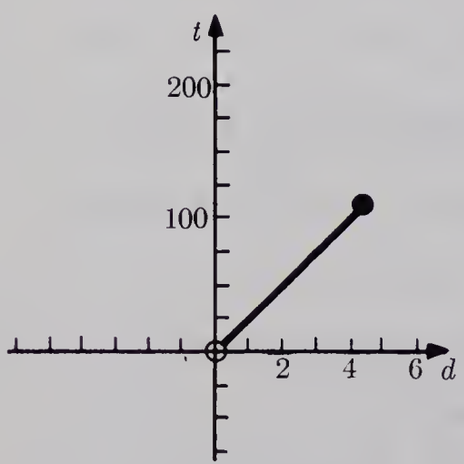
Let $t = x$, and let $s = y$
$m = \frac{1}{2} = \frac{1}{2} = \frac{1}{2}$; $m = \frac{1}{2}$
$0 = \frac{1}{2}(4) + b$; $b = -2$
$y = \frac{1}{2}x - 2$; $s = \frac{1}{2}t - 2$

10. As x increases by 1, y decreases by 3; y is 4 more than $-3x$; so $y = -3x + 4$

or

$m = \frac{-3}{1} = \frac{-3}{1} = \frac{-6}{2}$; $m = -3$
$1 = -3(1) + b$; $b = 4$
$y = -3x + 4$

11.



Domain = {real nos. greater than 0 and less than or equal to $4\frac{2}{5}$}

12. $S = 2wh + 2lh - 2ab - cd$

13. $V \doteq \frac{4}{3}\cdot\frac{22}{7}\cdot\frac{7}{2}\cdot\frac{7}{2}\cdot\frac{7}{2} = \frac{11\cdot 7\cdot 7}{3} = \frac{539}{3}$; $V \doteq 179\frac{2}{3}$, *Ans.*

14. $\frac{x_1}{y_1} = \frac{x_2}{y_2}$ **15.** $\frac{m_1}{m_2} = \frac{n_2^2}{n_1^2}$ **16.** $\frac{n_1}{n_2} = \frac{c_1 d_2}{c_2 d_1}$ **17.** $\frac{A_1}{h_1 b_1} = \frac{A_2}{h_2 b_2}$ **18.** $\frac{r_1}{r_2} = \frac{s_2}{s_1}$ **19.** $\frac{A_1}{B_1} = \frac{A_2}{B_2}$

20. $h = kt$ *or* $\frac{h}{t} = k$

21. $VP = k$ *or* $V = \frac{k}{P}$

22. $xy = k$ *or* $x = \frac{k}{y}$

23. $A = ks^2$ *or* $\frac{A}{s^2} = k$

24. $\frac{f_1}{w_1} = \frac{f_2}{w_2}$; $\frac{81}{18} = \frac{90}{w_2}$; $81w_2 = 90(18)$; $w_2 = \frac{90(18)}{81} = \frac{10(2)}{1} = 20$, *Ans.*

25. $\frac{s_1}{t_1^2} = \frac{s_2}{t_2^2}$; $\frac{64}{2^2} = \frac{100}{t_2^2}$; $64t_2^2 = 400$; $t_2^2 = \frac{400}{64} = \frac{25}{4}$; $t_2 = \frac{5}{2}$, *Ans.*

26. $\frac{x_1}{x_2} = \frac{y_1}{y_2} = \frac{z_1}{z_2}$; $x_1 z_2 = x_2 z_1$ *or* $\frac{x_1}{z_1} = \frac{x_2}{z_2}$; $\frac{6}{9} = \frac{8}{z_2}$; $6z_2 = 72$; $z_2 = 12$, *Ans.*

27. $\frac{r_1}{r_2} = \frac{s_2}{s_1}$; $\frac{6}{9} = \frac{s_2}{\frac{2}{3}}$; $9s_2 = 4$; $s_2 = \frac{4}{9}$, *Ans.*

28. $\frac{F_1}{F_2} = \frac{d_2^2}{d_1^2}$; $\frac{28}{F_2} = \frac{9^2}{6^2}$; $81F_2 = 28(36)$; $F_2 = \frac{28(36)}{81} = \frac{28(4)}{9} = \frac{112}{9} = 12\frac{4}{9}$, *Ans.*

29. $\frac{L_1}{L_2} = \frac{R_2}{R_1}$; $\frac{6}{14} = \frac{420}{R_1}$; $6R_1 = 14(420)$; $R_1 = \frac{14(420)}{6} = \frac{14(70)}{1} = 980$ rpm, *Ans.*

30. $\frac{T_1}{T_2} = \frac{R_2}{R_1}$; $\frac{46}{48} = \frac{600}{R_1}$; $46R_1 = 48(600)$; $R_1 = \frac{48(600)}{46} = \frac{24(600)}{23} = \frac{14{,}400}{23} = 626\frac{2}{23}$ rpm, *Ans.*

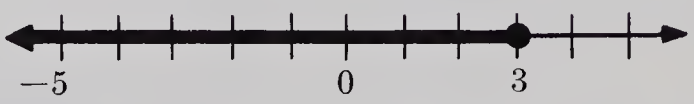
31. $\frac{T_1}{d_1 l_1} = \frac{T_2}{d_2 l_2}$; $\frac{3}{6\cdot 32} = \frac{T_2}{7\cdot 20}$; $6\cdot 32\cdot T_2 = 3\cdot 7\cdot 20$; $T_2 = \frac{3\cdot 7\cdot 20}{6\cdot 32} = \frac{7\cdot 5}{2\cdot 8} = \frac{35}{16} = 2\frac{3}{16}$ hr, *Ans.*

32. a. $\frac{4.6}{4} = \frac{m}{n}$; $4m = 4.6n$; $m = \frac{4.6}{4}n = 1.15n = kn$; $k = 1.15$, *Ans.*

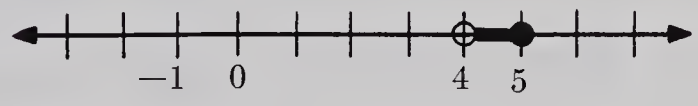
b. $m = 1.15n$; $207 = 1.15n$; $n = \frac{207}{1.15} = \frac{20{,}700}{115} = 180$ naut. mi, *Ans.*

Page 532 · INEQUALITIES

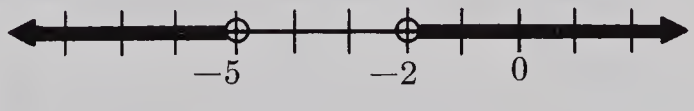
1. $-4v \geq -12$; $4v \leq 12$; $v \leq 3$



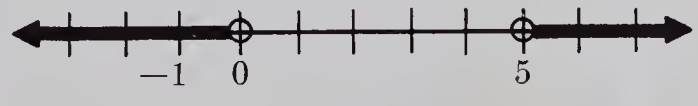
2. $5 < 2n - 3$ and $2n - 3 \leq n + 2$
$8 < 2n$ and $n \leq 5$
$4 < 2n$ and $n \leq 5$
$4 < n \leq 5$



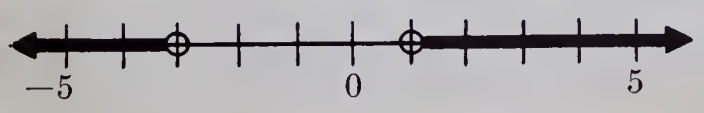
3. $2t + 7 > 3$ or $-(2t + 7) > 3$
$2t > -4$ or $2t + 7 < -3$
$t > -2$ or $2t < -10$
$t > -2$ or $t < -5$



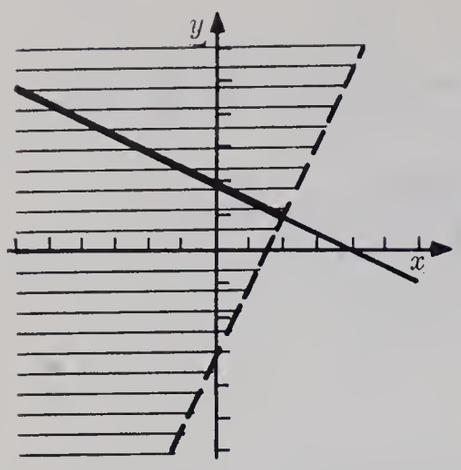
4. $2k - 5 > 5$ or $-(2k - 5) > 5$
$2k > 10$ or $2k - 5 < -5$
$k > 5$ or $2k < 0$
$k > 5$ or $k < 0$



5. $x^2 + 2x - 3 > 0$; $(x + 3)(x - 1) > 0$;
$x + 3 > 0$ and $x - 1 > 0$ or $x + 3 < 0$ and $x - 1 < 0$
$x > -3$ and $x > 1$ or $x < -3$ and $x < 1$
$x > 1$ or $x < -3$



6.



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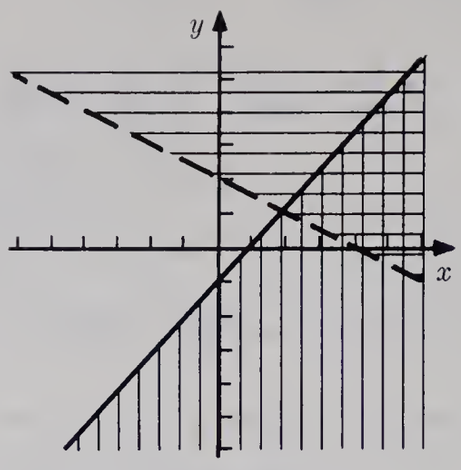
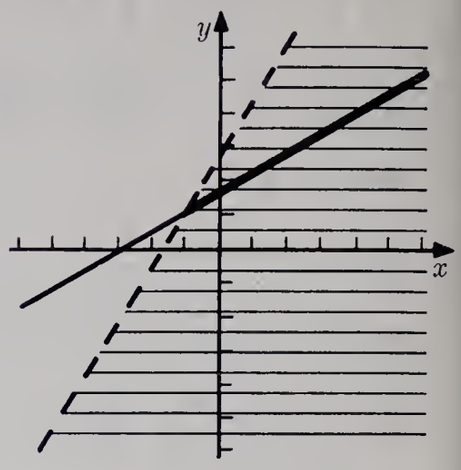


8.



Pages 532–535 · **PROBLEMS**

1. $l + s = 100$; $l = 3s + 4$; $3s + 4 + s = 100$; $4s = 96$; $s = 24$; $l = 76$, *Ans.*

2. Let $4x$ and $5x$ = the original nos.; $\dfrac{4x + 4}{5x - 4} = \dfrac{13}{14}$; $14(4x + 4) = 13(5x - 4)$;
$56x + 56 = 65x - 52$; $-9x = -108$; $x = 12$; $4x = 48$, $5x = 60$, *Ans.*

3. $c^2 = a^2 + b^2$; $a = x$; $b = x + 2$; $c = x + 4$; $(x + 4)^2 = x^2 + (x + 2)^2$;
$x^2 + 8x + 16 = x^2 + x^2 + 4x + 4$; $x^2 - 4x - 12 = 0$; $(x + 2)(x - 6) = 0$; $x + 2 = 0$ or $x - 6 = 0$; $x = -2$ (reject); $x = 6$; $a = 6$, $b = 8$, $c = 10$, *Ans.*

4. $\dfrac{1}{x} + \dfrac{1}{3x} = 2$; L.C.D. $= 3x$; $3 + 1 = 6x$; $6x = 4$; $x = \frac{2}{3}$, $3x = 2$, *Ans.*

5. Let x = smaller no.; $38 - x$ = larger no.;
$\dfrac{38 - x}{x} = 3 + \dfrac{2}{x}$; $38 - x = 3x + 2$; $-4x = -36$; $x = 9$, $38 - x = 29$, *Ans.*

6. $\dfrac{n + 7}{d + 7} = \dfrac{4}{5}$; $5n + 35 = 4d + 28$; $5n - 4d = -7$

$\dfrac{n - 7}{d - 7} = \dfrac{2}{3}$; $3n - 21 = 2d - 14$; $3n - 2d = 7$

$$\begin{array}{rr} 5n - 4d = & -7 \\ 6n - 4d = & 14 \\ \hline -n \qquad = & -21 \end{array}$$

$n = 21$

$3(21) - 2d = 7$; $-2d = -56$; $d = 28$; $\dfrac{n}{d} = \dfrac{21}{28}$, *Ans.*

7. Let x = Ethelynn's age; then $3x$ = Dan's age;
$3x + 4 = 2(x + 4)$; $3x + 4 = 2x + 8$; $x = 4$ yr, $3x = 12$ yr, *Ans.*

8. Let w = width of walk; then $30 - 2w$ = width of garden, and $36 - 2w$ = length of garden;
$(36 - 2w)(30 - 2w) = 720$; $1080 - 132w + 4w^2 = 720$; $4w^2 - 132w + 360 = 0$;
$w^2 - 33w + 90 = 0$; $(w - 30)(w - 3) = 0$; $w - 30 = 0$ or $w - 3 = 0$; $w = 30$ (reject); $w = 3$ ft, *Ans.*

9. Let x = size of second angle; $\dfrac{3x}{2}$ = size of third angle; $\dfrac{x}{3} - 7$ = size of smallest angle;

$x + \dfrac{3x}{2} + \dfrac{x}{3} - 7 = 180$; $\dfrac{6x + 9x + 2x}{6} = 187$; $17x = 6(187)$; $x = 6(11) = 66$;
$\dfrac{x}{3} - 7 = 22 - 7 = 15°$, *Ans.*

10. $(180 - x) = 3(90 - x) - 10$; $180 - x = 270 - 3x - 10$; $2x = 80$; $x = 40°$, *Ans.*

11. $\dfrac{10}{6} = \dfrac{h}{15}$; $6h = 150$; $h = 25$ ft, *Ans.*

12. $p_1 = 8 + 10 + 12 = 30$; $\dfrac{30}{p_2} = \dfrac{12}{54}$; $12p_2 = 1620$; $p_2 = 135$, *Ans.*

13. $\dfrac{6}{7} = \dfrac{x}{140}$; $7x = 6(140)$; $x = 120$ lb, *Ans.*

14. $\dfrac{245}{100} = \dfrac{96}{x}$; $245x = 9600$; $x \doteq 39.2$ grams, *Ans.*

15. $1 : 10 = \frac{15}{8} : l$; $l = \frac{150}{8} = 18\frac{3}{4}$; $l = 18$ ft 9 in, *Ans.*
$1 : 10 = \frac{11}{8} : w$; $w = \frac{110}{8} = 13\frac{3}{4}$; $w = 13$ ft 9 in, *Ans.*

16. $\frac{80}{100} = \frac{x}{60}$; $100x = 4800$; $x = 48$, *Ans.*

17. $\frac{5}{2} : \frac{15}{8} = 100 : x$; $\frac{5x}{2} = \frac{1500}{8}$; $5x = \frac{1500}{4}$; $x = 75$ ft, *Ans.*

18. Let r = rate of local; then $r + 18$ = rate of express;
$\frac{5}{2}r + \frac{5}{2}(r + 18) = 205$; $5r + 5r + 90 = 410$; $10r = 320$; $r = 32$ mph, $r + 18 = 50$ mph, *Ans.*

19. $\frac{d}{12} + \frac{d}{36} = 1$; L.C.D. $= 36$; $3d + d = 36$; $4d = 36$; $d = 9$ mi, *Ans.*

20. $\frac{d}{5+1} + \frac{d}{5-1} = 3$; $12\left[\frac{d}{6} + \frac{d}{4}\right] = 12\ [3]$; $2d + 3d = 36$; $5d = 36$; $d = 7\frac{1}{5}$ mi, *Ans.*

21. Let x = Carl's rate; $x + 3$ = Mark's rate;
$1(x) + 1(x + 3) = 15$; $2x = 12$; $x = 6$ mph, $x + 3 = 9$ mph, *Ans.*

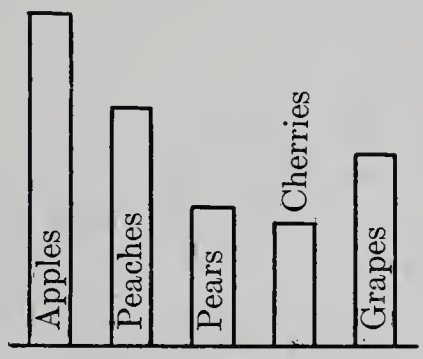
22. $.05x + .035y = 65$; $50x + 35y = 65{,}000$; $10x + 7y = 13{,}000$
$.035x + .05y = 71$; $35x + 50y = 71{,}000$; $7x + 10y = 14{,}200$

$$\begin{array}{r} 100x + 70y = 130{,}000 \\ 49x + 70y = 99{,}400 \\ \hline 51x \quad = 30{,}600 \end{array}$$

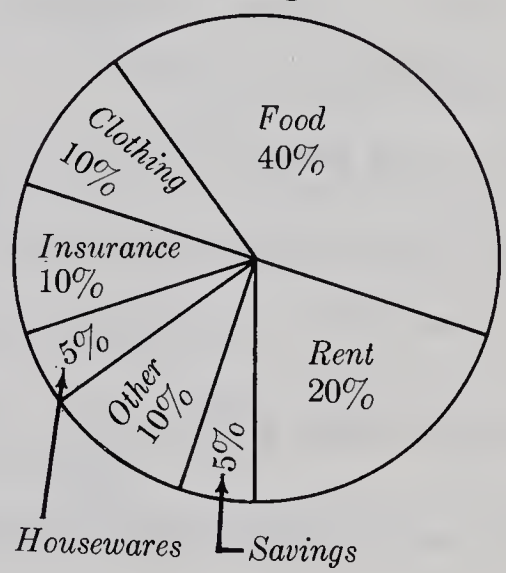
$x = \$600$, at 5%
$10(600) + 7y = 13{,}000$; $7y = 7000$; $y = \$1000$, at $3\frac{1}{2}\%$

23. **One Day's Fruit Sales**

Scale: $\frac{1}{8}$ in $= 4$ lb



24. **A Budget**



25. Let x = price per yd of silk; y = price per yd of cotton;

$$\begin{array}{r} 2x + 8y = 8.30 \\ 3x + \frac{16}{3}y = 7.85 \end{array}' \qquad \begin{array}{r} 4x + 16y = 16.60 \\ 9x + 16y = 23.55 \\ \hline -5x \quad = -6.95 \end{array}$$

$x = \$1.39$

$2(1.39) + 8y = 8.30$; $8y = 5.52$; $y = \$.69$. Silk, \$1.39/yd, cotton, \$.69/yd, *Ans.*

26. $.07x + .02(1000 - x) = 32$; $.07x + 20 - .02x = 32$; $.05x = 12$; $5x = 1200$;
$x = \$240$, $1000 - x = \$760$, *Ans.*

27. $x(44) + 10(80) = (10 + x)59$; $44x + 800 = 590 + 59x$; $15x = 210$; $x = 14$ lb, *Ans.*

28. Let x = no. of quarters; then $52 - x$ = no. of dimes;
$25x + 10(52 - x) = 1000$; $15x = 480$; $x = 32$; $52 - x = 20$. 32 quarters, 20 dimes, *Ans.*

29. $.8(12) = .6(12 + x)$; $9.6 = 7.2 + .6x$; $.6x = 2.4$; $x = 4$ milliliters, *Ans.*

30. $.05(85) + x = .15(85 + x)$; $4.25 + x = 12.75 + .15x$; $.85x = 8.50$; $x = 10$ pt, *Ans.*

31. $u = t - 6$; $10t + u = 9(t + u) - 1$; $t - 8u = -1$; $t - 8(t - 6) = -1$; $-7t = -49$; $t = 7$;
$u = 7 - 6 = 1$; $10t + u = 71$, *Ans.*

32. $x(\frac{1}{12}) + x(\frac{1}{18}) = 1$; $3x + 2x = 36$; $5x = 36$; $x = 7\frac{1}{5}$ hr *or* 7 hr 12 min, *Ans.*

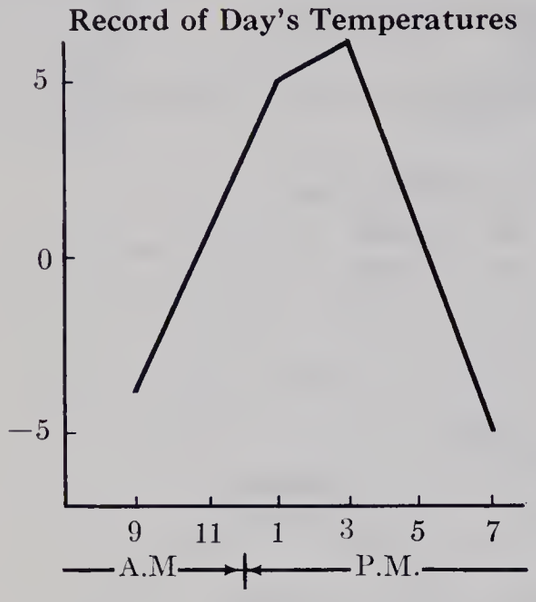
33. $\frac{96}{n} + 4 = \frac{96}{n-2}$; $96(n - 2) + 4n(n - 2) = 96n$; $96n - 192 + 4n^2 - 8n = 96n$;
$n^2 - 2n - 48 = 0$; $(n + 6)(n - 8) = 0$; $n + 6 = 0$ or $n - 8 = 0$; $n = -6$ (reject); $n = 8$, *Ans.*

34. $\frac{x}{4}+\frac{x+2}{6}+\frac{x+4}{8}=29$; $6x+4(x+2)+3(x+4)=24(29)$; $6x+4x+8+3x+12=696$; $13x=676$; $x=52$, $x+2=54$, $x+4=56$, *Ans.*

35. Let c = length of diagonal of base; d = length of diagonal of cube; $c^2=s^2+s^2$; $d^2=c^2+s^2$; $d^2=3s^2$; $d^2=3\cdot 4^2$; d $=\sqrt{3\cdot 4^2}=4\sqrt{3}\doteq 4(1.732)=6.928\doteq 6.9$ in, *Ans.*

36. $\frac{720}{n}+1=\frac{720}{n-10}$; $720(n-10)+n(n-10)=720n$; $n^2-10n-7200=0$; $(n+80)(n-90)=0$; $n+80=0$ or $n-90=0$; $n=-80$ (reject); $n=90$, *Ans.*

37.



38. $t=u-2$; $3t^2+t+u=10t+u$; $3t^2-9t=0$; $t^2-3t=0$; $t(t-3)=0$; $t=0$ or $t-3=0$; $t=0$ (reject) or $t=3$; $3=u-2$; $u=5$; $10t+u=35$, *Ans.*

39. $h=u$; $t=h+u+1$; $100h+10t+u=28(h+t+u)$; $72h-18t-27u=0$; $72h-18(h+u+1)-27u=0$; $54h-45u-18=0$; $6h-5u-2=0$; $6u-5u-2=0$; $u=2$; $h=2$; $t=5$; $100h+10t+u=252$, *Ans.*

40. Let x = no. of degrees in $\angle B$; $2x+20$ = no. of degrees in $\angle A$; $2x+20+x\le 179$; $3x\le 159$; $x\le 53$; $2x+20\le 126$. 126°, *Ans.*

41. $\frac{160}{10}\le l\le\frac{160}{8}$; 16 ft $\le l\le$ 20 ft, *Ans.*

42. Let x = no. of hr for apprentice to do job alone; $6\left(\frac{1}{x}\right)+6\left(\frac{1}{\frac{x}{2}}\right)=1$; $\frac{6}{x}+\frac{12}{x}=1$; $6+12=x$; $x=18$; $\frac{x}{2}=9$ hr; $6\left(\frac{1}{x}\right)+6\left(\frac{1}{\frac{x}{3}}\right)=1$; $\frac{6}{x}+\frac{18}{x}=1$; $6+18=x$; $x=24$; $\frac{x}{3}=8$ hr

It takes master more than 8 but at most 9 hr to do job alone.

Pages 535–536 · INDIRECT MEASUREMENT: VECTORS

1. .7547 **2.** 3.7321 **3.** .8572 **4.** 56° **5.** 50° **6.** 77° **7.** $\frac{5}{12}=.4\overline{167}$; 23°

8. $\sin A=\frac{a}{c}$; $A=52°$, $a=CB$, $c=AB=100$; $\sin 52°=\frac{a}{100}$; $.7880\doteq\frac{a}{100}$; $a\doteq 78.80$; $CB\doteq 78.8$, *Ans.*

9. $\cos A=\frac{b}{c}$; $A=27°$, $b=AC$, $c=AB=100$; $\cos 27°=\frac{b}{100}$; $.8910\doteq\frac{b}{100}$; $b\doteq 89.10$; $AC\doteq 89.1$, *Ans.*

10. $\tan A=\frac{a}{b}$; $a=CB=4$, $b=AC=5$; $\tan A=\frac{4}{5}=.8$; $A\doteq 39°$, *Ans.*

11. a. Let $AB=x+y$; $\tan 39°=\frac{10}{x}$; $.8098\doteq\frac{10}{x}$; $x\doteq\frac{10}{.8098}\doteq 12.3\doteq 12$ in; $\tan 73°=\frac{10}{y}$; $3.2709\doteq\frac{10}{y}$; $y\doteq\frac{10}{3.2709}\doteq 3.0\doteq 3$ in; $AB=15$ in, *Ans.*

b. $A=\frac{1}{2}bh$; $A=\frac{1}{2}(15)(10)=75$ sq in, *Ans.*

c. $\sin 39°=\frac{10}{AC}$; $.6293\doteq\frac{10}{AC}$; $AC\doteq\frac{10}{.6293}\doteq 15.8$ in; $\sin 73°=\frac{10}{BC}$; $.9563\doteq\frac{10}{BC}$; $BC\doteq\frac{10}{.9563}\doteq 10.4$ in; $P=AB+AC+BC\doteq 15.3+15.8+10.4\doteq 42$ in, *Ans.*

12. $\tan x = \frac{750}{4300} \doteq .1744$; $x \doteq 10°$, *Ans.*

13. $\sin 52° = \frac{h}{609}$; $.7880 \doteq \frac{h}{609}$; $h \doteq 479.8 \doteq 480$ ft, *Ans.*

14. $c = 24$ ft, $b = 8$ ft; $\cos A = \frac{b}{c} = \frac{8}{24}$; $\cos A \doteq .3333$; $A \doteq 71°$, *Ans.*

15. $\sin 52° = \frac{20}{c}$; $.7880 \doteq \frac{20}{c}$; $c \doteq \frac{20}{.7880} \doteq 25.3 \doteq 25$ in, *Ans.*

16. $\tan 8° = \frac{120}{b}$; $.1405 \doteq \frac{120}{b}$; $b \doteq \frac{120}{.1405} \doteq 854.0 \doteq 854$ ft, *Ans.*

17. $r^2 = (150)^2 + (180)^2 = 54{,}900$; $r = \sqrt{54{,}900} = \sqrt{10^2 \cdot 3^2 \cdot 61} = 30\sqrt{61} \doteq 30(7.81) = 234.3 \doteq 234$ lb; $\tan x° = \frac{150}{180} = \frac{5}{6} \doteq .8333$; $x \doteq 40°$, *Ans.*

18. $t^2 + (3t)^2 = (50)^2$; $10t^2 = 2500$; $t^2 = 250$; $t = \sqrt{250} = 5\sqrt{10} \doteq 5(3.162) = 15.81 \doteq 16$ lb; $3t \doteq 48$ lb, *Ans.*

19. $\sin 4° = \frac{w}{370}$; $.0698 \doteq \frac{w}{370}$; $w \doteq 25.8 \doteq 26$ knots, *Ans.*

20. $r^2 = 5^2 + 12^2 = 169$; $r = 13$ mph; $\cos x° = \frac{5}{13} \doteq .3846$; $x \doteq 67°$; $90° + 67° = 157°$. 157° SE at 13 mph, *Ans.*

Pages 536–537 · TRUE–FALSE TEST

1. T. $(x-1)^2 = (x-1)(x-1)$; $x^2 - 1 = (x+1)(x-1)$

2. T. $m\left(\frac{x-y}{m}\right) = x - y$; $m\left(\frac{x}{m} - \frac{y}{m}\right) = m\left(\frac{x}{m}\right) - m\left(\frac{y}{m}\right) = x - y$

3. T. $\frac{a}{m-n} \cdot \frac{-1}{-1} = \frac{-a}{-m+n} = \frac{-a}{n-m}$

4. F. $a^{3m} \div a^{3m} = a^{3m-3m} = a^0$ *or* 1

5. F. $\frac{2x-y}{2m} = \frac{2x}{2m} - \frac{y}{2m} = \frac{x}{m} - \frac{y}{2m}$; $\frac{x-y}{m} = \frac{x}{m} - \frac{y}{m}$

6. F. $(a+b)^2 = a^2 + 2ab + b^2$

7. F. $2^m \cdot 2^n = 2^{m+n}$

8. F. $\frac{a^3}{a^3} = 1$

9. T, by defn.

10. F, it may have no real roots.

11. F. $3(4) - (-6) = 12 + 6 = 18 \neq 6$

12. F. $2x + 2y = 6$ reduces to $x + y = 3$; graphs are parallel lines

13. T. $3(0) = 2(0)$; $2(0) = 3(0)$

14. T. $(2x)(2x) = 4(x \cdot x)$

15. F, the perimeter is doubled; $3(2s) = 2(3s)$

16. F, the circumference becomes 4 times as long; $2\pi(4r) = 4(2\pi r)$

17. T. For $(90 - x)$ to increase, x must decrease; as x decreases, $(180 - x)$ increases

18. T. $\frac{n}{2n} = \frac{3h}{6h}$; $\frac{1}{2} = \frac{3}{6}$

19. F. $30m = 6$; $m = \frac{6}{30} = .2$

20. T. $m^2 = (2\sqrt{3})(3\sqrt{3}) = 18$; $m = \sqrt{18} = 3\sqrt{2}$

21. F, the set of rational fractions is everywhere dense.

22. T

23. T, the sum of any two sides of a triangle must exceed the third side.

24. T. $(180 - x) - (90 - x) = 180 - x - 90 + x = 90$

25. T. $(90 - x) + (90 - y) = 180 - x - y =$ third angle

26. F. $ay = -x + b$, $y = -\frac{1}{a}x + \frac{b}{a}$; slope $= -\frac{1}{a}$

27. T. $y = mx + b_1$ and $y = mx + b_2$ are parallel if $b_1 \neq b_2$ and coincident if $b_1 = b_2$.

28. T. $y = mx + b$ and $m = 2$

29. F, total distance $= 2AB$; total time $= \frac{AB}{45} + \frac{AB}{30}$;

$$\text{average rate} = \frac{2AB}{\frac{AB}{45} + \frac{AB}{30}} = \frac{2AB}{\frac{5AB}{90}} = \frac{2AB(90)}{5AB} = \frac{180AB}{5AB} = 36 \text{ mph}$$

30. T, since $\angle A + \angle B = 90°$, $\angle C = 90°$ **31.** T. $\dfrac{\frac{1}{a}}{\frac{1}{a}} = \dfrac{1(a)}{\frac{1}{a}(a)} = \dfrac{a}{1} = a$

32. F. $y = 3x + 1$ would give $(1, 4)$, $(2, 7)$, and $\frac{1}{4} \neq \frac{2}{7}$; true only if $b = 0$

33. T. $(x - 3)(x - 1) = 0$; $x - 3 = 0$ or $x - 1 = 0$; $x = 3$ or $x = 1$

34. F, it is an equation whose solution set is $\emptyset$. **35.** T

36. F. $1 + 1 = 2$, not in set **37.** F, $\{0, 1, 2, 3, \ldots, 40\}$

38. F, an irrational no. is expressed by a non-terminating, non-repeating decimal.

39. F, the distrib. prop. is not needed.

40. T, for any point (x, y) the distances from the two axes are the x and y coordinates.

Pages 538–539 · COMPLETION TEST

1. $p = 2(l + w)$; $36 = 2(6 + w)$; $w + 6 = 18$; $w = 12$. 12, *Ans.*

2. Let a = any number, $-1 = \dfrac{-a}{a}$; $-a + a = 0$, *Ans.*

3. $w = ksd^2$ **4.** $\dfrac{H_1}{H_2} = \dfrac{d_2^2}{d_1^2}$ **5.** $m - 1$, $m + 1$

6. $2(2) - 3y = 7$; $-3y = 3$; $y = -1$. -1, *Ans.*

7. $2 + 3 + 5 = 10$; $2 : 10 = x : 25$; $10x = 50$; $x = 5$ lb, *Ans.*

8. $k = 5 - 6 = -1$, *Ans.* **9.** $|8| = 8$

10. $s^2 = 28$; $s = \sqrt{28} = 2\sqrt{7} \doteq 2(2.646) = 5.292 \doteq 5.3$ in

11. $8(m + 7) = 12(3 - m)$; $8m + 56 = 36 - 12m$; $20m = -20$; $m = -1$, *Ans.*

12. $(2\sqrt{5} - 3)(2\sqrt{5} + 3) = (2\sqrt{5})^2 - 9$; $4 \cdot 5 - 9 = 11$, *Ans.*

13. $\dfrac{3(82) + x}{4} = 85$; $246 + x = 340$; $x = 94\%$, *Ans.*

14. $x\sqrt{3} = 6\sqrt{48}$; $x = \dfrac{6\sqrt{48}}{\sqrt{3}} = 6\sqrt{16}$; $x = 6 \cdot 4 = 24$, *Ans.*

15. Total distance $= 20 + rh$; total time $= 4 + h$; average rate $= \dfrac{20 + rh}{4 + h}$, *Ans.*

16. decreases **17.** increases **18.** $xy = 48$; $12y = 48$; $y = 4$, *Ans.*

19. $\dfrac{16}{20} = \dfrac{x}{60}$; $20x = 16(60)$; $x = 48$ days, *Ans.*

20. Let c = diag. of rectangle $\doteq$ diam. of table; $c^2 = 4^2 + 7^2$; $c^2 = 65$; $c = \sqrt{65} \doteq 8.062$; $c \doteq 8$ ft, *Ans.*

21. The y-intercept is 0; $b = 0$, *Ans.* **22.** $a(2) = 3(0) - 4$; $2a = -4$; $a = -2$, *Ans.*

23. $\dfrac{4x^2}{4x^2} + \dfrac{y^2}{4x^2} = 1 + \dfrac{y^2}{4x^2} = 2$; $\dfrac{y^2}{4x^2} = 1$; $\dfrac{y^2}{x^2} = 4$; $\dfrac{y}{x} = 2$, *Ans.*

24. $(4s)^2 + s^2 = (200)^2$; $17s^2 = 40{,}000$; $s^2 = \dfrac{40{,}000}{17}$; $c^2 = s^2 + s^2$; $c^2 = 2s^2 = \dfrac{2(40{,}000)}{17}$;

$c = \sqrt{\dfrac{40{,}000 \cdot 2}{17} \cdot \dfrac{17}{17}} = \frac{200}{17}\sqrt{34} \doteq \frac{200}{17}(5.831) = \dfrac{1166.2}{17} = 68.6$ cm, *Ans.*

25. $-r \leq -2$; $r \geq 2$. $\{2, 3\}$

26. $(c - d)^2 = c^2 - 2cd + d^2 = 49$; $c^2 - 2(5) + d^2 = 49$; $c^2 + d^2 = 59$, *Ans.*

27. $\dfrac{r}{4} = \dfrac{s}{7} = \dfrac{t}{15}$, $15r = 4t$, $15s = 7t$; $2r + s = 45$, $s = 45 - 2r$; $15(45 - 2r) = 7t$, $675 - 30r = 7t$, $675 - 2(15r) = 7t$; $675 - 2(4t) = 7t$, $15t = 675$, $t = 45$, *Ans.*

28. $(h + k)^2 = h^2 + 2hk + k^2 = 20$; $h^2 + 2(1) + k^2 = 20$; $h^2 + k^2 = 18$;
$(h - k)^2 = h^2 - 2hk + k^2 = (h^2 + k^2) - 2hk = 18 - 2(1) = 16$; $h - k = \sqrt{16} = 4$, *Ans.*

29. $\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2} = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$, *Ans.*

30. $\frac{1}{4} = \frac{2}{k}$; $k = 8$ *or* $m_1 = -\frac{1}{2}$, $m_2 = -\frac{4}{k}$, $-\frac{1}{2} = -\frac{4}{k}$; $k = 8$, *Ans.*

31. $\frac{1}{2}(-5) = -\frac{5}{2}$; $(-\frac{5}{2})^2 = \frac{25}{4}$, *Ans.*

32. $x^2 - 9 = (x + 3)(x - 3)$; $(x - 3)$, *Ans.*

33. $a = 0$ and $b \neq 0$

34.
$$\begin{aligned} 100N &= 63.63\overline{63} \\ N &= \quad .63\overline{63} \\ \hline 99N &= 63 \\ N &= \tfrac{63}{99} = \tfrac{7}{11}, \textit{ Ans.} \end{aligned}$$

35. function

36. $A_1 = \pi r_1^2$; $A_2 = \pi(2r_1)^2$; $A_2 = 4\pi r_1^2 = 4A_1$; 4, *Ans.*

37. $y = 2x - 1$

38. $(AC)^2 + 1^2 = 2^2$; $(AC)^2 = 3$; $(AC) = \sqrt{3}$; $AC : BC = \sqrt{3} : 1$, *Ans.*

39. $\frac{x-1}{2} > 4$ and $\frac{x-1}{2} < 6$

$x - 1 > 8$ and $x - 1 < 12$

$x > 9$ and $x < 13$ $\{10, 11, 12\}$, *Ans.*

40. $x^2 = 3$; $x = \sqrt{3}$; $x^3 = (\sqrt{3})^3 = 3\sqrt{3}$, *Ans.*

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